

Imprints of non-trivial inflationary dynamics on small scales

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Plan of the talk

- 1 Constraints on inflationary dynamics over large scales
- 2 Enhancing scalar power on small scales
- 3 Implications for PBH formation and secondary GWs
- 4 Non-Gaussianities generated in ultra slow roll and punctuated inflation
- 5 Implications for inflationary magnetogenesis
- 6 Outlook

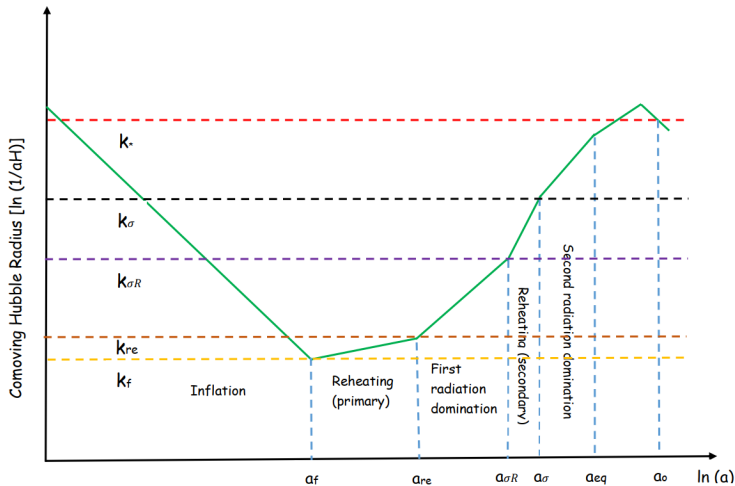


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Behavior of the comoving wave numbers and Hubble radius

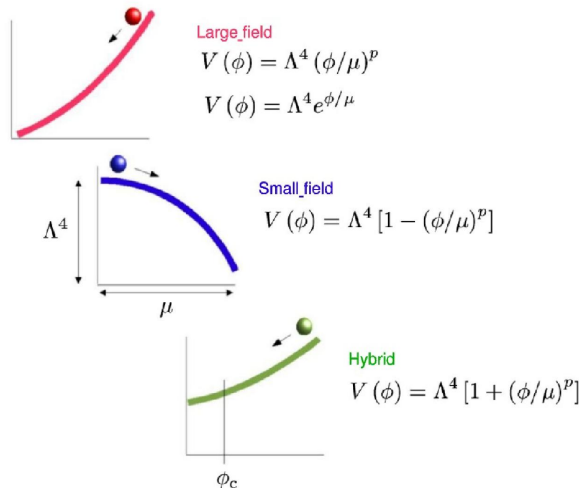


Behavior of the comoving wave number k (horizontal lines in different colors) and the comoving Hubble radius $d_H/a = (aH)^{-1}$ (in green) across different epochs¹.

¹Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, Phys. Rev. D **104**, 063513 (2021).



Variety of potentials can drive inflation



A variety of scalar field potentials have been considered to drive inflation². Often, these potentials are classified as small field, large field and hybrid models.

²Image from [W. Kinney, astro-ph/0301448](https://arxiv.org/abs/astro-ph/0301448).



Scalar spectral index and the tensor-to-scalar ratio

While comparing with the observations, for convenience, one often uses the following power law, template scalar and the tensor spectra:

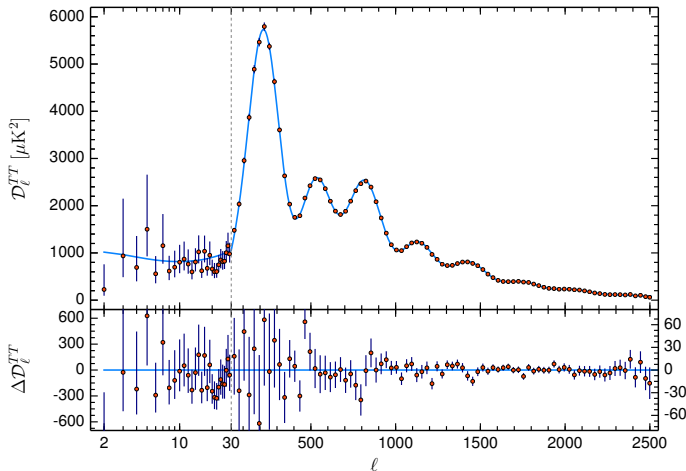
$$\mathcal{P}_S(k) = A_S \left(\frac{k}{k_*} \right)^{n_S - 1}, \quad \mathcal{P}_T(k) = A_T \left(\frac{k}{k_*} \right)^{n_T},$$

