Gravitational waves and the stochastic background

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Prof. M. C. Valsakumar Memorial Lecture

Academy of Physics Teachers, Kerala

Cochin University of Science and Technology, Kochi

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Plan of the talk

1. Newtonian gravity

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2. Electromagnetism and special theory of relativity

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6. Observations by PTAs and the stochastic GW background
7. Outlook

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Newton's second law and Galilean relativity

Recall that, the Newton's second law governing the motion of a particle under the influence of a force $\boldsymbol{F}$ is given by

$$m\,\boldsymbol{a} = \boldsymbol{F},$$

where m is the mass of the particle and $\boldsymbol{a} = \mathrm{d}^2\boldsymbol{x}/\mathrm{d}t^2$ is its acceleration.

In Newtonian mechanics, time is absolute. The time and the spatial coordinates of, say, a particle as observed by two inertial frames can be related as

$$t = t', \qquad \boldsymbol{x} = \boldsymbol{x}' + \boldsymbol{V}\,t',$$

where $\boldsymbol{V}$ is the relative velocity between the two frames. This relation leads to the Galilean addition of velocities[1].

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[1] See, for instance, J. R. Taylor, *Classical Mechanics* (University Science Books, Mill Valley, California, 2004).

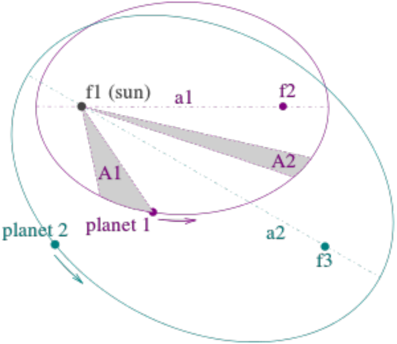
Newton’s inverse square law of gravitation

As is well known, the gravitational force $\boldsymbol{F}$ due to a massive body of mass M on, say, a test particle of mass m, which is located at a distance r is given by the following inverse square law due to Newton:

$$\boldsymbol{F} = -\frac{G\,M\,m}{r^2}\,\hat{\boldsymbol{r}},$$

where G is the gravitational constant and $\hat{\boldsymbol{r}}$ is the unit radial vector pointing from the massive object towards the test particle.

Kepler's laws of planetary motion[2]



1. The orbit of a planet is an ellipse with the Sun at one of the two foci.
2. A line segment joining the Sun and a planet sweeps out equal areas during equal intervals of time.
3. The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

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[2] Image from https://en.wikipedia.org/wiki/Kepler's_laws_of_planetary_motion.

Poisson equation for gravitation

In electrostatics, the potential V satisfies the following Poisson equation:

$$\boldsymbol{\nabla}^2 V = -\frac{\rho}{\epsilon_0},$$

where ρ is the charge density and ϵ_0 is the permittivity of free space.

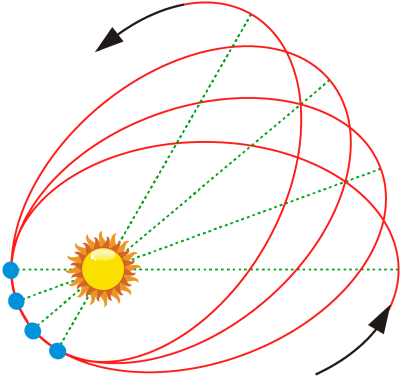
Since the Coulomb's law and the inverse square of gravitation have the same form, it should be clear that the gravitational potential, say, ϕ, will satisfy a similar Poisson equation given by

$$\boldsymbol{\nabla}^2 \phi = 4\,\pi\, G\, \rho,$$

where ρ now denotes the mass density that generates the gravitational field.

Note that the above equation for the gravitational potential ϕ implies that the force propagates *instantaneously*.

Precession of the perihelion on Mercury



The perihelion of Mercury has been observed to precess by about $5600''$ per century. Of this, the precession of $5557''$ per century can be explained within Newtonian gravity, when one takes into account the effects due to the other planets and the fact that the Sun is not a perfect sphere. But, at the turn of the twentieth century, there had remained the discrepancy of about $43''$ per century[3].

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[3] See, J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Pearson Education, Delhi, 2003).

Plan of the talk

Electromagnetism and Maxwell's equations

The Maxwell's equations govern the dynamics of electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$, and are given by

$$\begin{aligned}
\boldsymbol{\nabla}\cdot\boldsymbol{E} &= \frac{\rho}{\epsilon_0},\\
\boldsymbol{\nabla}\times\boldsymbol{E} &= -\frac{\partial\boldsymbol{B}}{\partial t},\\
\boldsymbol{\nabla}\cdot\boldsymbol{B} &= 0,\\
\boldsymbol{\nabla}\times\boldsymbol{B} &= \mu_0\,\boldsymbol{j}+\frac{1}{\mu_0\,\epsilon_0}\,\frac{\partial\boldsymbol{E}}{\partial t},
\end{aligned}$$

where ρ and $\boldsymbol{j}$ denote the charge and the current densities, while ϵ_0 and μ_0 represent the permittivity and permeability of free space[4].

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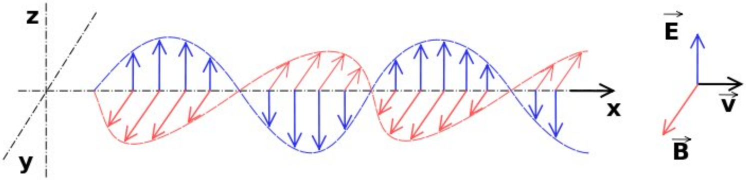
[4] See, for instance, D. J. Griffiths, *Introduction to Electrodynamics*, Third Edition (Prentice Hall of India, Delhi, 2002).

Electromagnetic waves



In the regions of space where there is no charge or current, the electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$ satisfy the following wave equation:

$$\Box\, f = \frac{1}{c^2}\,\frac{\partial^2 f}{\partial t^2} - \boldsymbol{\nabla}^2 f = 0,$$

where $c = 1/\sqrt{\epsilon_0\,\mu_0}$ denotes the speed of light in the vacuum.

Electromagnetic waves travel at the speed of light. They are transverse in nature (*i.e.* they oscillate in a plane perpendicular to their direction of propagation), with their polarization being determined by the behavior of the electric field[5].

