Gravitational waves from the early universe

L. Sriramkumar

Centre for Strings, Gravitation and Cosmology, Department of Physics,
Indian Institute of Technology Madras, Chennai

From Big Bang to Now: A Theory-Experiment Dialogue

Department of Physics, SRM University, Andhra Pradesh

January 23–25, 2025

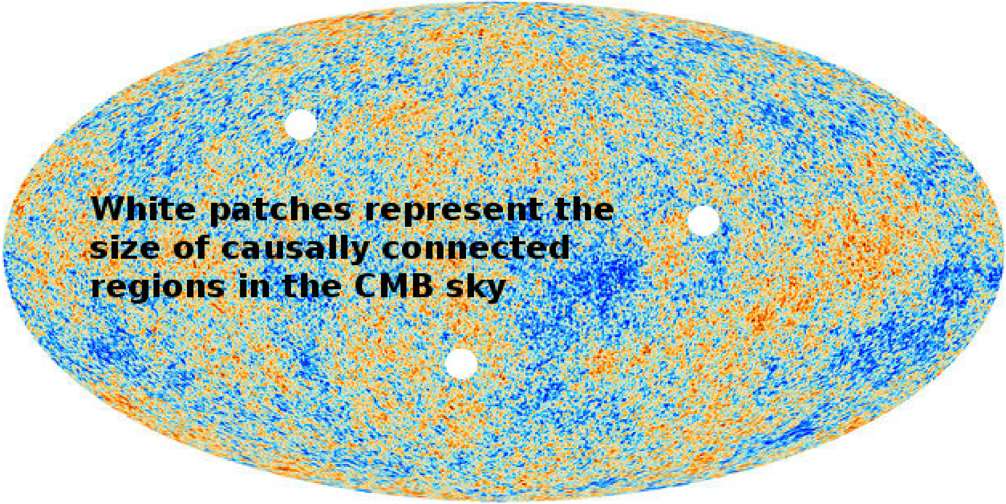
Plan of the talk

This talk is based on. . .

- M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, *Generating PBHs and small-scale GWs in two-field models of inflation*, JCAP **08**, 001 (2020) [arXiv:2005.02895 [astro-ph.CO]].
- H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, *Primordial black holes and secondary gravitational waves from ultra slow roll and punctuated inflation*, Phys. Rev. D **103**, 083510 (2021) [arXiv:2008.12202 [astro-ph.CO]].
- Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, *Decoding the phases of early and late time reheating through imprints on primordial gravitational waves*, Phys. Rev. D **104**, 063513 (2021) [arXiv:2105.09242 [astro-ph.CO]].
- H. V. Ragavendra and L. Sriramkumar, *Observational imprints of enhanced scalar power on small scales in ultra slow roll inflation and associated non-Gaussianities*, Galaxies **11**, 34 (2023) [arXiv:2301.08887 [astro-ph.CO]].
- S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, *Constraining the history of reheating with the NANOGrav 15-year data*, arXiv:2403.16963 [astro-ph.CO], accepted for publication in JCAP.

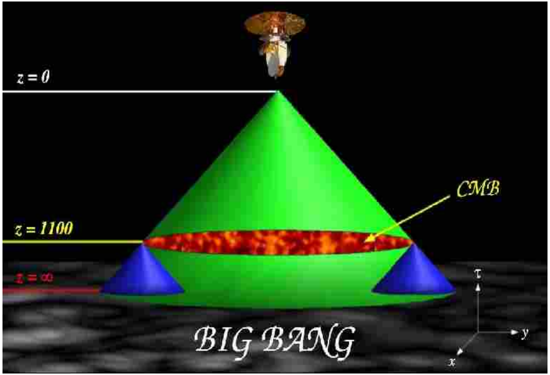
Plan of the talk

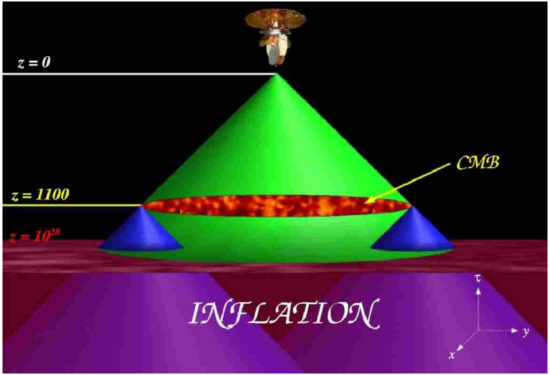
The horizon problem



The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the surface of last scattering, which subtends an angle of about 1° today, could not have interacted before decoupling.

Resolution of the horizon problem in the inflationary scenario

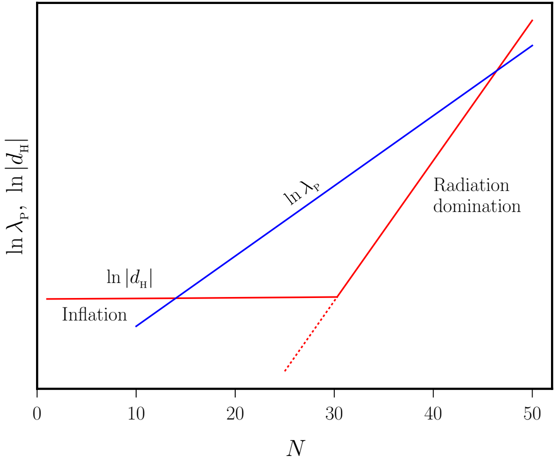




Another illustration of the horizon problem (on the left), and an illustration of its resolution (on the right) through an early and sufficiently long epoch of inflation[1].

[1] Images from W. Kinney, astro-ph/0301448.

Bringing the modes inside the Hubble radius

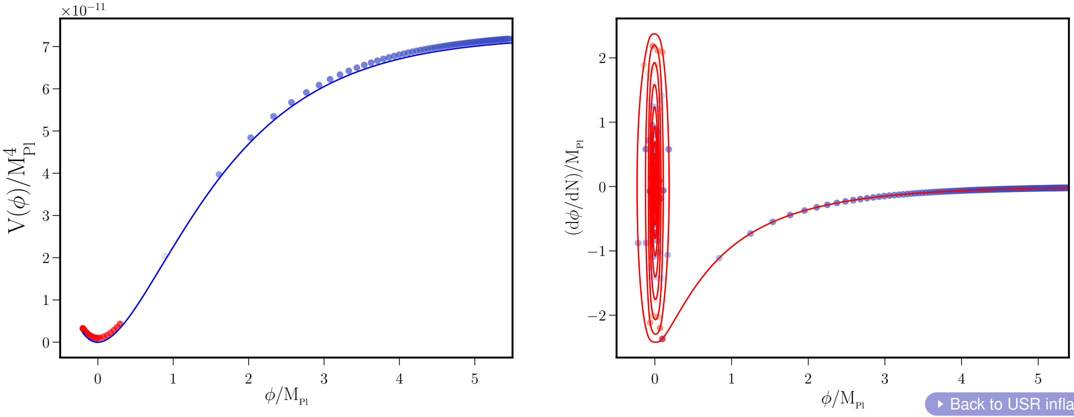


▸ Evolution of comoving lengths

The physical wavelength $\lambda_{\rm P} \propto a$ (in blue) and the Hubble radius $d_{\rm H} = H^{-1}$ (in red) in the inflationary scenario[2]. The scale factor is expressed in terms of e-folds N as $a(N) \propto {\rm e}^N$.

[2] See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.