with the spectral indices n_S and n_T assumed to be constant. The tensor-to-scalar ratio r is defined as

$$r(k) = \frac{\mathcal{P}_T(k)}{\mathcal{P}_S(k)}.$$



CMB angular power spectrum from Planck

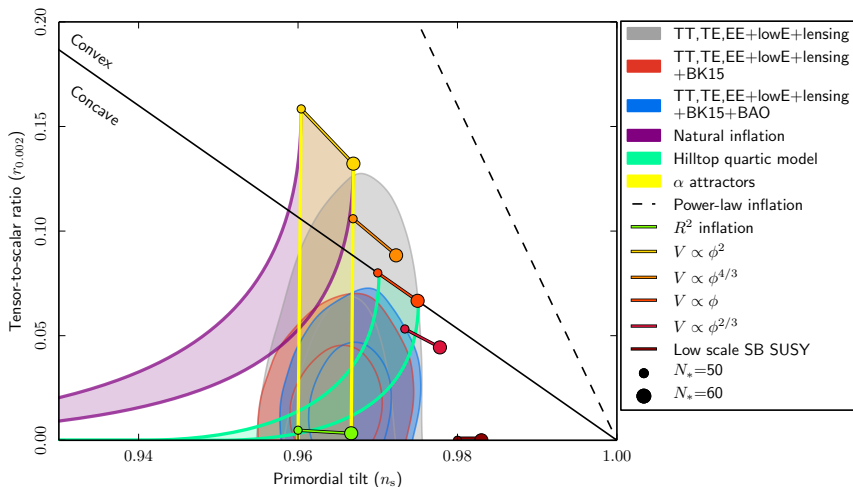


The CMB TT angular power spectrum from the Planck 2018 data (red dots with error bars) and the best fit Λ CDM model with a power law primordial spectrum (solid blue curve)³.

³Planck Collaboration (N. Aghanim *et al.*), *Astron. Astrophys.* **641**, A6 (2020).



Performance of inflationary models in the n_s - r plane



Joint constraints on n_s and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of some of the popular inflationary models⁴.

⁴Planck Collaboration (Y. Akrami *et al.*), *Astron. Astrophys.* **641**, A10 (2020).

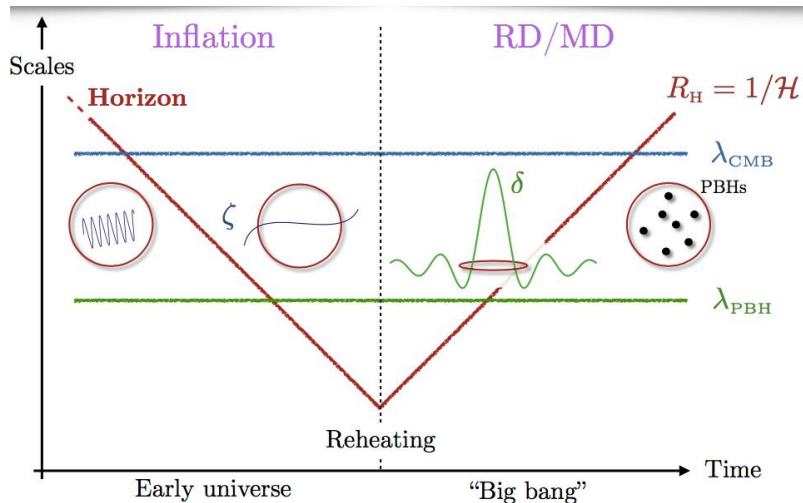


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Formation of BHs in the early universe