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[5] D. J. Griffiths, *Introduction to Electrodynamics*, Third Edition, Prentice Hall of India, Delhi, 2002).

Sources of electromagnetic waves

It can be shown that, due to charge conservation, an electric monopole does not radiate. For example, a charged sphere, with a total charge Q, which oscillates radially in and out does not radiate, since, according to the Gauss's law, the field outside the sphere is always $Q\,\hat{\boldsymbol{r}}/(4\,\pi\,\epsilon_0\,r^2)$.

The most dominant type of radiation is due to a charge distribution with a time-dependent electric dipole moment. A simple source with such a property is an accelerating charge[6].

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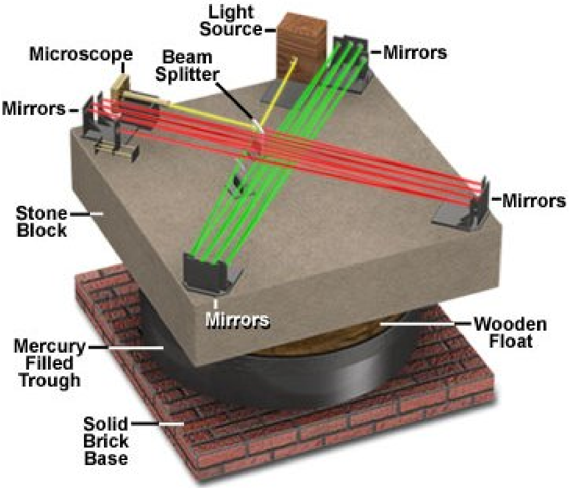
[6] D. J. Griffiths, *Introduction to Electrodynamics*, Third Edition, Prentice Hall of India, Delhi, 2002).

Light, ether, and the Michelson-Morley interferometer



The Michelson-Morley interferometer consists of two arms of equal length along which light is made to propagate before interfering with each other[7].

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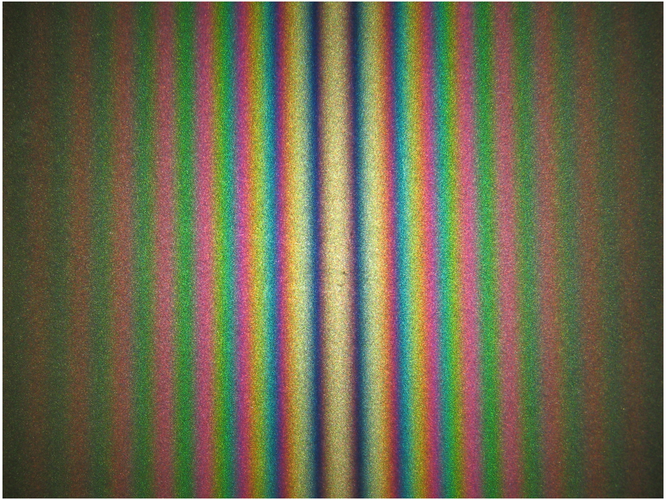
[7] Image from http://olympus.magnet.fsu.edu/primer/lightandcolor/speedoflight.html.

Fringes in the Michelson-Morley interferometer



The interference fringes in a Michelson-Morley interferometer[8]. Any difference in the two path lengths will be reflected in the shift of the fringes.

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[8] Image from https://en.wikipedia.org/wiki/Michelson%E2%80%93Morley_experiment.

Postulates of the special theory of relativity

The special theory of relativity is essentially based on the following two postulates[9]:

1. The laws of physics take the same form in every inertial frame.
2. The speed of light is the same in all inertial frames.

The first of these postulates is essentially the relativistic principle originally due to Galileo.

The second, evidently, is in conflict with the Galilean addition of velocities.

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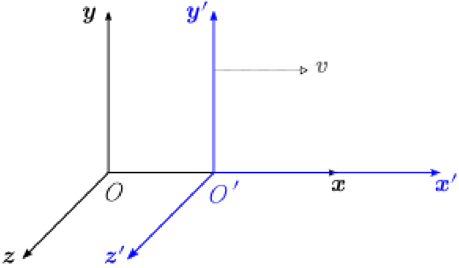
[9] See, for instance, A. P. French, *Special Relativity* (W. W. Norton, New York, 1968).

Lorentz transformations and consequences of special relativity



The time and the spatial coordinates of the above two inertial frames are related by the following Lorentz transformations:

$$t = \gamma\,(t' + v\,x'/c^2), \quad x = \gamma\,(x' + v\,t'), \quad y = y', \quad z = z',$$

where $\gamma = 1/\sqrt{1-(v^2/c^2)}$ and c denotes the velocity of light.

The consequences of special relativity include relativity of simultaneity, Lorentz contraction, and time dilation[10].

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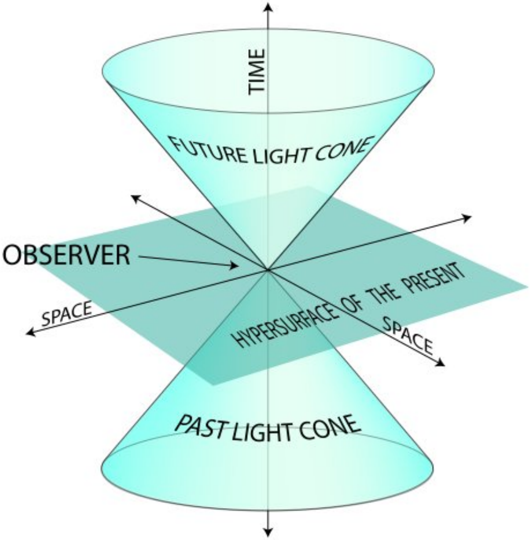
[10] A. P. French, *Special Relativity* (W. W. Norton, New York, 1968).

Concept of spacetime in special relativity



Spacetime is viewed as a single entity in special relativity.

