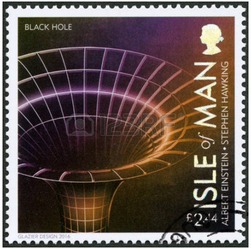
Hawking’s genius

L. Sriramkumar

Department of Physics, Indian Institute of Technology Madras, Chennai



Institute colloquium

Indian Institute of Technology, Palakkad

April 4, 2018

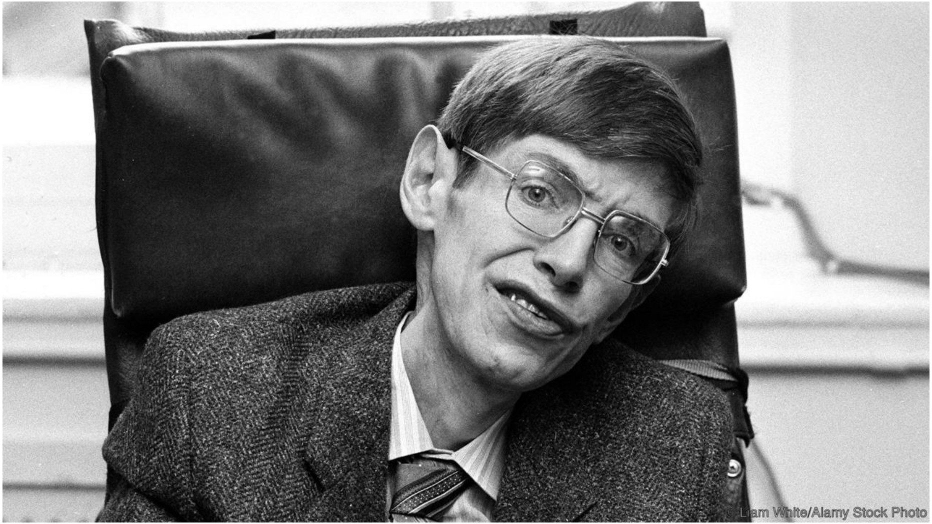
Plan of the talk

1. Hawking's academic history
2. Evolution of stars
3. General relativity and black holes
4. Black hole thermodynamics
5. Hawking radiation
6. Hawking's other contributions
7. Summary and outlook

Plan of the talk

1. Hawking's academic history
2. Evolution of stars
3. General relativity and black holes
4. Black hole thermodynamics
5. Hawking radiation
6. Hawking's other contributions
7. Summary and outlook

Stephen Hawking



Stephen Hawking: January 8, 1942 – March 14, 2018[1].

---

[1] Photo from http://www.bbc.com/earth/story/20160107-these-are-the-discoveries-that-made-stephen-hawking-famous.

Academic history[2]

- B.A., University College, Oxford, England, 1959–1962
- M.A., Ph.D., Department of Applied Mathematics and Theoretical Physics (DAMTP), University of Cambridge, Cambridge, England, 1962–66
  Thesis title: Properties of expanding universes
  Ph.D. supervisor: Dennis Sciama
- Research Fellow, Gonville and Caius College (Caius), Cambridge, England, 1966–69
- Professorial Fellow, Caius, 1969
- Research Assistant, Institute of Astronomy, University of Cambridge, Cambridge, England, 1969–73
- Research Assistant, DAMTP, 1973–75
- Reader, DAMTP, 1975–77
- Professor, DAMTP, 1977–79
- Lucasian Professor of Mathematics, DAMTP, 1979–2009
- Director of Research, DAMTP, 2009–18

---

[2] Source http://www.hawking.org.uk/ and https://www.ast.cam.ac.uk/content/stephen.hawking.1942–2018.

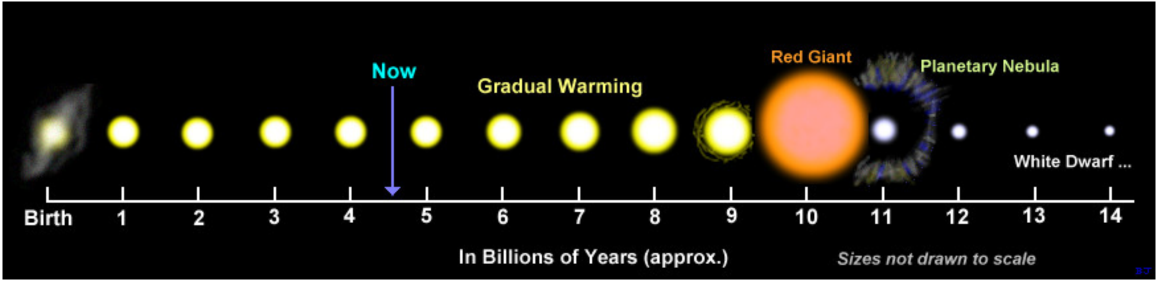
This talk is largely based on...