The inflationary attractor

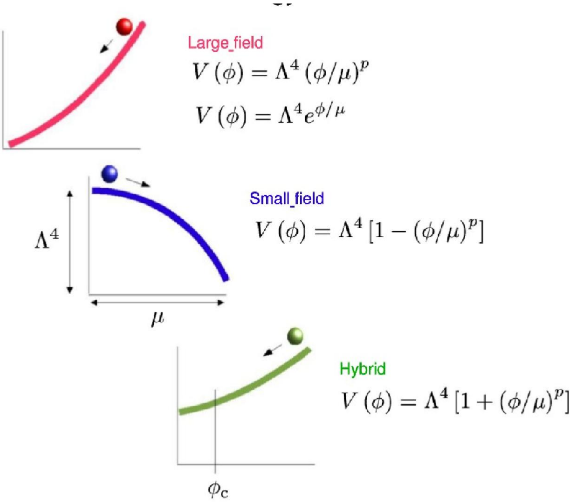


Back to USR inflation

Evolution of the scalar field in the popular Starobinsky model, which leads to slow roll inflation, is indicated (as circles, in blue and red) at regular intervals of time (on the left). Illustration of the behavior of the scalar field in phase space (on the right)[3].

[3]Figure from H. V. Ragavendra, *Observational imprints of non-trivial inflationary dynamics over large and small, scales*, Ph.D. Thesis, Indian Institute of Technology Madras, Chennai, India (2022).

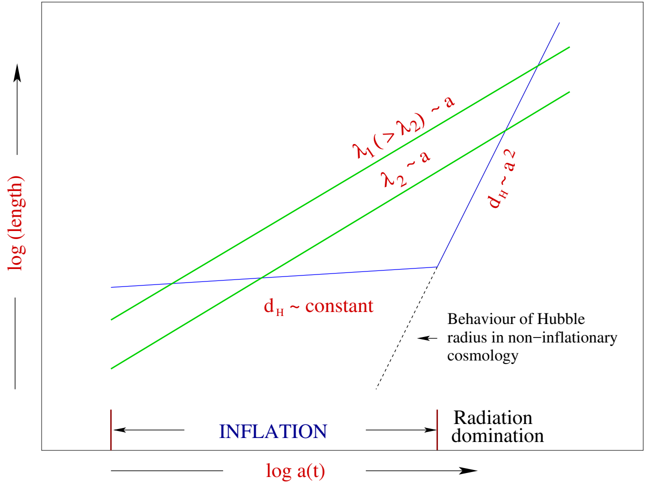
A variety of potentials to choose from



A variety of scalar field potentials have been considered to drive inflation[4]. Often, these potentials are classified as small field, large field and hybrid models.

[4] Image from W. Kinney, astro-ph/0301448.

From inside the Hubble radius to super-Hubble scales



The initial conditions are imposed in the sub-Hubble regime when the Fourier modes of the perturbations are well inside the Hubble radius (*viz.* when $k/(a\,H) \gg 1$) and the power spectra are evaluated when they sufficiently outside (*i.e.* as $k/(a\,H) \ll 1$).

Spectral indices and the tensor-to-scalar ratio

While comparing with the observations, for convenience, one often uses the following power law, template scalar and the tensor spectra:

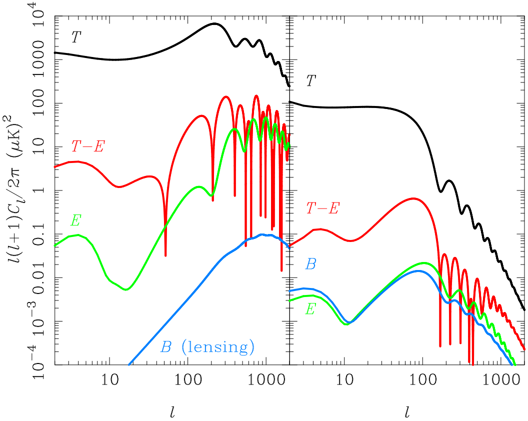
$$\mathcal{P}_{\mathrm{S}}(k) = A_{\mathrm{S}} \left(\frac{k}{k_*}\right)^{n_{\mathrm{S}}-1}, \quad \mathcal{P}_{\mathrm{T}}(k) = A_{\mathrm{T}} \left(\frac{k}{k_*}\right)^{n_{\mathrm{T}}},$$

with the spectral indices n_{S} and n_{T} assumed to be constant. The tensor-to-scalar ratio r is defined as

$$r(k) = \frac{\mathcal{P}_{\mathrm{T}}(k)}{\mathcal{P}_{\mathrm{S}}(k)}.$$

Plan of the talk

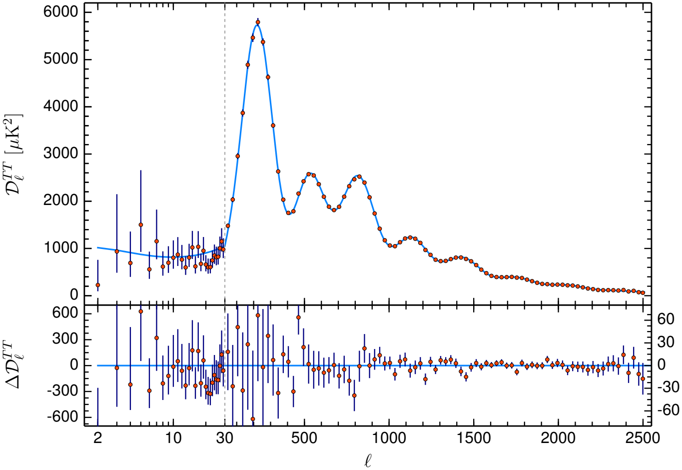
Theoretical angular power spectra[5]



The *theoretically* computed, angular power and cross-correlation spectra of the CMB arising due to scalars (on the left) and tensors (on the right) corresponding to a tensor-to-scalar ratio of $r = 0.24$. The B-mode spectrum induced by weak gravitational lensing has also been shown (in blue) in the panel on the left.

[5]Figure from, A. Challinor, arXiv:1210.6008 [astro-ph.CO].

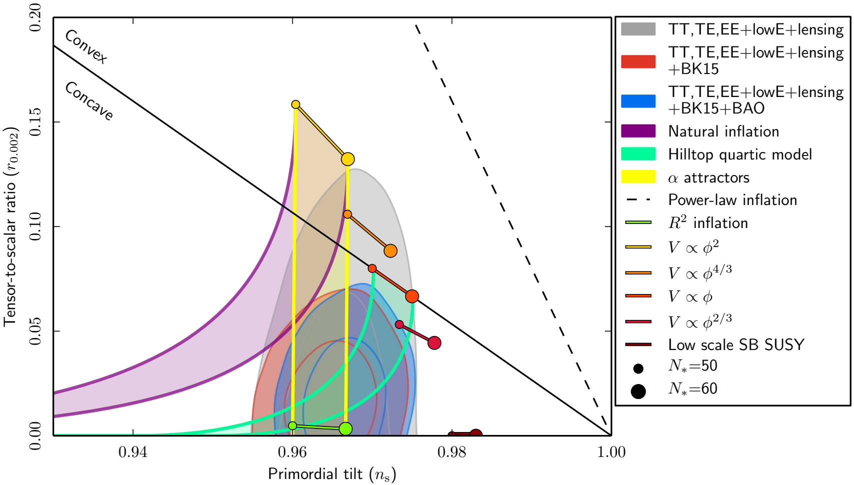
CMB angular power spectrum from Planck



The CMB TT angular power spectrum from the Planck 2018 data (red dots with error bars) and the best fit ΛCDM model with a power law primordial spectrum (solid blue curve)[6].

[6] Planck Collaboration (N. Aghanim *et al.*), Astron. Astrophys. **641**, A6 (2020).