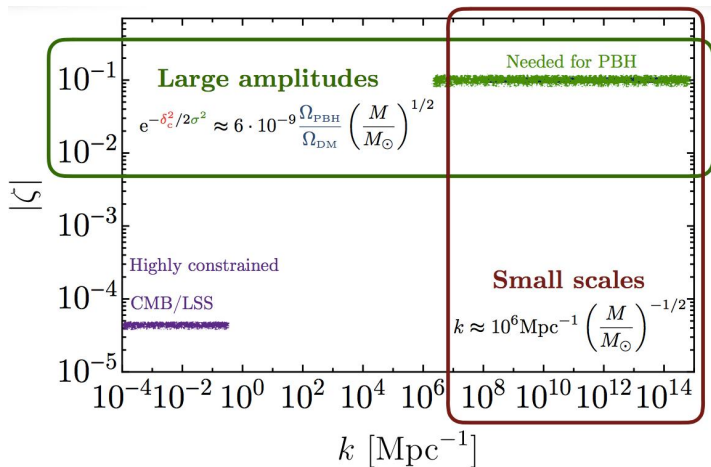


BHs can form when perturbations with significant amplitudes reenter the Hubble radius during the radiation dominated epoch⁵.

⁵Figure from G. Franciolini, [arXiv:2110.06815 \[astro-ph.CO\]](https://arxiv.org/abs/2110.06815).



Amplitude required to form significant number of PBHs

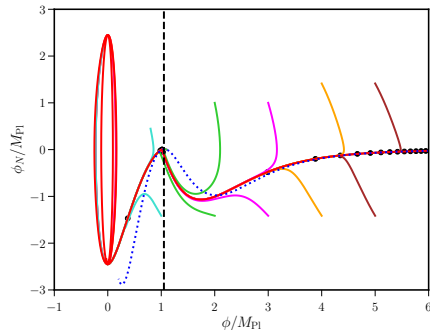
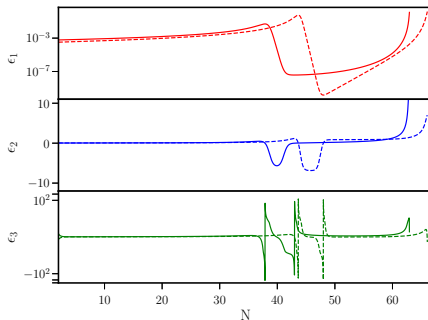


In order to form significant number of black holes, the amplitude of the perturbations on small scales has to be large enough such that the dimensionless amplitude of the scalar perturbation is close to unity⁶.

⁶Figure credit G. Franciolini.



Potentials admitting ultra slow roll inflation



Potentials leading to ultra slow roll inflation (with $x = \phi/v$, v being a constant)⁷:

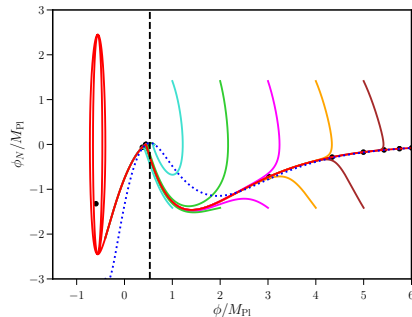
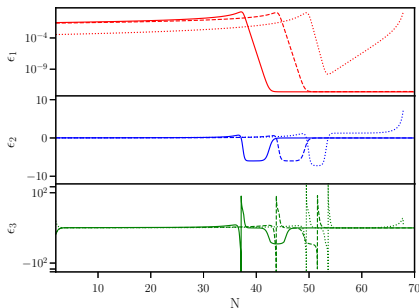
$$\text{USR1} : V(\phi) = V_0 \frac{6x^2 - 4\alpha x^3 + 3x^4}{(1 + \beta x^2)^2},$$

$$\text{USR2} : V(\phi) = V_0 \left\{ \tanh\left(\frac{\phi}{\sqrt{6} M_{\text{Pl}}}\right) + A \sin\left[\frac{\tanh[\phi/(\sqrt{6} M_{\text{Pl}})]}{f_\phi}\right] \right\}^2.$$

⁷J. Garcia-Bellido and E. R. Morales, Phys. Dark Univ. **18**, 47 (2017);
I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).