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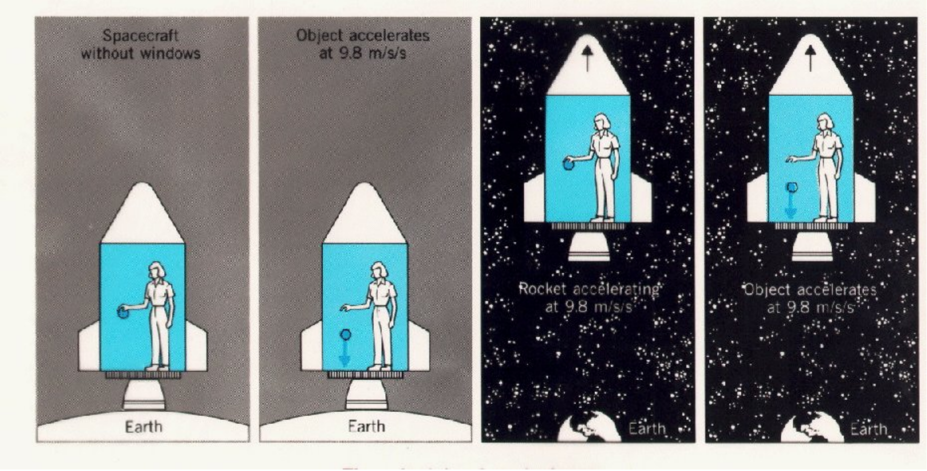
The equivalence principle



An inertial frame in a uniform gravitational field is completely equivalent to a uniformly accelerated frame[11]. Note that the coordinates of the uniformly accelerated frame are related to those of the inertial frame by a non-linear coordinate transformation.

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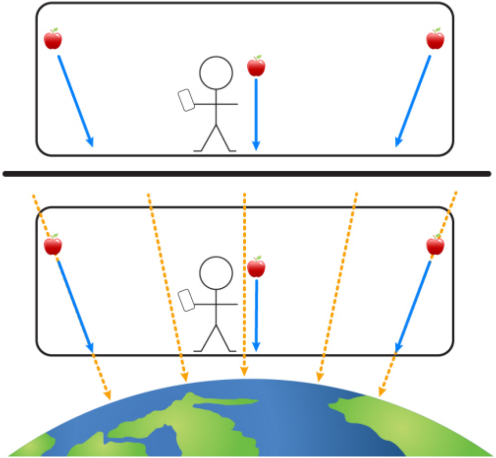
[11] Image from http://hendrix2.uoregon.edu/~imamura/FPS/week-6/week-6.html.

Tidal effects in a gravitational field



While a gravitational field is equivalent to a uniformly accelerating frame ‘locally’, it is not so ‘globally’[12]. The tidal effects make the paths of the falling apples, referred to as geodesics, converge, due to the curvature of Earth.

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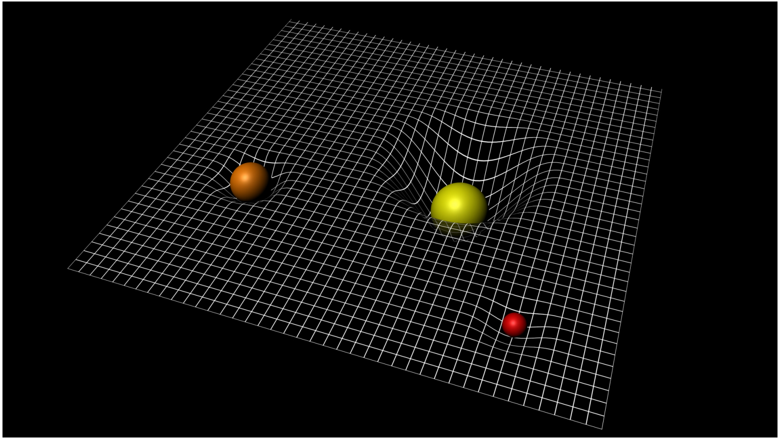
[12] Image from https://writescience.wordpress.com/tag/albert-einstein/.

The Einstein's equations[14]



Matter tells spacetime how to curve[13]. The curvature of spacetime (as described by the Einstein tensor $G_{\mu\nu}$) is related to the matter content (as represented by the stress-energy tensor $T_{\mu\nu}$) through the Einstein's equations, *viz.* $G_{\mu\nu} = (8\,\pi\,G/c^4)\,T_{\mu\nu}$.

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[13] J. A. Wheeler, *Geons, Black Holes, and Quantum Foam: A Life in Physics* (W. W. Norton, New York, 2010).

[14] Image from http://www.esa.int/spaceinimages/Images/2015/09/Spacetime_curvature.

Precession of the perihelion of Mercury

When the general relativistic effects are taken into account, it can be shown that the perihelion of Mercury indeed precesses exactly by the extent of the discrepancy with the Newtonian theory, *viz.* by $43''$ per century[15].

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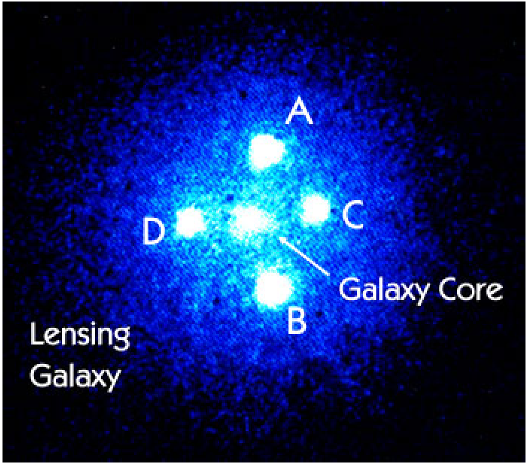
[15] J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Pearson Education, Delhi, 2003).

Gravitational bending of light



According to general relativity, gravity bends light, a phenomenon that is regularly observed by astronomers. The image above is that of the Einstein cross, which is a quasar that is gravitationally lensed quasar by a foreground galaxy[16].

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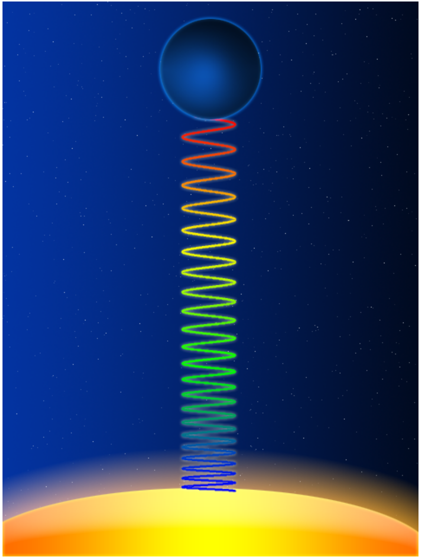
[16] Image from http://www.skyhound.com/observing/archives/sep/Q2237+0305A.html.