- J. M. Bardeen, B. Carter and S. W. Hawking, *The four laws of black hole mechanics*, Comm. Math. Phys. **31**, 161 (1973).
- J. D. Bekenstein, *Black holes and entropy*, Phys. Rev. D **7**, 2333 (1973).
- S. W. Hawking, *Black hole explosions?*, Nature **248**, 30 (1974).
- S. W. Hawking, *Particle creation by black holes*, Comm. Math. Phys. **43**, 199 (1975).

Plan of the talk

1. Hawking's academic history
2. Evolution of stars
3. General relativity and black holes
4. Black hole thermodynamics
5. Hawking radiation
6. Hawking's other contributions
7. Summary and outlook

Life cycle of the Sun

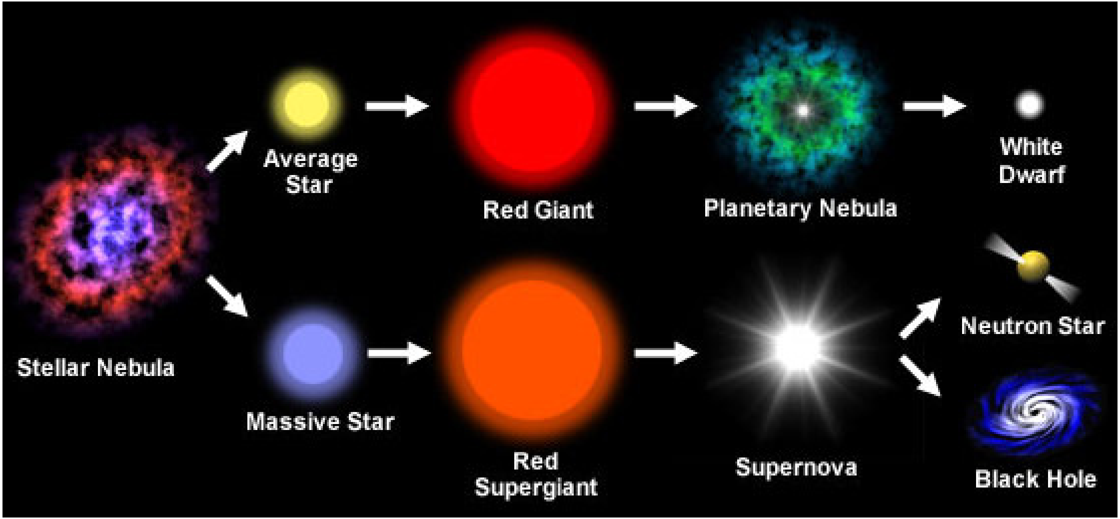


The life cycle of the Sun[3].

---

[3] Image from http://openhighschoolcourses.org/mod/book/tool/print/index.php?id=7202.

Life cycle of stars



The life cycle of a star depends on its initial mass[4].

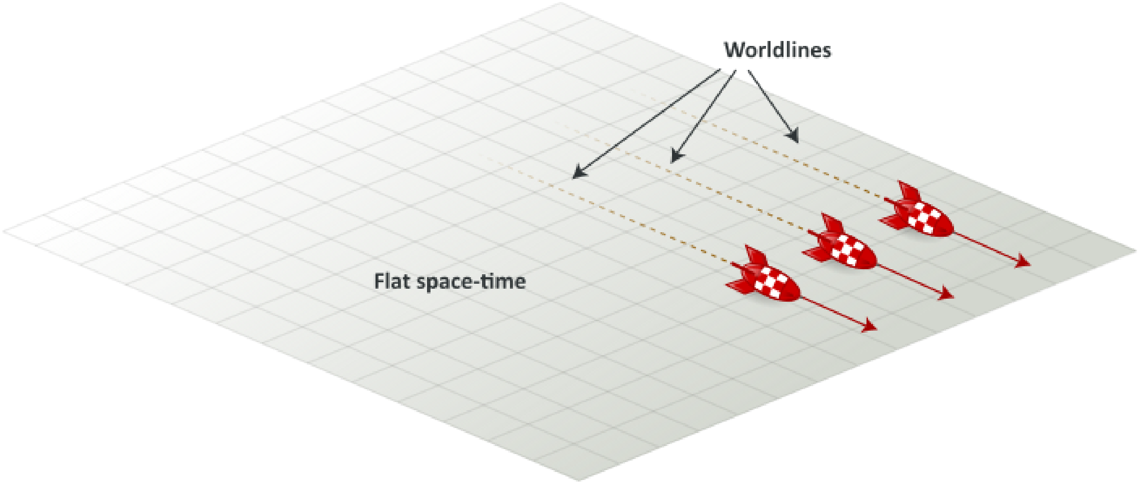
---

[4] Image from https://www.schoolsobservatory.org/learn/astro/stars/cycle.