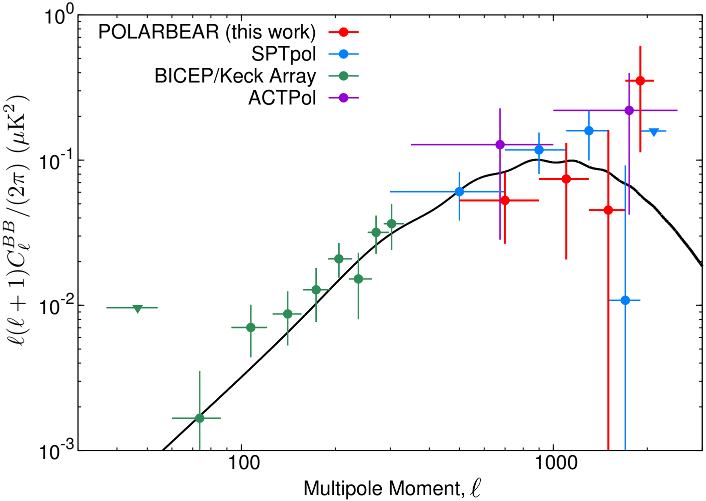
Performance of inflationary models in the n_s-r plane



Joint constraints on n_s and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of some of the popular inflationary models[7].

[7] Planck Collaboration (Y. Akrami *et al.*), Astron. Astrophys. **641**, A10 (2020).

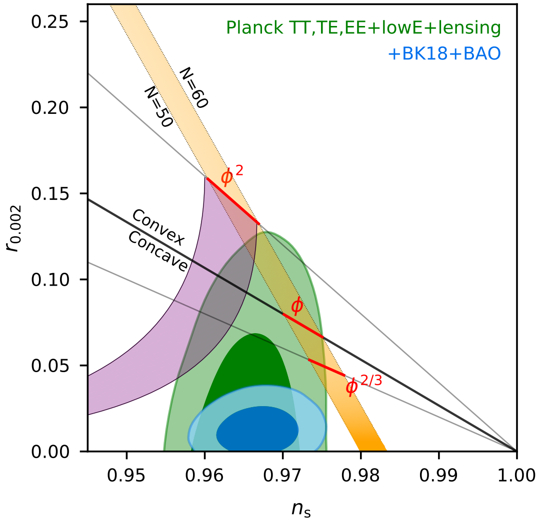
Constraints on the B-mode polarization of the CMB



Constraints on the B-mode, angular power spectrum of the CMB from two years of POLARBEAR data[8].

[8] POLARBEAR Collaboration (P. A. R. Ade *et al.*), Ap. J. **848**, 141, (2017).

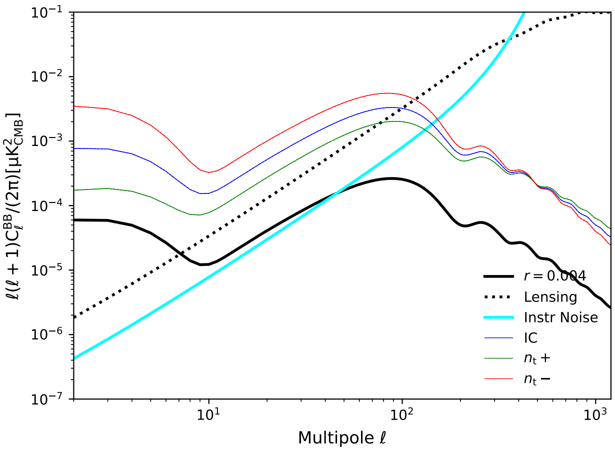
Latest constraints on the tensor-to-scalar ratio r



Latest constraints on the tensor-to-scalar ratio r from the BICEP/Keck telescopes[9].

[9] BICEP/Keck Collaboration (P. A. R. Ade *et al.*), arXiv:2203.16556 [astro-ph.CO].

Prospects of observing the imprints of the tensor perturbations

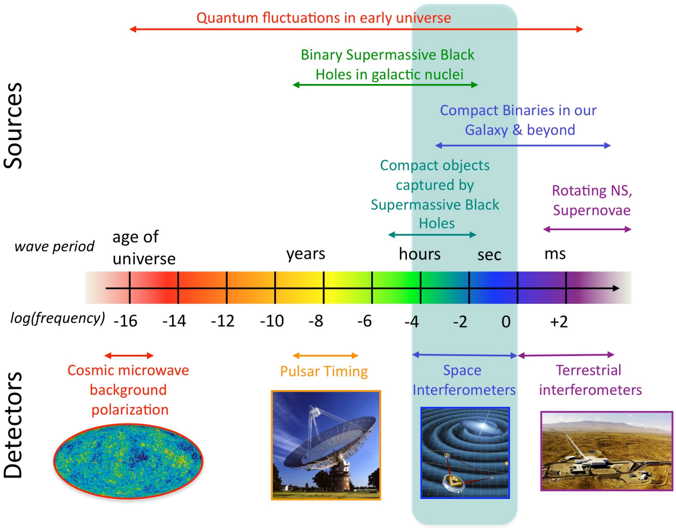


The B-mode angular power spectra of the CMB resulting from the primordial tensor perturbations for three models with $r_{0.05} = 0.05$ have been plotted, along with the CMB lensing signal and the instrumental noise of a LiteBIRD-like configuration[10].

[10] D. Paoletti, F. Finelli, J. Valiviita and M. Hazumi, Phys. Rev. D **106**, 083528 (2022).

Plan of the talk

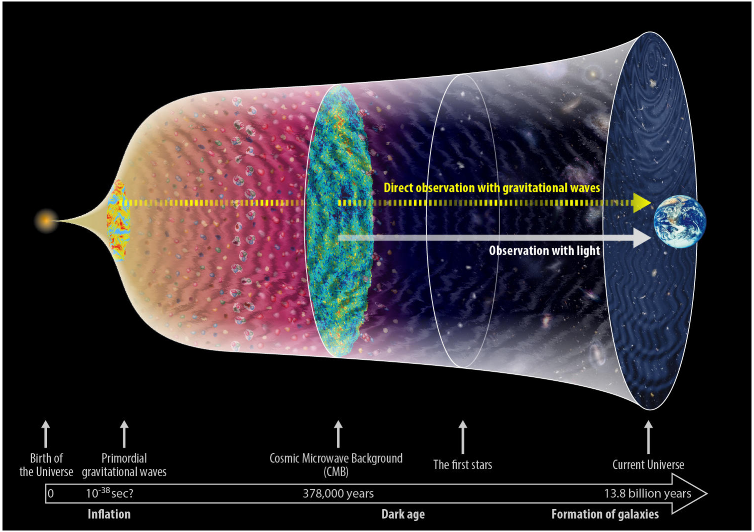
The spectrum of GWs



Different sources of GWs and corresponding detectors[11].

[11] J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Pearson Education, Delhi, 2003).

Probing the primordial universe through GWs

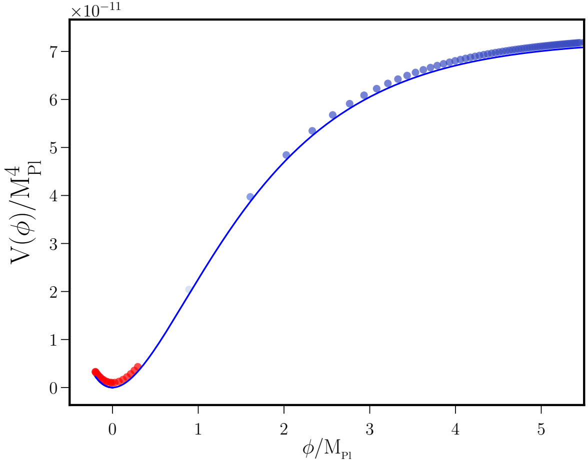


GWs provide a unique window to probe the primordial universe[12].

[12] Image from https://gwpo.nao.ac.jp/en/gallery/000061.html.