Potentials permitting punctuated inflation



Potentials admitting punctuated inflation⁸:

$$\text{PI1} : V(\phi) = V_0 (1 + B \phi^4), \quad \text{PI2} : V(\phi) = \frac{m^2}{2} \phi^2 - \frac{2m^2}{3\phi_0} \phi^3 + \frac{m^2}{4\phi_0^2} \phi^4,$$

$$\text{PI3} : V(\phi) = V_0 \left[c_0 + c_1 \tanh \left(\frac{\phi}{\sqrt{6\alpha} M_{\text{Pl}}} \right) + c_2 \tanh^2 \left(\frac{\phi}{\sqrt{6\alpha} M_{\text{Pl}}} \right) + c_3 \tanh^3 \left(\frac{\phi}{\sqrt{6\alpha} M_{\text{Pl}}} \right) \right]^2.$$

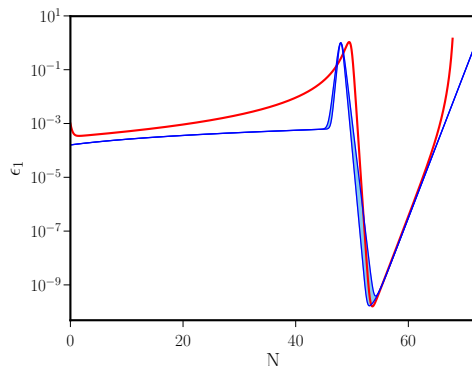
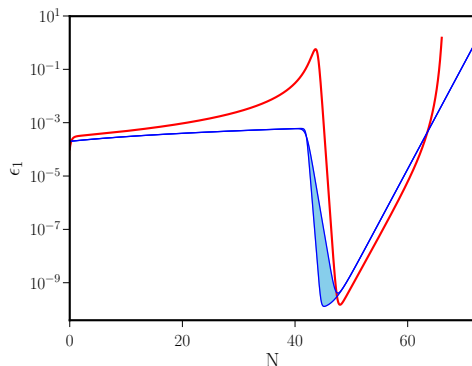
⁸D. Roberts, A. R. Liddle and D. H. Lyth, Phys. Rev. D **51**, 4122 (1995);

R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **01**, 009 (2009);

I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).



Reconstructing scenarios of ultra slow roll and punctuated inflation



Behavior of the first slow roll parameter $\epsilon_1(N)$ leading to ultra slow and punctuated inflation⁹:

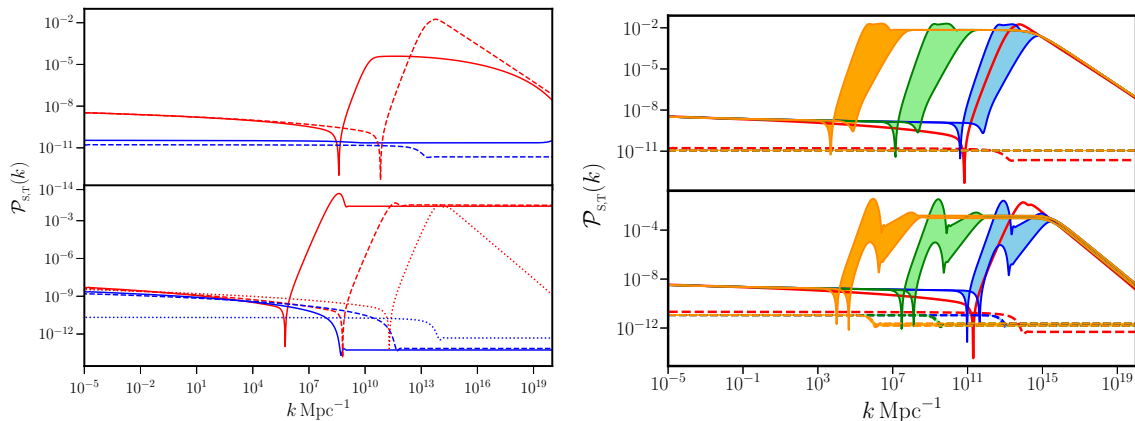
$$\text{RS I} : \epsilon_1^{\text{I}}(N) = [\epsilon_{1a} (1 + \epsilon_{2a} N)] \left[1 - \tanh \left(\frac{N - N_1}{\Delta N_1} \right) \right] + \epsilon_{1b} + \exp \left(\frac{N - N_2}{\Delta N_2} \right),$$