Plan of the talk

1. Hawking's academic history
2. Evolution of stars
3. General relativity and black holes
4. Black hole thermodynamics
5. Hawking radiation
6. Hawking's other contributions
7. Summary and outlook

Flat or Minkowski spacetime

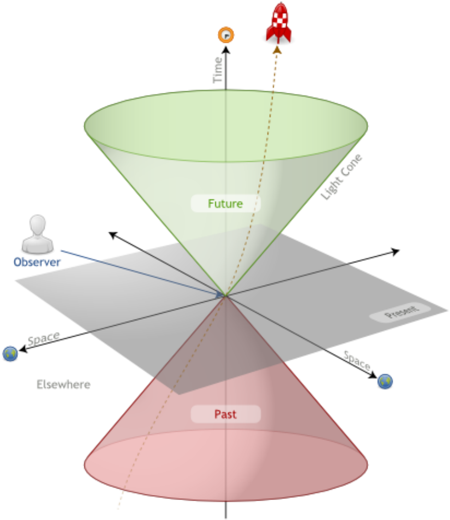


The spacetime is flat in the absence of any matter[5].

---

[5] Image from https://www.quantum-bits.org/?p=963.

Light cones in flat spacetime

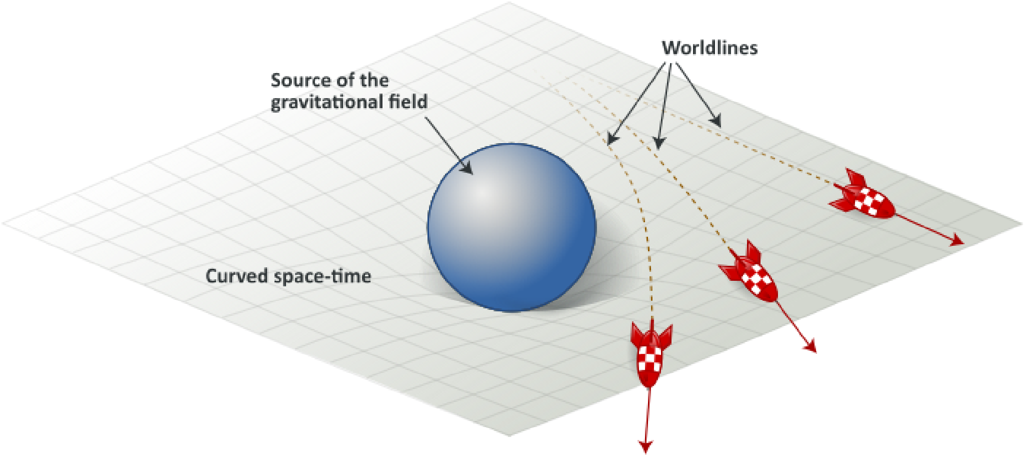


The behavior of light cones in flat spacetime[6].

---

[6] Image from https://www.quantum-bits.org/?p=963.

Curved spacetimes[8]



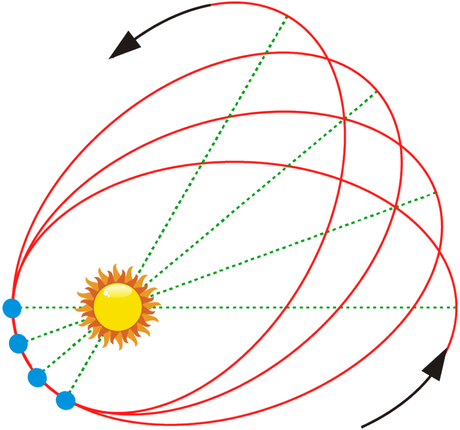
According to general relativity, matter tells spacetime how to curve and the spacetime tells matter how to move[7].

---

[7] J. A. Wheeler, *Geons, Black Holes, and Quantum Foam: A Life in Physics* (W. W. Norton, New York, 2010).

[8] Image from https://www.quantum-bits.org/?p=963.

Precession of the perihelion of Mercury

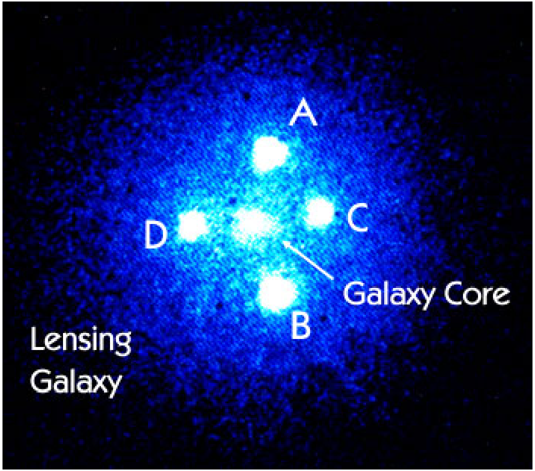


When the general relativistic effects are taken into account, it can be shown that the perihelion of Mercury precesses exactly by the extent of the discrepancy with Newtonian gravity, *viz.* by $43''$ per century[9].

---

[9] Image from https://writescience.wordpress.com/2015/02/07/gravity-04-testing-the-new-gravity/.