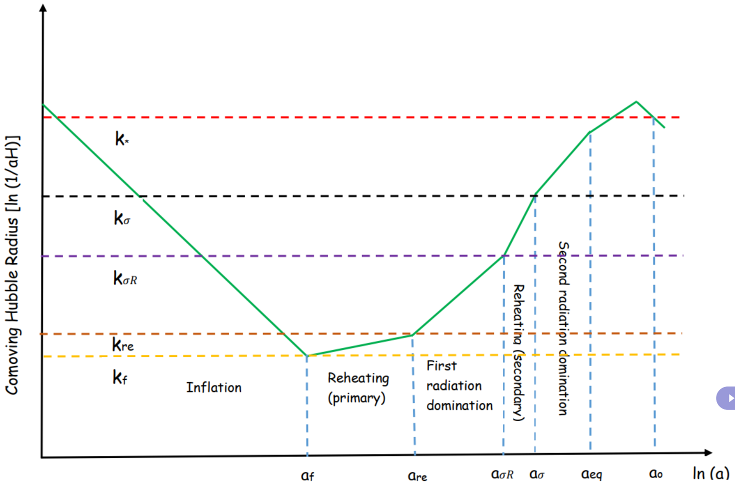
Plan of the talk

Evolution of the scalar field in an inflationary potential



The evolution of the scalar field in the so-called Starobinsky model has been indicated (as circles, in blue and red) at regular intervals of time. Inflation is terminated as the field approaches the bottom of the potential (near the light blue dot). Thereafter, the field oscillates at the bottom of the potential (indicated by the red dots).

Behavior of the comoving wave number and Hubble radius



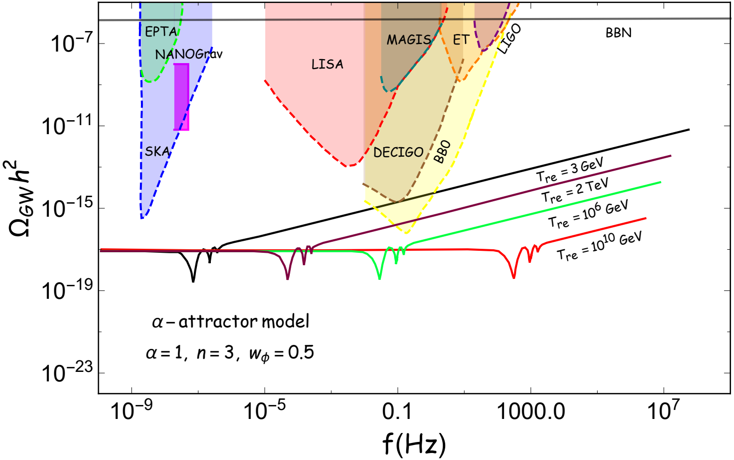
Evolution of physical lengths

Back to secondary GWs

Behavior of the comoving wave number k (horizontal lines in different colors) and the comoving Hubble radius $d_{\mathrm{H}}/a = (a\,H)^{-1}$ (in green) across different epochs[13].

[13] Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, Phys. Rev. D **104**, 063513 (2021).

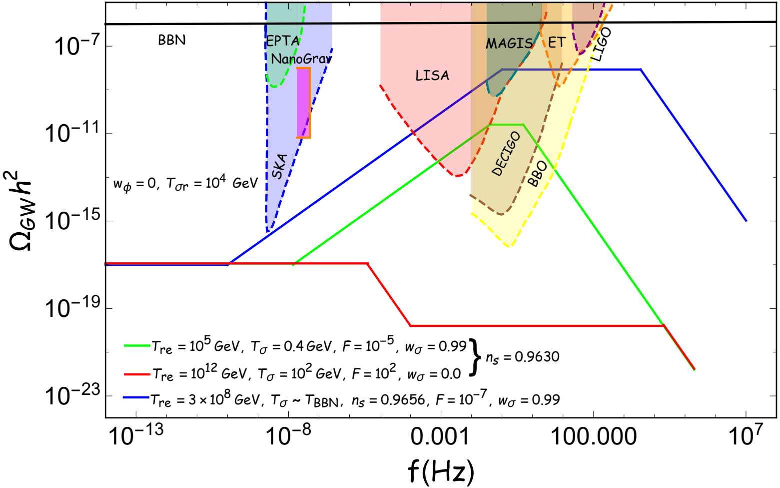
Effects on $\Omega_{\rm GW}(f)$ due to reheating



The behavior of the dimensionless spectral energy density of primary GWs today, viz. $\Omega_{\rm GW}$, has been plotted, over a wide range of frequency f, for different reheating temperatures (in red, green, brown and black)[14].

[14] Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, Phys. Rev. D **104**, 063513 (2021).

Effects on $\Omega_{\rm GW}(f)$ due to late time entropy production

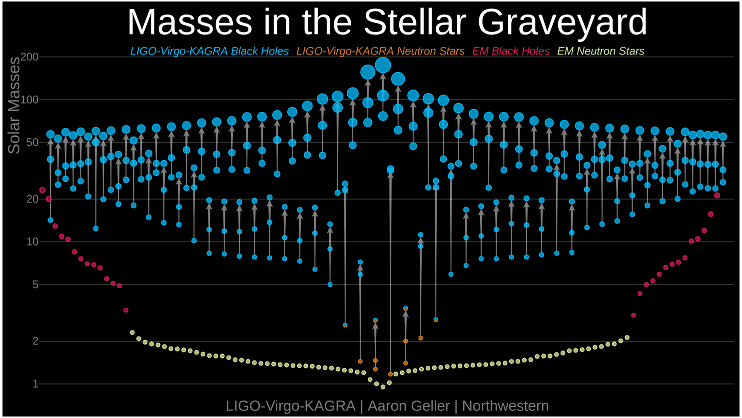


The dimensionless spectral energy density of primary GWs observed today $\Omega_{\rm GW}(f)$ has been plotted in a scenario involving late time production of entropy[15].

[15] Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, Phys. Rev. D **104**, 063513 (2021).

Plan of the talk

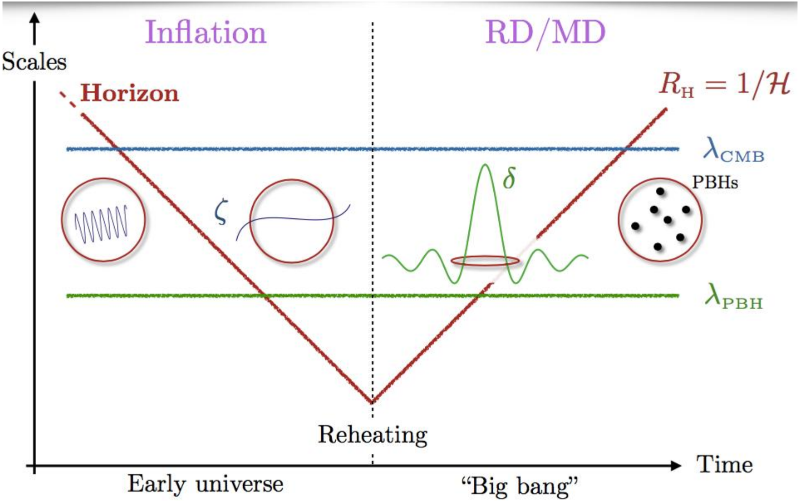
Coalescence of compact binaries observed by LIGO



In November 2021, the LIGO-Virgo-KAGRA Collaboration released the results of the second-half of their third observing run. This third GW Transient Catalog is the largest catalog of mergers involving black holes and neutron stars released thus far and includes events released in prior observing runs[16].

[16] Image from https://www.ligo.caltech.edu/LA/image/ligo20211107a.