Gravitational redshift[17]



In general relativity, a photon loses energy as it climbs out of a gravitational field. This phenomenon has been experimentally confirmed in the laboratory.

An equivalent phenomenon corresponds to clocks running slow in a gravitational field. This phenomenon and the relativistic time dilation has to accounted for in the Global Positioning System (GPS), if it is to meet the accuracy (of 2 m) desired for military applications.

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[17] J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Pearson Education, Delhi, 2003).

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Transverse nature of GWs

The GWs are small disturbances in a given spacetime (very much like ripples in water), which travel at the speed of light. They satisfy the wave equation in the given background.

The GWs are transverse in nature and are characterized by two degrees of polarization[18].

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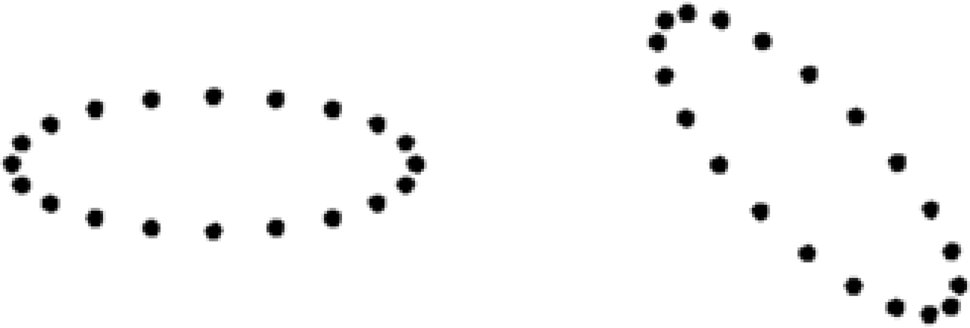
[18] J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Pearson Education, Delhi, 2003).

Polarization of GWs



A GW impinging on a ring of masses leads to oscillations of the particles depending on the polarization of the wave: plus (on the left) and cross (on the right)[19].

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[19] J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Pearson Education, Delhi, 2003).

Quadrupole radiation

Recall that, in the case of electromagnetism, a varying monopole moment is not possible due to conservation of charge. As a result, it is the time-dependent dipole moment of a charge distribution that leads to the dominant contribution to radiation from the system.

In the context of gravitation, conservation of mass forbids monopole radiation, while conservation of energy and momentum rules out dipole radiation. Therefore, the leading contribution to the gravitational radiation from a source arises due to a varying quadrupole moment[20].

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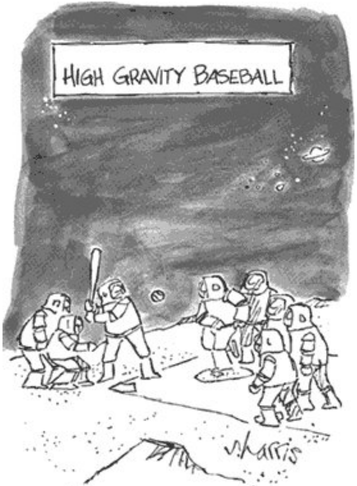
[20] J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Pearson Education, Delhi, 2003).

Sources of GWs[21]

In order to generate GWs of detectable amplitude, the gravitational fields of the sources ought to be very strong.



Strong sources of GWs include

- Rotating neutron stars
- Exploding supernovae
- Coalescing binary neutron stars or black holes
- Supermassive binary black holes at the centre of galaxies
- Quantum fluctuations in the early universe

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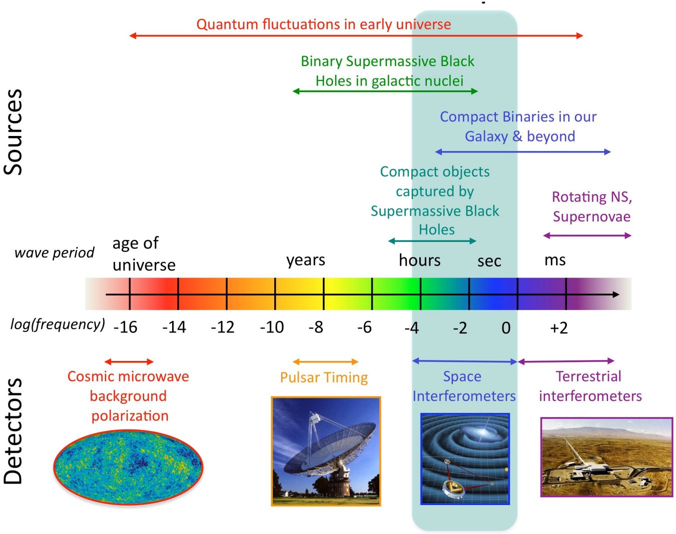
[21] Cartoon from http://www.sciencecartoonsplus.com/gallery/physics/galphys2b.php.

The spectrum of GWs



Sources

Quantum fluctuations in early universe

Binary Supermassive Black Holes in galactic nuclei

Compact Binaries in our Galaxy & beyond

Compact objects captured by Supermassive Black Holes

Rotating NS, Supernovae

wave period age of universe years hours sec ms

log(frequency) -16 -14 -12 -10 -8 -6 -4 -2 0 +2

Detectors

Cosmic microwave background polarization

Pulsar Timing

Space Interferometers

Terrestrial interferometers

Different sources of GWs and corresponding detectors[22].

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[22] J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Pearson Education, Delhi, 2003).

Plan of the talk

Laser Interferometer Gravitational-Wave Observatory (LIGO)

Views of LIGO at Hanford (on the left) and at Livingston (on the right). These observatories are essentially Michelson-Morley interferometers with rather long arms (of length about 4 km) that are extremely sensitive to the smallest disturbances of the mirrors[23].

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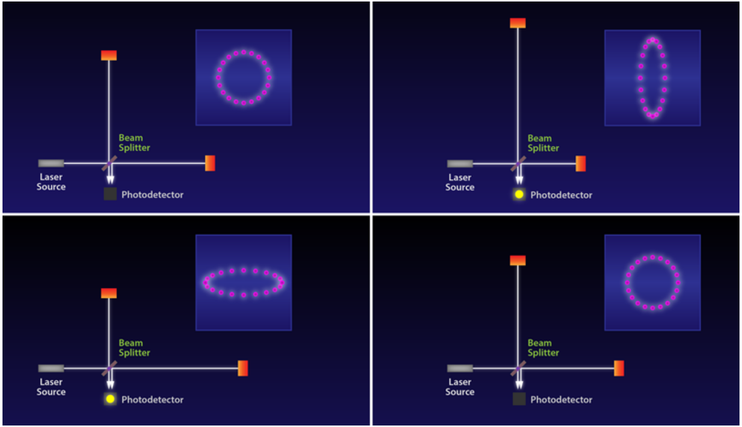
[23] Images from https://www.advancedligo.mit.edu/summary.html.