Gravitational bending of light[10]

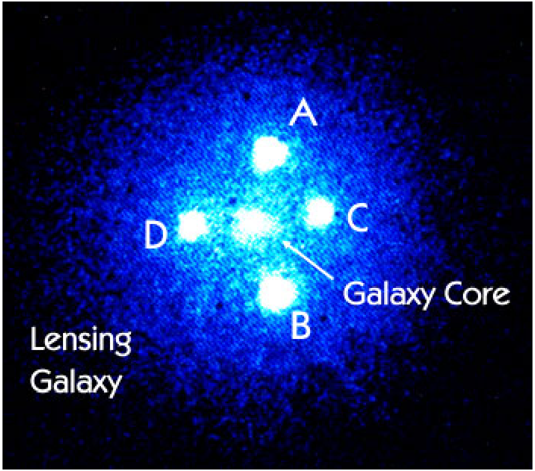


According to general relativity, gravity bends light, a phenomenon that is regularly observed by astronomers.

---

[10] Image from http://www.skyhound.com/observing/archives/sep/Q2237+0305A.html.

Gravitational bending of light[10]



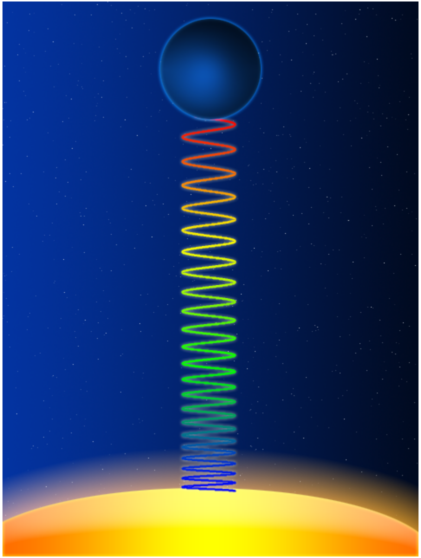
According to general relativity, gravity bends light, a phenomenon that is regularly observed by astronomers.

The image on the left is that of the Einstein cross, which is a quasar that is gravitationally lensed quasar by a foreground galaxy. The strong gravitational field of the foreground galaxy bends the light of the distant quasar creating four images of it.

---

[10] Image from http://www.skyhound.com/observing/archives/sep/Q2237+0305A.html.

Gravitational redshift[11]

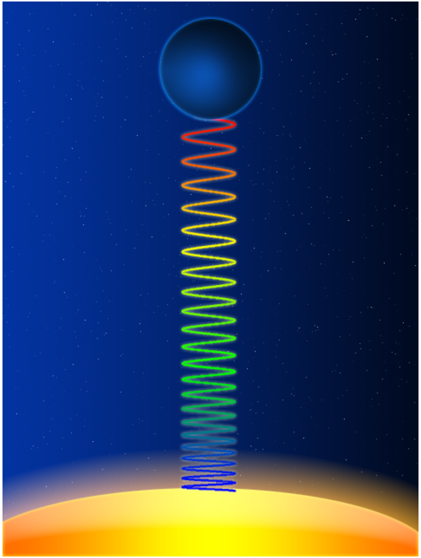


In general relativity, a photon loses energy as it climbs out of a gravitational field. This phenomenon has been experimentally confirmed in the laboratory.

---

[11] Image from https://en.wikipedia.org/wiki/Gravitational_redshift.

Gravitational redshift[11]



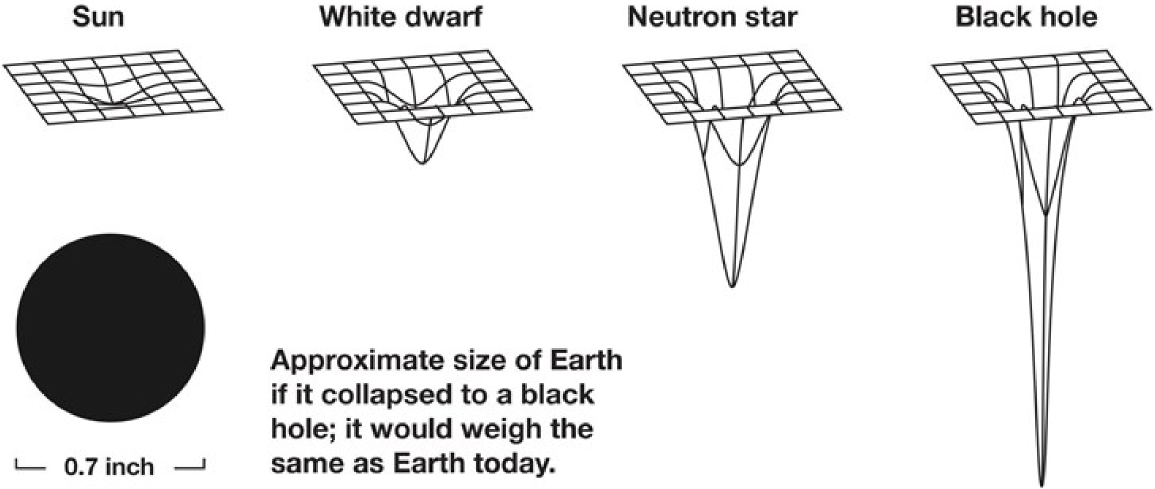
In general relativity, a photon loses energy as it climbs out of a gravitational field. This phenomenon has been experimentally confirmed in the laboratory.