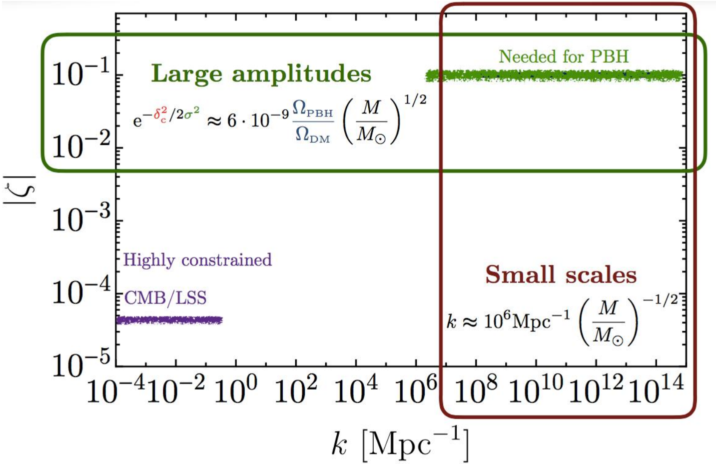
Formation of primordial black holes (PBHs)



BHs can form in the primordial universe when perturbations with significant amplitudes on small scales reenter the Hubble radius during the radiation dominated epoch[17].

[17] Figure from G. Franciolini, arXiv:2110.06815 [astro-ph.CO].

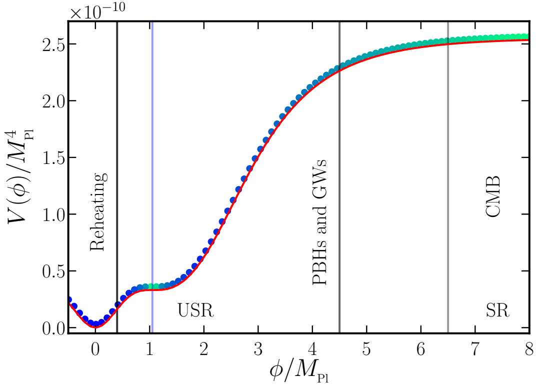
Amplitude required to form significant number of PBHs

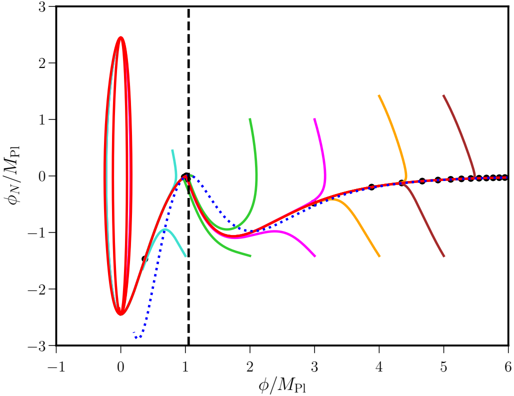


In order to form significant number of black holes, the amplitude of the perturbations on small scales has to be large enough such that the dimensionless amplitude of the scalar perturbation is close to unity[18].

[18] Figure credit G. Franciolini.

Single-field models admitting ultra slow roll inflation



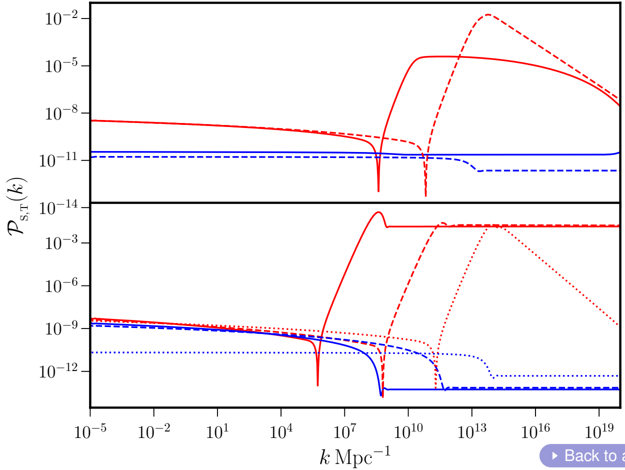


Inflationary attractor

Potentials which contain a point of inflection lead to ultra slow roll (USR) inflation[19].

[19] See, for example, J. Garcia-Bellido and E. R. Morales, Phys. Dark Univ. **18**, 47 (2017); I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).
Figures credits, S. Maity and H. V. Ragavendra.

Power spectra in models permitting USR inflation



Back to analytical form of the power spectrum

Scalar (in red) and the tensor (in blue) power spectra arising in different single-field models that permit a period of USR inflation[20].

[20] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021); Also see H. V. Ragavendra and L. Sriramkumar, Galaxies **11**, 34 (2023).

The two field model of interest

It has been shown that two scalar fields ϕ and χ governed by the following action:

$$S[\phi, \chi] = \int \mathrm{d}^4x \sqrt{-g} \left[-\frac{1}{2}\, \partial^\mu \phi\, \partial_\mu \phi - \frac{f(\phi)}{2}\, \partial^\mu \chi\, \partial_\mu \chi - V(\phi, \chi)\right]$$

described by the potential

$$V(\phi,\, \chi) = V_0\, \frac{\phi^2}{\phi_0^2 + \phi^2} + \frac{m_\chi^2}{2}\, \chi^2$$

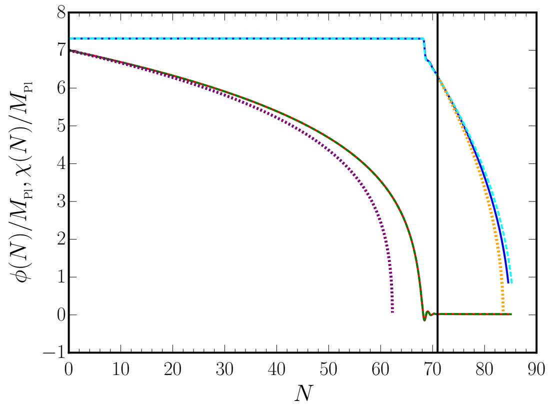
and the non-canonical coupling functions

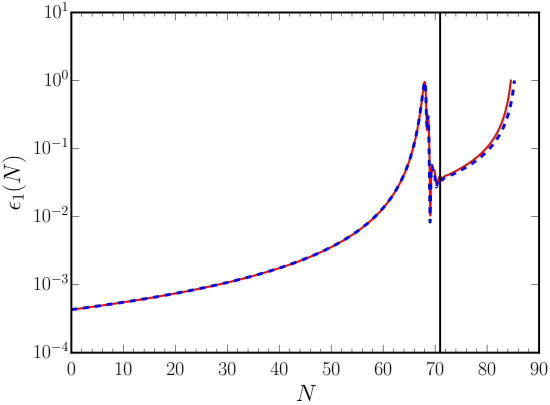
$$f_1(\phi) = \mathrm{e}^{2\, b_1\, \phi} \quad \text{or} \quad f_2(\phi) = \mathrm{e}^{2\, b_2\, \phi^2}$$

can lead to strong features in the scalar power spectrum[21].

[21] M. Braglia, D. K. Hazra, L. Sriramkumar and F. Finelli, JCAP **08** 025 (2020).

Non-trivial inflationary dynamics in the two-field model

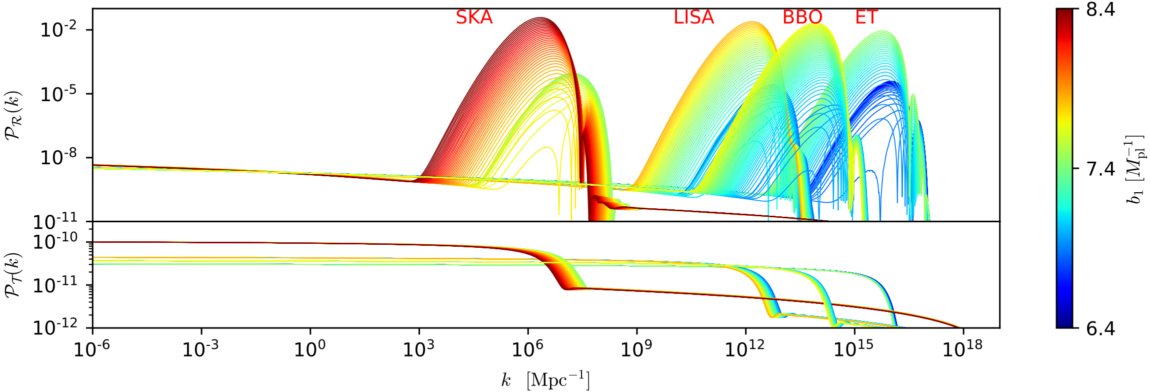




Behavior of the two scalar fields ϕ and χ (in blue and red, on the left) and the first slow roll parameter ϵ_1 (on the right) in the two field model of our interest[22]. Note that there arises a turn in the field space around $N = 70$, when the first slow roll parameter begins to decrease before increasing again, leading to the termination of inflation.