Response of LIGO to an incoming GW



LIGO is designed so that, in the absence of GWs (top left), light takes the same time to travel back and forth along the two arms and interferes destructively at the photodetector, producing no signal. As the wave passes (moving clockwise from top right) the travel times for the lasers change, and a signal appears in the photodetector[24].

▸ Play LIGO interferometer movie

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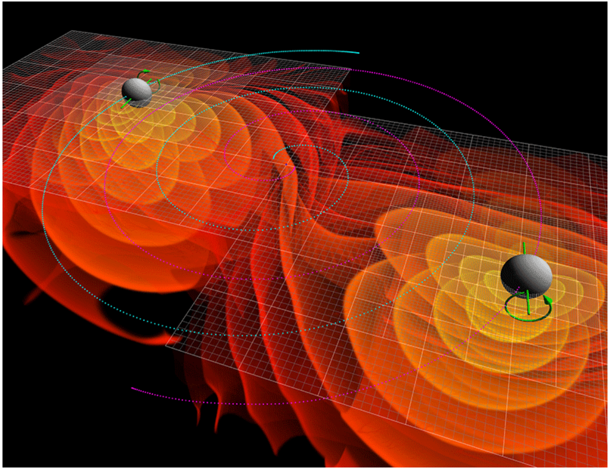
[24] Image from E. Berti, Physics **9**, 17 (2016).

GWs from inspiralling black holes



Numerical simulations of the GWs emitted by the coalescence of two black holes. The orange contours represent the amplitude of the GWs and the blue lines represent the orbits of the black holes[25].

▸ Play black hole coalescence movie

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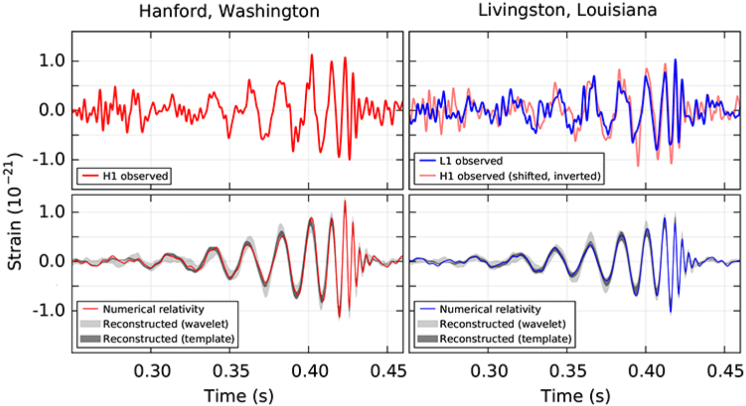
[25] Image from E. Berti, Physics **9**, 17 (2016).

A binary black hole merger I: Signals at the two detectors



On September 14, 2015, similar signals were observed in both of LIGO's interferometers. The top panels show the measured signal in the Hanford (top left) and Livingston (top right) detectors. The bottom panels show the expected signal produced by the merger of two black holes, based on numerical simulations[26].

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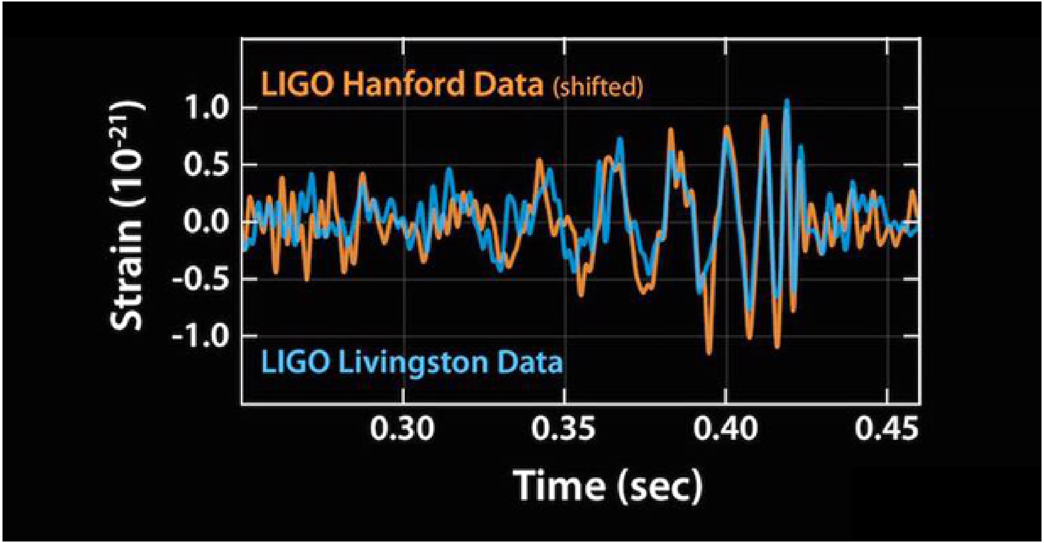
[26] Figure from B. P. Abbott *et al.*, Phys. Rev. Lett. **116**, 061102 (2016).

A binary black hole merger II: Superimposing the signals



The signals from the two LIGO's interferometers superimposed[27].

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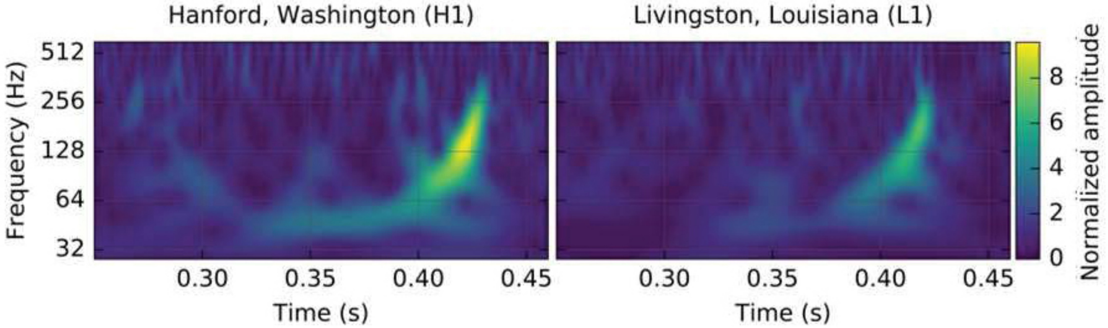
[27] Figure from B. P. Abbott *et al.*, Phys. Rev. Lett. **116**, 061102 (2016).