An equivalent phenomenon corresponds to clocks running slow in a gravitational field. This phenomenon and the relativistic time dilation has to accounted for in the Global Positioning System (GPS), if it is to meet the accuracy (of 2 m) desired for military applications.

---

[11] Image from https://en.wikipedia.org/wiki/Gravitational_redshift.

End stages of gravitational collapse

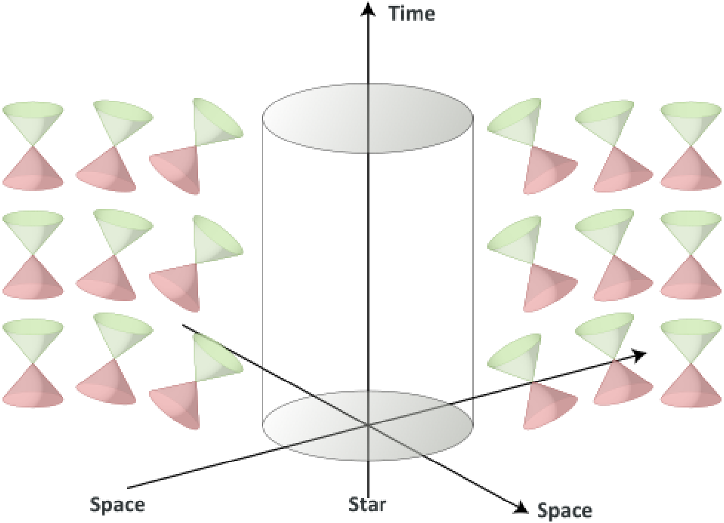


The behavior of spacetime near compact objects[12].

---

[12] Image from http://www.nationalgeographic.com.au/space/star-eater.aspx.

Light cones tip near a massive object

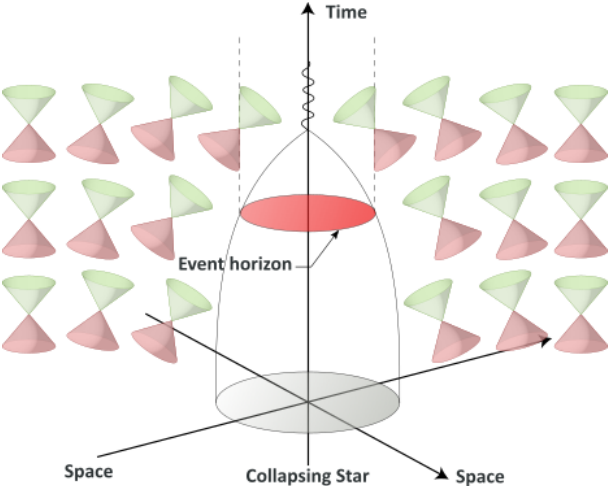


The light cones tip strongly near massive, compact objects[13].

---

[13] Image from https://www.quantum-bits.org/?p=963.

Formation of the event horizon



The behavior of the light cones as a mass collapses into its Schwarzschild radius (i.e. $r_{\rm S} = 2\,G\,M/c^2$) leading to the formation of the black hole's event horizon[14].

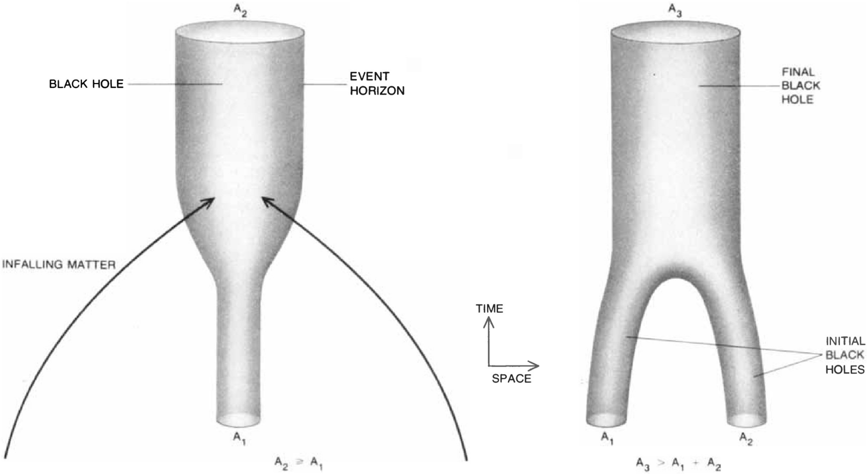
---

[14] Image from https://www.quantum-bits.org/?p=963.

Plan of the talk

1. Hawking's academic history
2. Evolution of stars
3. General relativity and black holes
4. Black hole thermodynamics
5. Hawking radiation
6. Hawking's other contributions
7. Summary and outlook

The area theorem of black holes



The area of a black hole never decreases, suggesting that it behaves like the entropy of a thermodynamic system[15].

---

[15] Image from https://blogs.scientificamerican.com/sa-visual/a-visual-information-paradox/.