[22] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

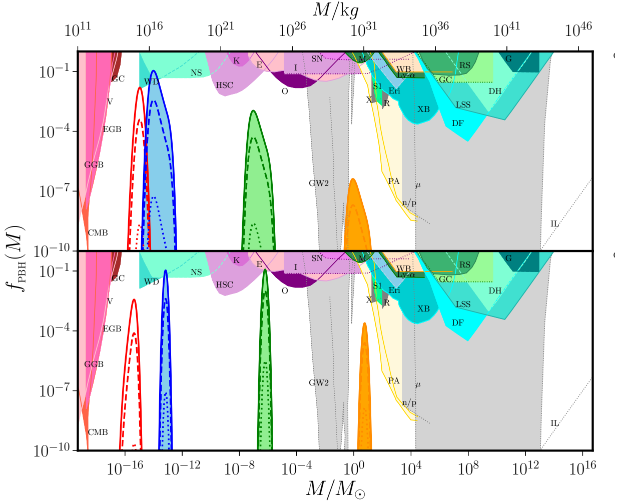
Enhanced power on small scales in the two-field model



The scalar (on top) and the tensor (at the bottom) power spectra evaluated at the end of inflation have been plotted for a few different sets of initial conditions for the fields and a range of values of a particular parameter[23].

[23] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

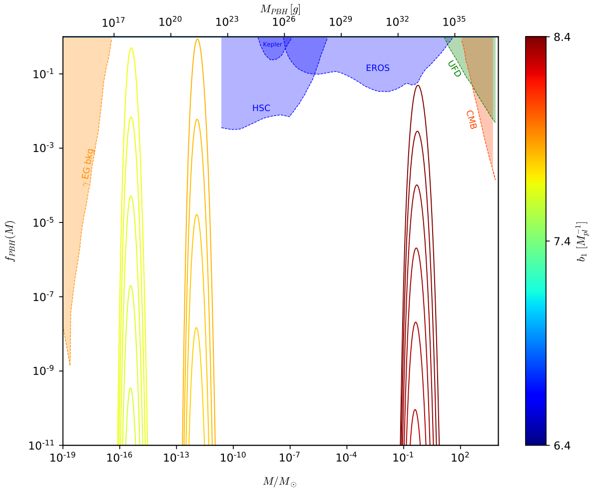
$f_{\rm PBH}(M)$ in ultra slow roll and punctuated inflation



The fraction of PBHs contributing to the cold dark matter density today $f_{\rm PBH}(M)$ has been plotted for different models, viz. USR2 (on top, in red) and PI3 (at the bottom, in red)[24].

[24] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

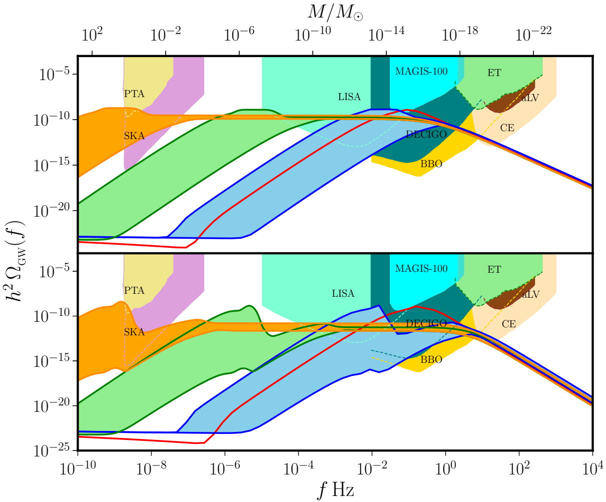
$f_{\rm PBH}(M)$ in the two-field model



The fraction of PBHs contributing to the dark matter density today $f_{\rm PBH}(M)$ in the two-field model of our interest[25].

[25] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

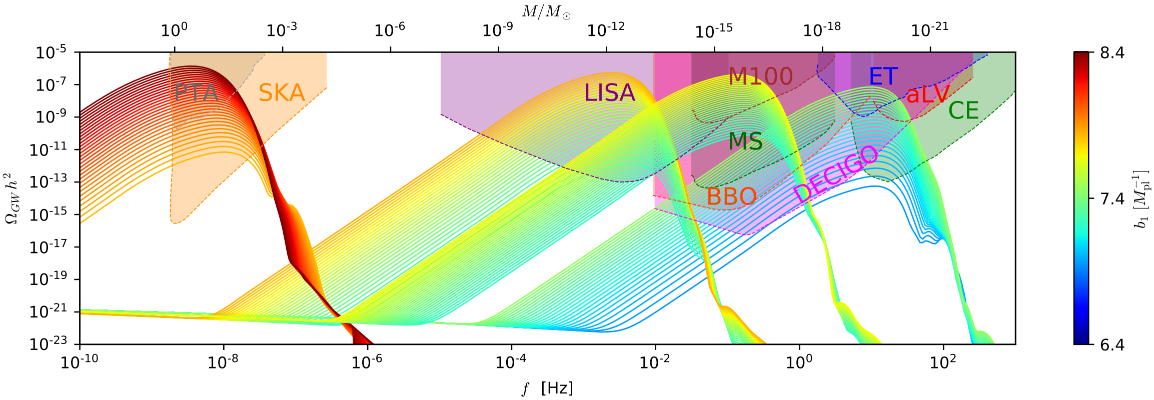
$\Omega_{\rm GW}(f)$ in ultra slow roll and punctuated inflation



The dimensionless spectral density of GWs $\Omega_{\rm GW}(f)$ arising in the models of USR2 (in red, on top) as well as PI3 (in red, at the bottom)[26].

[26] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

$\Omega_{\mathrm{GW}}(f)$ in the two-field model

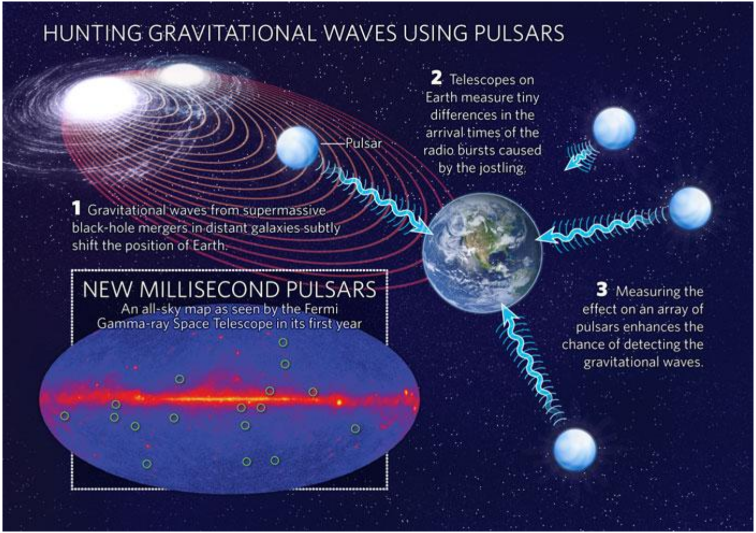


The dimensionless spectral density of GWs $\Omega_{\mathrm{GW}}(f)$ arising in the two-field model has been plotted for a set of initial conditions for the background fields as well as a range of values of the parameter b_1[27].