A binary black hole merger III: The chirp



The behavior of the frequency of the observed signal[28]. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}.

▸ Play the chirping of black holes movie

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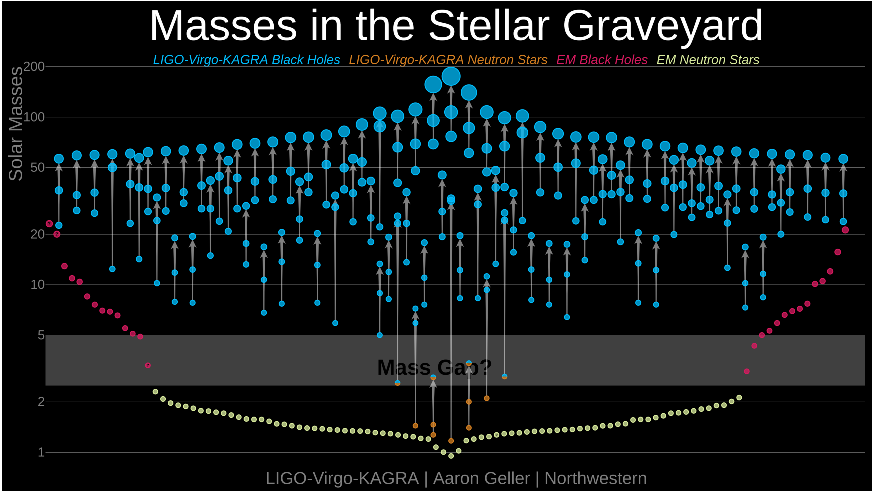
[28] Figure from B. P. Abbott *et al.*, Phys. Rev. Lett. **116**, 061102 (2016).

Coalescence of compact binaries observed by LIGO



Catalog of mergers involving black holes and neutron stars released by the LIGO-Virgo-KAGRA Collaboration[29].

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[29] Image from https://media.ligo.northwestern.edu.

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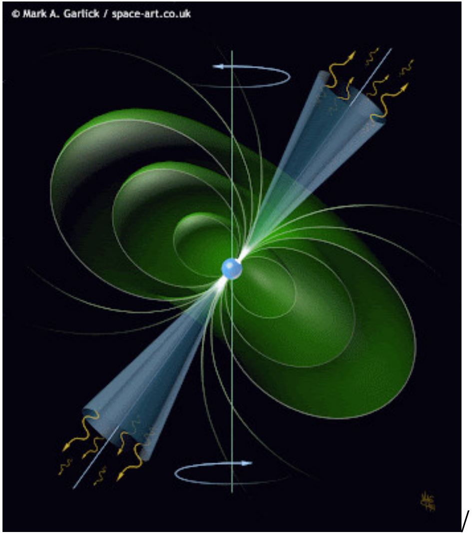
Pulsars



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Pulsars are dense and rotating neutron stars that emit regular beams of light[30].

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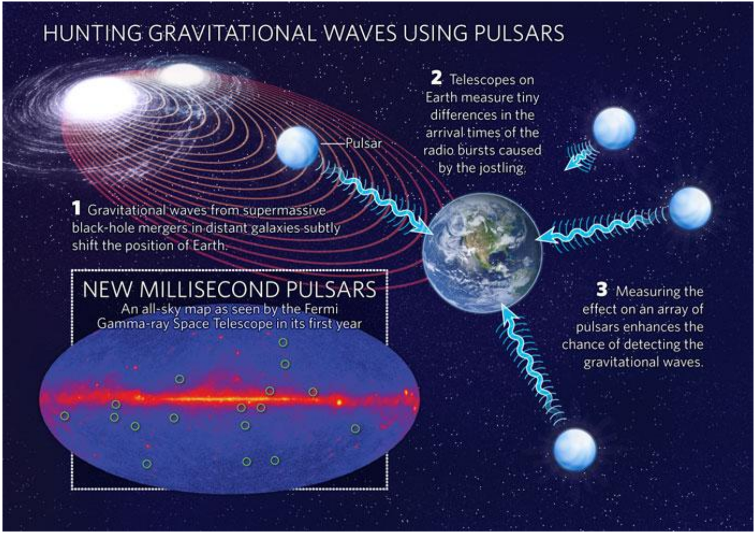
[30] Image from https://dmr-astronomersclub.blogspot.com/2012/07/what-is-pulsar.html.

Pulsar timing arrays (PTAs)



The PTAs monitor an array of millisecond pulsars[31].

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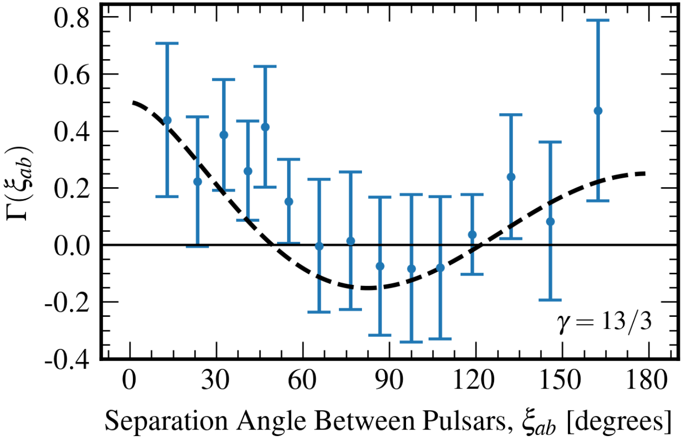
[31] See https://ipta.github.io/mock_data_challenge/.

Hellings-Downs curve



The inter-pulsar correlations measured from 2,211 distinct pairings in the 67-pulsar array of the 15-year NANOGrav data set. The dashed black line shows the Hellings-Downs correlation pattern[32].