Laws of black hole thermodynamics[16]

- ✦ Zeroth law: The surface gravity κ (i.e. the acceleration due to gravity in the Newtonian sense) is always a constant over the horizon of a stationary black hole. This is reminiscent of thermodynamics wherein a system in thermal equilibrium has a uniform temperature everywhere.
- ✦ First law: For a rotating and charged black hole, one finds that

$$\mathrm{d}M = \kappa\,\frac{\mathrm{d}A}{8\,\pi\,G} + \Omega\,\mathrm{d}J + \Phi\,\mathrm{d}Q,$$

  where M, A, J and Q denote the mass, area, angular momentum and charge associated with the black hole. This is similar to the first law of thermodynamics wherein $\mathrm{d}U = T\,\mathrm{d}S - P\,\mathrm{d}V + \mu\,\mathrm{d}N$.
- ✦ Second law: In a manner similar to thermodynamic entropy, the area of the event horizon never decreases (assuming cosmic censorship and positive energy condition).
- ✦ Third law: The surface gravity of a black hole horizon cannot be reduced to zero in a finite number of steps.

---

[16] See, for example, http://www.physics.umd.edu/grt/taj/776b/lectures.pdf.

Black hole entropy and the generalized second law

Famously, John Wheeler had asked Jacob Bekenstein whether one can violate the second law of thermodynamics by dropping a hot cup of tea into a black hole[17].

---

[17] See, for example, K. Thorne, *Black Holes and Time Warps* (W. W. Norton, New York, 1994).

Black hole entropy and the generalized second law

Famously, John Wheeler had asked Jacob Bekenstein whether one can violate the second law of thermodynamics by dropping a hot cup of tea into a black hole[17].

Bekenstein proposed the *generalized second law of thermodynamics*, which says that it is the *total* entropy, *including the entropy of black holes*, that never decreases.

---

[17] See, for example, K. Thorne, *Black Holes and Time Warps* (W. W. Norton, New York, 1994).

Black holes have entropy but no temperature!

While the analogy between the laws of black hole mechanics and the laws of thermodynamics is striking, two glaring issues had remained:

Black holes have entropy but no temperature!

While the analogy between the laws of black hole mechanics and the laws of thermodynamics is striking, two glaring issues had remained:

- The temperature of a black hole (since it cannot emit anything) seems to be zero.

Black holes have entropy but no temperature!

While the analogy between the laws of black hole mechanics and the laws of thermodynamics is striking, two glaring issues had remained:

- The temperature of a black hole (since it cannot emit anything) seems to be zero.
- Entropy is a dimensionless quantity, whereas the area of the horizon has dimensions of the square of length.

Plan of the talk

1. Hawking's academic history
2. Evolution of stars
3. General relativity and black holes
4. Black hole thermodynamics
5. Hawking radiation
6. Hawking's other contributions
7. Summary and outlook

Uncertainty in quantum mechanics

In classical mechanics, the position and the velocity (or momentum) of a particle can be specified simultaneously.

This cannot be done in quantum mechanics, which involves fluctuations and hence uncertainties. For instance, the uncertainties in position and momentum, say, Δx and Δp, satisfy the following so-called uncertainty principle:

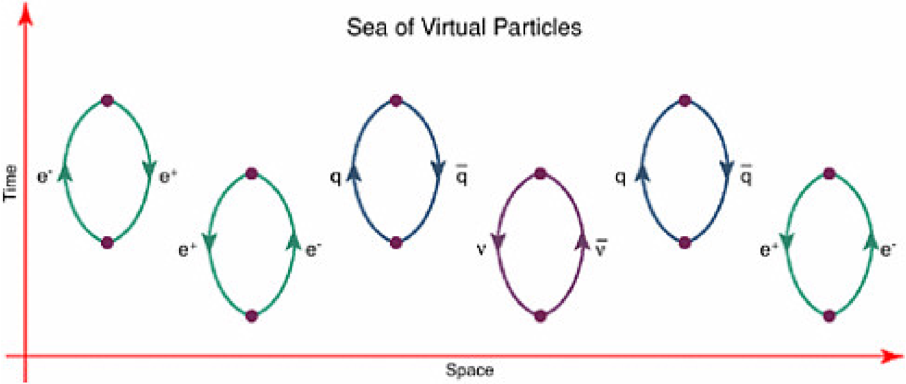
$$\Delta x \, \Delta p \geq \frac{\hbar}{2},$$

where $\hbar = h/(2\,\pi)$, with h being the Planck constant.

These arguments can be translated to uncertainties in the energy and time of a particle to be

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}.$$

Vacuum fluctuations in quantum field theory

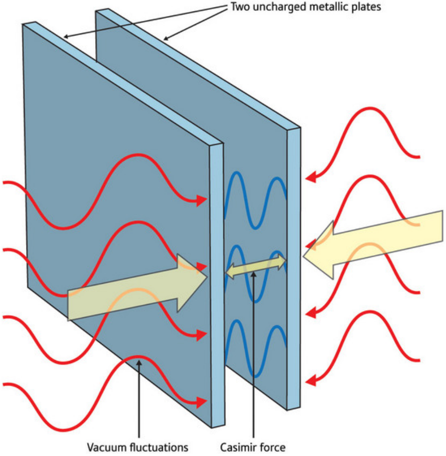


Quantum field theory involves extensions of the principles of quantum mechanics to fields such as the electromagnetic field. In quantum field theory, vacuum is not devoid of activity[18], but is actually buzzing with *virtual* particle-anti-particle pairs (for example, an electron and a position) that emerge out of nowhere (with energy, say, ΔE), only to disappear again very soon (within, say, Δt), such that $\Delta E\, \Delta t \geq \hbar/2$.