[27] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

Plan of the talk

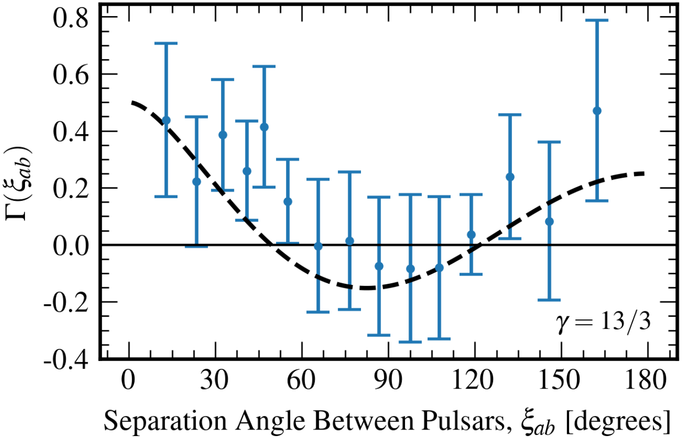
Pulsar timing arrays (PTAs)



The PTAs monitor an array of millisecond pulsars[28].

[28] See https://ipta.github.io/mock_data_challenge/.

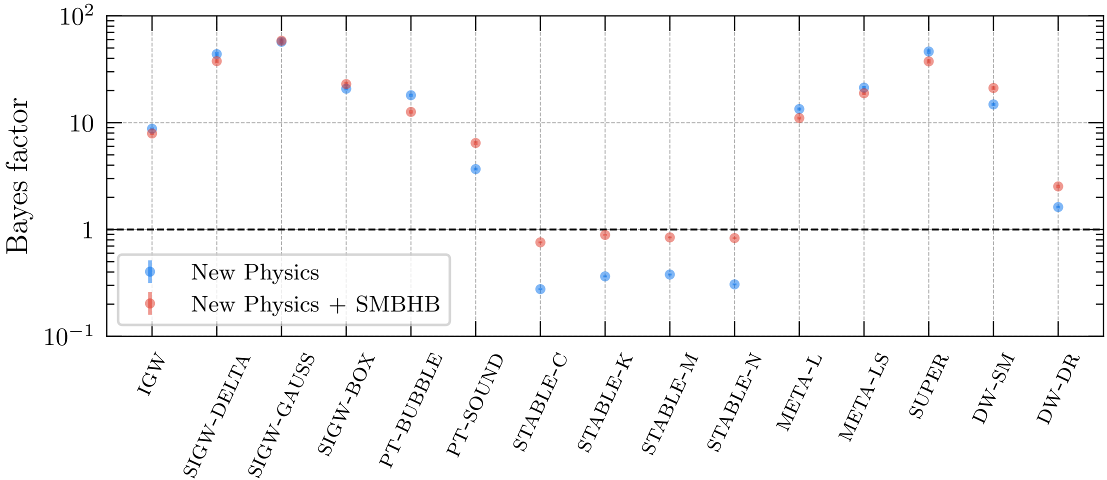
Hellings-Downs curve



The inter-pulsar correlations measured from 2,211 distinct pairings in the 67-pulsar array of the NANOGrav 15-year data. The dashed black line shows the Hellings-Downs correlation pattern[29].

[29] NANOGrav Collaboration (G. Agazie *et al.*), Astrophys. J. Lett. **951**, 1 (2023).

Stochastic GW background observed by pulsar timing arrays (PTAs)



The Bayesian evidence for a variety of astrophysical and cosmological sources for the stochastic GW background suggested by the observations of the PTAs [30].

[30] NANOGrav Collaboration, Astrophys. J. Lett. **951**, L11 (2023).

Shape of the inflationary scalar power spectrum

We assume that the inflationary scalar power spectrum is given by[31]

$$\mathcal{P}_{_{\mathrm{S}}}(k) = A_{_{\mathrm{S}}} \left(\frac{k}{k_\ast}\right)^{n_{_{\mathrm{S}}}-1} + A_0 \begin{cases} \left(\dfrac{k}{k_{\mathrm{peak}}}\right)^4 & k \leq k_{\mathrm{peak}}, \\ \left(\dfrac{k}{k_{\mathrm{peak}}}\right)^{n_0} & k \geq k_{\mathrm{peak}}, \end{cases}$$

where $A_{_{\mathrm{S}}}$ and $n_{_{\mathrm{S}}}$ are the amplitude and spectral index of the power spectrum at the CMB pivot scale of $k_\ast = 0.05\,\mathrm{Mpc}^{-1}$.

▸ Power spectra in USR inflation

We set the reheating temperature to the rather low value of $T_{\mathrm{re}} = 50\,\mathrm{MeV}$.

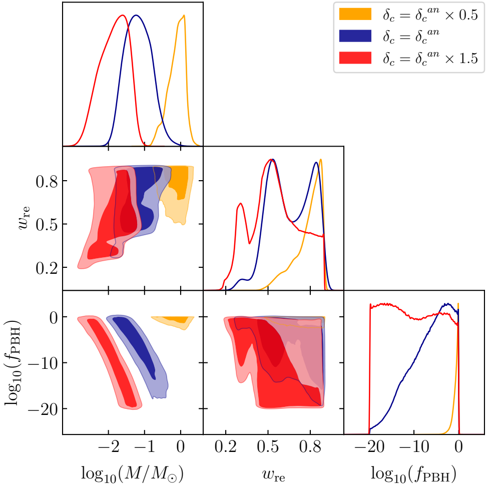
We shall assume that the threshold value of the density contrast for the formation of PBHs is given by[32]:

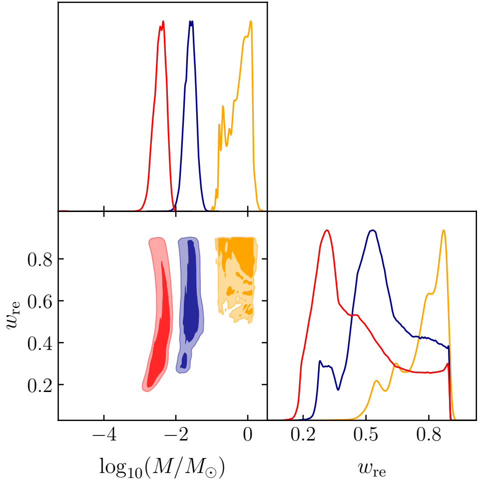
$$\delta_{\mathrm{c}}^{\mathrm{an}} = \frac{3\,(1+w_{\mathrm{re}})}{5+3\,w_{\mathrm{re}}}\,\sin^2\left(\frac{\pi\,\sqrt{w_{\mathrm{re}}}}{1+3\,w_{\mathrm{re}}}\right).$$

[31] For other forms of spectra, see G. Domènech, S. Pi, A. Wang and J. Wang, arXiv:2402.18965 [astro-ph.CO].

[32] In this context, see T. Harada, C.-M. Yoo, and K. Kohri, Phys. Rev. D **88**, 084051 (2013).

Constraints on the epoch of reheating

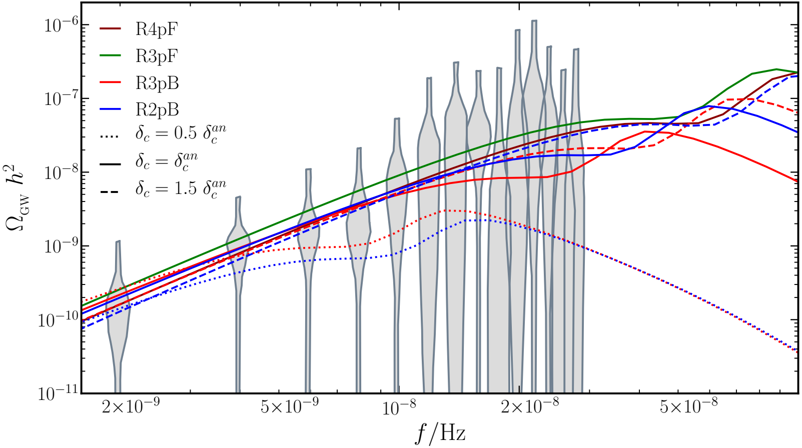




Constraints on the parameters in the models `R3pB` (on the left) and `R2pB` (on the right), arrived at upon comparison with the NANOGrav 15-year data[33].