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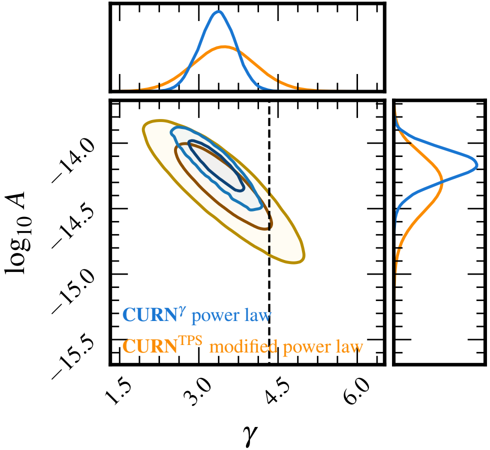
[32] NANOGrav Collaboration (G. Agazie *et al.*) Astrophys. J. Lett. **951**, 1 (2023).

Constraints on the spectral amplitude and index of GWs



Constraints on the amplitude A and the index γ of the stochastic background of GWs from the 15-year NANOGrav data[33].

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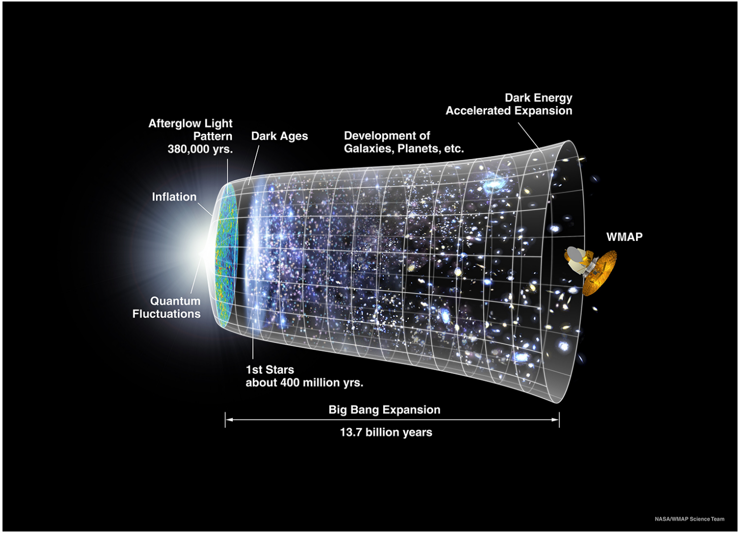
[33] NANOGrav Collaboration (G. Agazie *et al.*) Astrophys. J. Lett. **951**, 1 (2023).

The timeline of the universe



A pictorial timeline of the universe[34].

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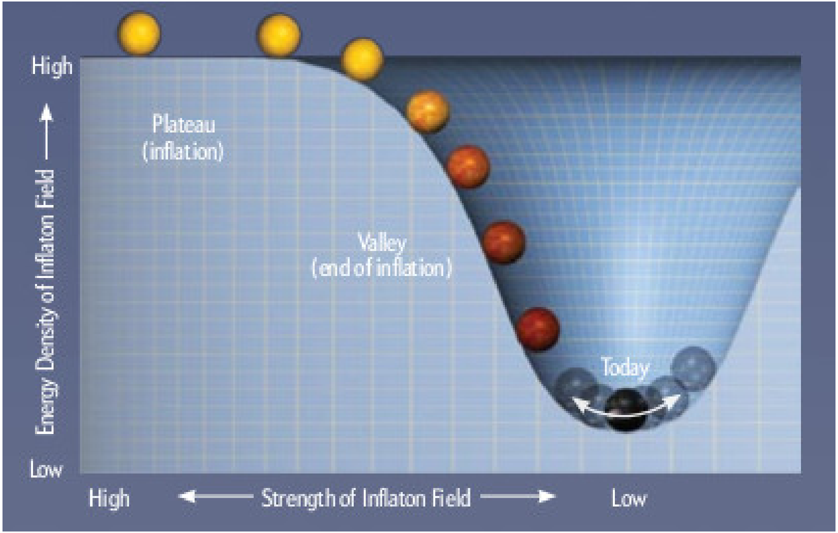
[34] See http://wmap.gsfc.nasa.gov/media/060915/060915_CMB_Timeline150.jpg.

Driving inflation with scalar fields



Inflation can be achieved with scalar fields encountered in high energy physics[35].

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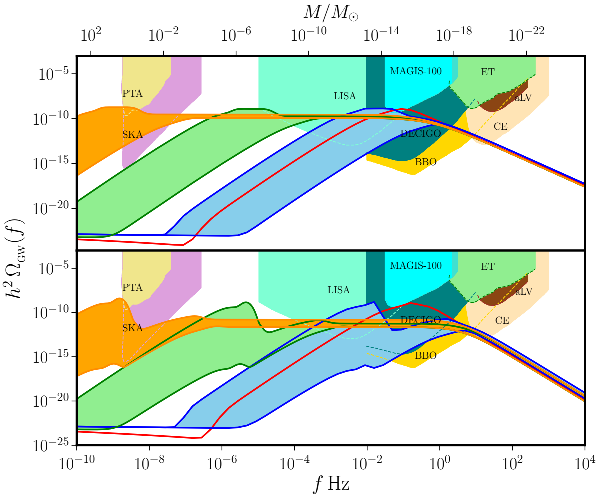
[35] Image from P. J. Steinhardt, Sci. Am. **304**, 34 (2011).

Secondary GWs induced by scalar perturbations in USR inflation



The dimensionless density parameter $\Omega_{\rm GW}(f)$ of secondary GWs arising in scenarios leading to ultra slow roll (USR) inflation[36].