---

[18] Image from https://www.nap.edu/read/10079/chapter/4#19.

Casimir effect



Using quantum field theory, the force per unit area between plates separated by a distance L can be determined to be

$$\frac{F}{A} = -\frac{\pi^2\,\hbar\,c}{240\,L^4} = \frac{0.0013}{L^4}\,\frac{\mathrm{N}\,\mu\mathrm{m}^4}{\mathrm{m}^2}.$$

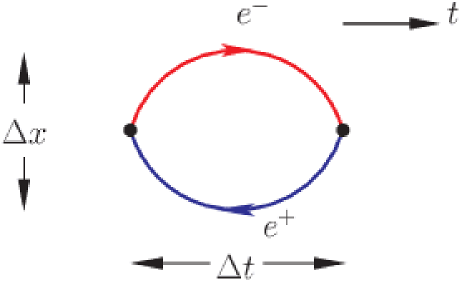
.

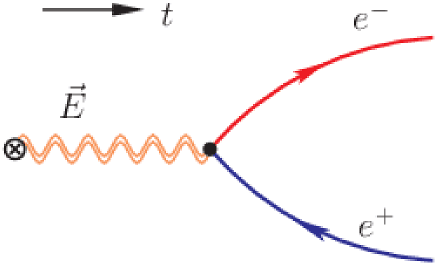
Vacuum fluctuations lead to an attraction between two metallic plates, called the Casimir effect[19], which has been measured in the laboratory.

---

[19] Image from http://www.riken.jp/en/research/rikenresearch/highlights/5668/.

Schwinger effect



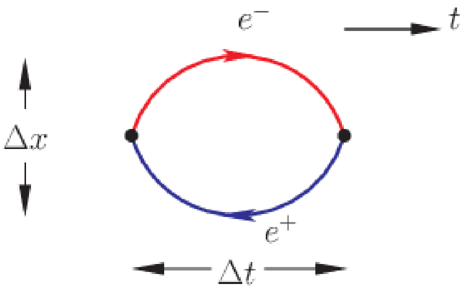


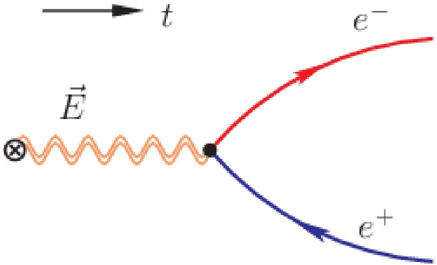
Left: Vacuum fluctuations in empty space, which we had discussed earlier.

---

[20] Images from http://naturalunits.blogspot.in/2015/04/the-super-critical-charge.html.

Schwinger effect



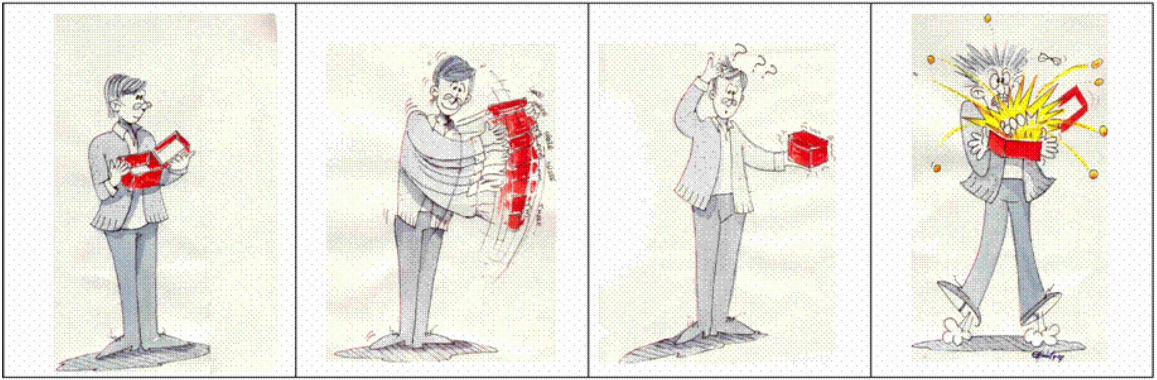


Left: Vacuum fluctuations in empty space, which we had discussed earlier.
Right: Vacuum fluctuations in the presence of an external electric field. The electric field converts virtual pairs into real pairs, leading to production of particles, an effect known as the Schwinger effect[20].

---

[20] Images from http://naturalunits.blogspot.in/2015/04/the-super-critical-charge.html.