[33] S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, arXiv:2403.16963 [astro-ph.CO], accepted in JCAP.

Generation of secondary GWs during the epoch of reheating

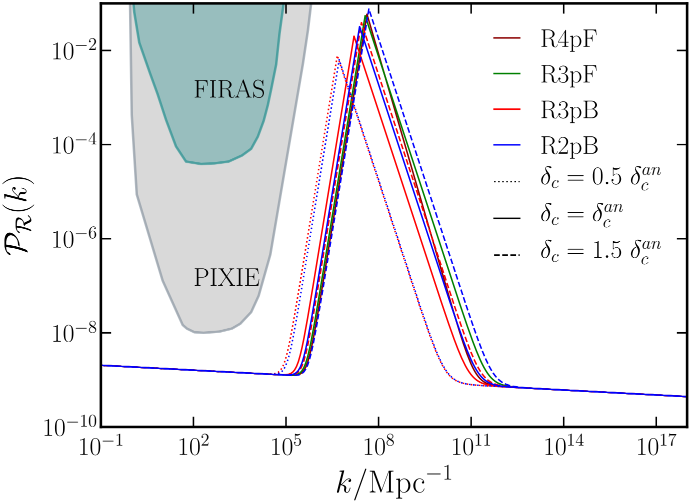


Epoch of reheating

The dimensionless spectral energy density of the secondary GWs today $\Omega_{\rm GW}(f)$ is plotted for a given reheating temperature and the best-fit values of the parameters in the different models[34].

[34] S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, arXiv:2403.16963 [astro-ph.CO], accepted in JCAP.

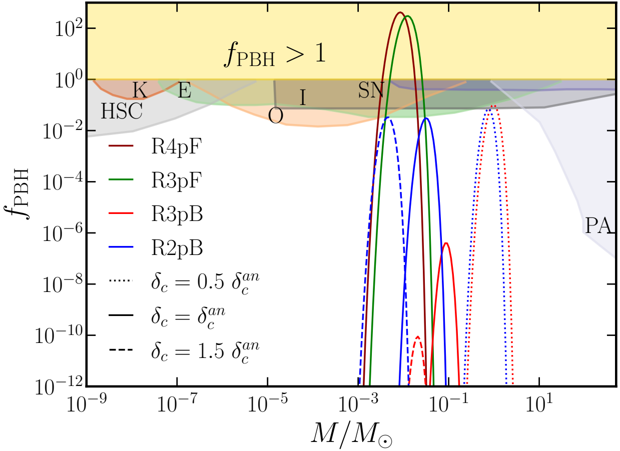
Constraints from spectral distortions



Constraints on the scalar power spectrum from spectral distortions in the CMB[35].

[35] S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, arXiv:2403.16963 [astro-ph.CO], accepted in JCAP.

Formation of PBHs during the epoch of reheating



The fraction of PBHs that constitute the dark matter density today, viz. $f_{\rm PBH}(M)$ is plotted for a given reheating temperature and the best-fit values of the parameters in the different models[36].

[36] S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, arXiv:2403.16963 [astro-ph.CO], accepted in JCAP.

Bayesian evidence

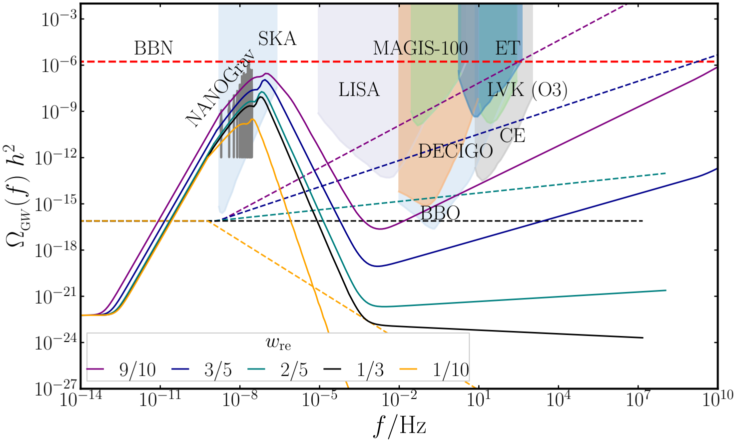
Model X	Model Y	$\mathrm{BF}_{Y,X}$		
		$\delta_\mathrm{c} = 0.5\,\delta_\mathrm{c}^\mathrm{an}$	$\delta_\mathrm{c} = \delta_\mathrm{c}^\mathrm{an}$	$\delta_\mathrm{c} = 1.5\,\delta_\mathrm{c}^\mathrm{an}$
SMBHB	R2pB	$1.7 \pm .06$	260.04 ± 19.21	350.61 ± 27.36

The Bayesian factors $\mathrm{BF}_{Y,X}$ for the model `R2pB` that invokes primordial physics as the source of the stochastic GW background observed by the NANOGrav 15-year data, when compared to the astrophysical scenario of merging supermassive binary black holes.

Bayesian factors $\mathrm{BF}_{Y,X}$ that far exceed unity indicate strong evidence for the model Y with respect to the model X.

Clearly, when $\delta_\mathrm{c} = \delta_\mathrm{c}^\mathrm{an}$ and $\delta_\mathrm{c} = 1.5\,\delta_\mathrm{c}^\mathrm{an}$, the NANOGrav 15-year data strongly favors the model `R2pB` when compared to the SMBHM model.

Generation of secondary GWs during the epoch of reheating



Epoch of reheating

The dimensionless spectral energy density of primary and secondary GWs today $\Omega_{\rm GW}(f)$ have been plotted for a given reheating temperature and different values of the parameter describing the EoS during reheating[37].

[37] S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, arXiv:2403.16963 [astro-ph.CO], accepted in JCAP.

Plan of the talk

Outlook

- If one of the future CMB missions—such as LiteBIRD (Lite, Light satellite for the studies of B-mode polarization and Inflation from cosmicbackground Radiation Detection), Primordial Inflation Explorer (PIXIE) or Exploring Cosmic History and Origin (ECHO, a proposed Indian effort)—detect the signatures of the primordial GWs, it will help us arrive at strong constraints on the dynamics during inflation and reheating.
- The observations by LIGO are a culmination of almost fifty years of effort to detect GWs. They have opened up a new window to observe the universe.
- The observations by the PTAs and their possible implications for the stochastic GW background offer a wonderful opportunity to understand the physics operating over a wider range of scales in the early universe.
- During the coming decades, GW observatories such as the Laser Interferometer Space Antenna (LISA), Einstein Telescope and Cosmic Explorer, can be expected to provide us with an unhindered view of the primordial universe.

Collaborators I

Matteo Braglia

Dhiraj Hazra

Fabio Finelli

George Smoot

Alexei Starobinsky

H. V. Ragavendra

Pankaj Saha

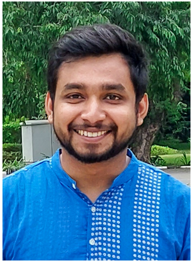
Joseph Silk

Collaborators II

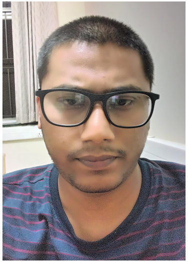
Md. Riajul Haque

Debaprasad Maity

Tanmoy Paul



Suvashis Maity



Nilanjandev Bhaumik

Thank you for your attention