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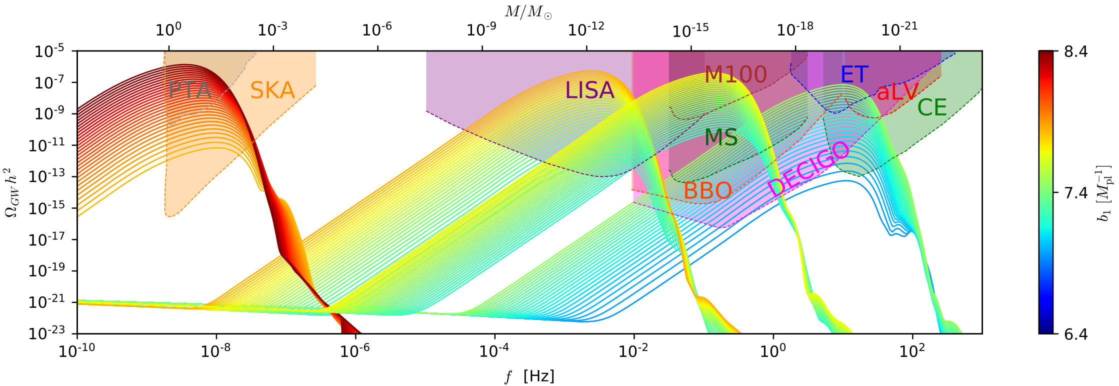
[36] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

Secondary GWs induced by scalar perturbations in two field models



The dimensionless density parameter $\Omega_{\rm GW}(f)$ of secondary GWs arising in a certain two field model has been plotted as a function of frequency for a set of initial conditions for the background fields as well as a range of values of a parameter involved[37].

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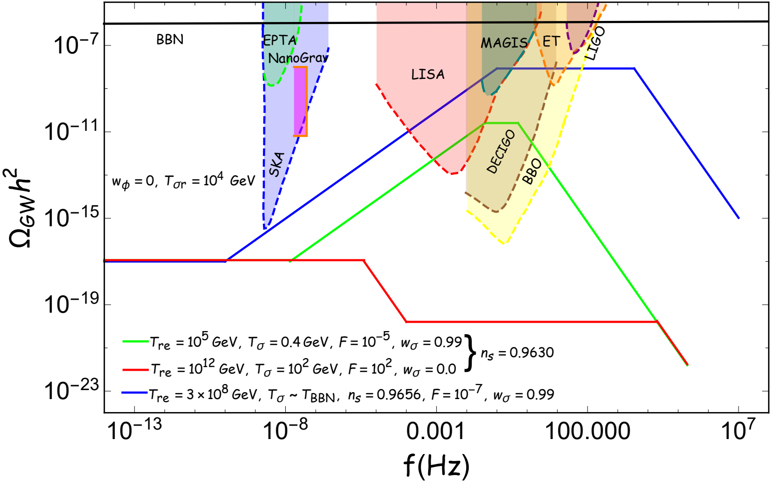
[37] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

Effects on primary GWs due to late time entropy production



The dimensionless spectral energy density of primary GWs observed today $\Omega_{\rm GW}(f)$ has been plotted in a scenario involving late time production of entropy[38].

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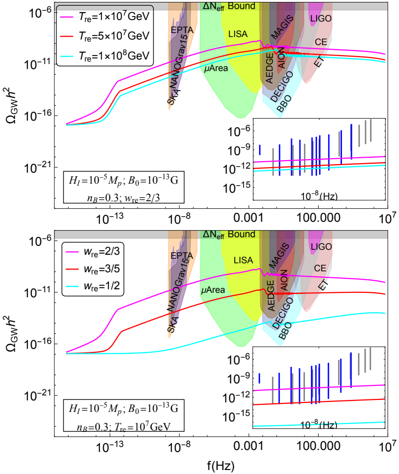
[38] Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, Phys. Rev. D **104**, 063513 (2021).

Secondary GWs induced by primordial magnetic fields (PMFs)



The dimensionless spectral energy density of secondary GWs observed today $\Omega_{\rm GW}(f)$, induced by the PMFs, have been plotted for different reheating temperatures (on top) and different equation-of-state parameters during reheating (at the bottom)[39].

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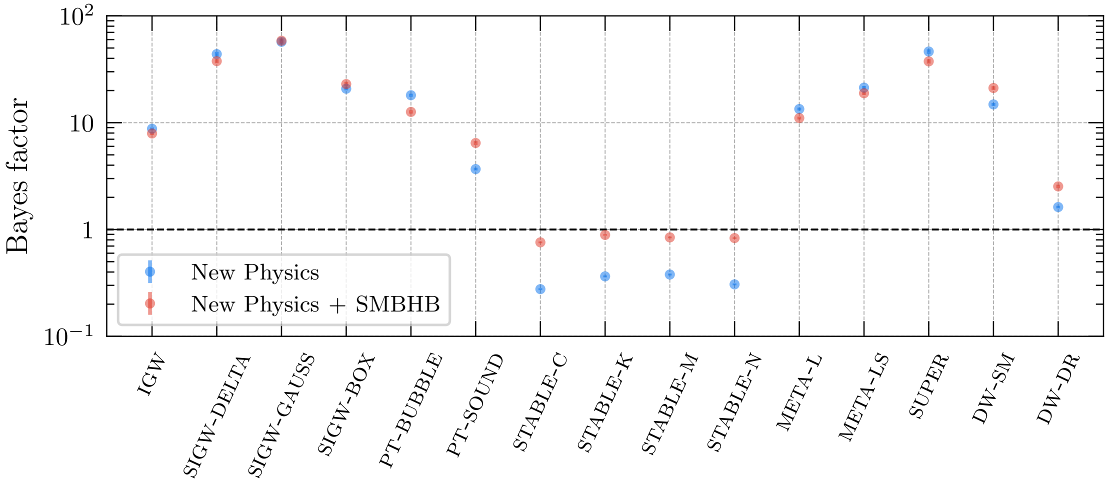
[39] S. Maiti, D. Maity and L. Sriramkumar, arXiv:2401.01864 [astro-ph.CO].

Stochastic GW background observed by pulsar timing arrays (PTAs)



The Bayesian evidence for a variety of astrophysical and cosmological sources for the stochastic GW background suggested by the observations of the PTAs [40].

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[40] NANOGrav Collaboration, Astrophys. J. Lett. **951**, L11 (2023).

Plan of the talk

1. Newtonian gravity
2. Electromagnetism and special theory of relativity
3. General theory of relativity
4. Gravitational waves (GWs)
5. Observations by LIGO
6. Observations by PTAs and the stochastic GW background
7. Outlook

Outlook

- The observations by LIGO are a culmination of almost fifty years of global effort to detect GWs. They are but the first step in a long process, and they have opened up a new window to observe the universe.
- The observations by the PTAs and its implications for the stochastic GW background provide a wonderful opportunity to understand the physics operating in the early universe.
- Observations of GWs along with observations of electromagnetic waves and, say, neutrinos, are expected to be complementary in nature. Such a *multi-messenger astronomy* can lead to a deeper understanding of the physics of the universe.

Thank you for your attention