Dynamical Casimir effect

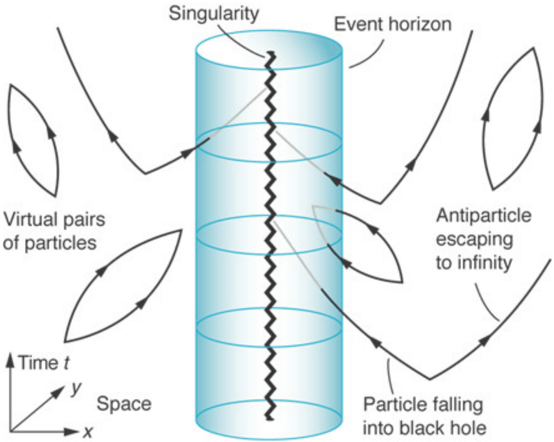


Moving mirrors can radiate, even in the vacuum[21].

---

[21] Images from http://www.pd.infn.it/casimir/Index.htm.

Particle creation by black holes

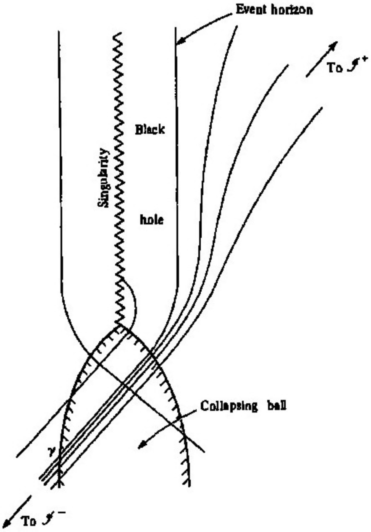


Vacuum fluctuations near a black hole event horizon[22]. One of the particles from a virtual pair can fall into the event horizon, while the other can escape to infinity, leading to the phenomenon of creation of particles by a black hole.

---

[22] Image from http://science.sciencemag.org/content/295/5559/1476.full?ijkey=fOy16bzxFKF5.&keytype=ref&siteid=sci.

Behavior of modes around a collapsing black hole[23]



The behavior of the modes of a quantum field near a black hole event horizon, as it is being formed. Modes that skim the event horizon as it is formed exhibit exponential redshift, which is responsible for the thermal nature of particles produced at late times.

The temperature of the thermal radiation, referred to as Hawking radiation, is found to be

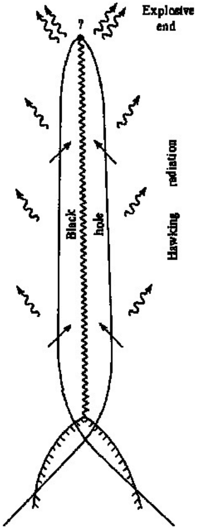
$$T_{\rm H} = \frac{\hbar\, c^3}{8\,\pi\,G\,k_{\rm B}} = 6.15\times 10^{-8}\,\frac{M_\odot}{M}\,{\rm K},$$

where $M_\odot$ denotes the mass of the Sun.

---

[23] Image from N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England).

Black holes evaporate and, eventually, explode[24]



Note that, the lighter the mass of a black hole, the higher is its temperature. Therefore, the temperature of black holes keeps increasing as they evaporate and the black holes are expected to eventually end in explosions.

---

[24] Image from N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England).

Entropy associated with Hawking radiation

The Hawking temperature implies that the black hole entropy is given by

$$S_{\rm BH} = \frac{k_{\rm B}\, c^3}{\hbar\, G}\, \frac{A_{\rm BH}}{4} = k_{\rm B}\, \frac{A_{\rm BH}}{4\, L_{\rm P}^2},$$

where $A_{\rm BH} = 4\,\pi\, r_{\rm S}^2 = 16\,\pi\, G^2\, M^2/c^4$ is the area of the event horizon and the quantity $L_{\rm P} = \sqrt{\hbar\, G/c^3}$ is known as the Planck length.

Plan of the talk

1. Hawking’s academic history
2. Evolution of stars
3. General relativity and black holes
4. Black hole thermodynamics
5. Hawking radiation
6. Hawking’s other contributions
7. Summary and outlook

Hawking's contributions

Hawking's ideas in theoretical physics include:

- Singularity theorems
- Black hole area theorem
- Black hole thermodynamics
- Hawking radiation
- Information loss paradox
- The concept of imaginary time
- Hartle-Hawking wavefunction of the universe
- Gibbons-Hawking temperature and entropy
- Hawking-Page phase transition

Plan of the talk

1. Hawking's academic history
2. Evolution of stars
3. General relativity and black holes
4. Black hole thermodynamics
5. Hawking radiation
6. Hawking's other contributions
7. Summary and outlook

Summary and outlook

- While black holes are completely black classically, they are found to emit a thermal spectrum of particles when the quantum effects are taken into account. The thermal emission from black holes is known as Hawking radiation.
- The thermal radiation leads to an entropy, referred to as the Bekenstein-Hawking entropy.
- As in statistical physics, the entropy of black holes possibly reflect the measure of microstates associated with a set of macroscopic parameters. A theory of quantum gravity is expected to help us understand the nature of the microstates and the origin of black hole entropy.

Thank you for your attention