$$\text{RS II} : \epsilon_1^{\text{II}}(N) = \epsilon_1^{\text{I}}(N) + \cosh^{-2} \left(\frac{N - N_1}{\Delta N_1} \right).$$

⁹H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).



Power spectra in the inflationary models and reconstructed scenarios



The scalar and the tensor power spectra arising in the various inflationary models (in red and blue on the left) and the reconstructed scenarios (in blue, green and orange, on the right)¹⁰.

¹⁰H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, *Phys. Rev. D* **103**, 083510 (2021).



The two field model of interest

It has been noticed that two scalar fields ϕ and χ governed by the following action:

$$S[\phi, \chi] = \int d^4x \sqrt{-g} \left[-\frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{f(\phi)}{2} \partial^\mu \chi \partial_\mu \chi - V(\phi, \chi) \right]$$

described by a potential such as

$$V(\phi, \chi) = V_0 \frac{\phi^2}{\phi_0^2 + \phi^2} + \frac{m_\chi^2}{2} \chi^2$$

and the non-canonical coupling functions

$$f_1(\phi) = e^{2b_1 \phi} \quad \text{or} \quad f_2(\phi) = e^{2b_2 \phi^2}$$

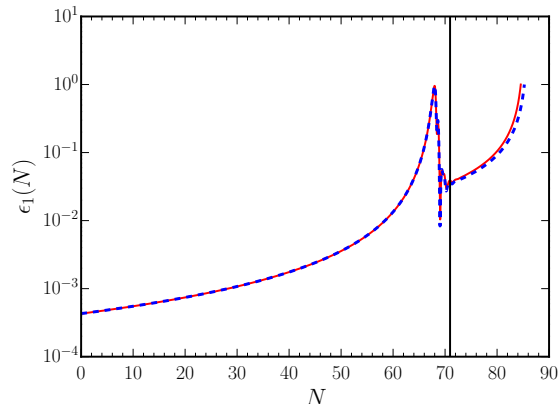
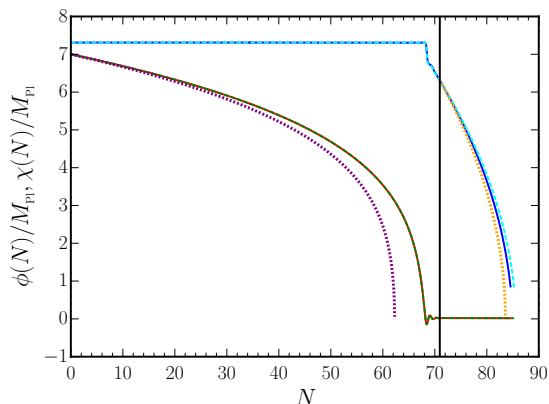
can lead to features in the scalar power spectrum¹¹.

[▶ Back to magnetogenesis](#)

¹¹M. Braglia, D. K. Hazra, L. Sriramkumar and F. Finelli, JCAP **08** 025 (2020).



Behavior of the scalar fields and the first slow roll parameter

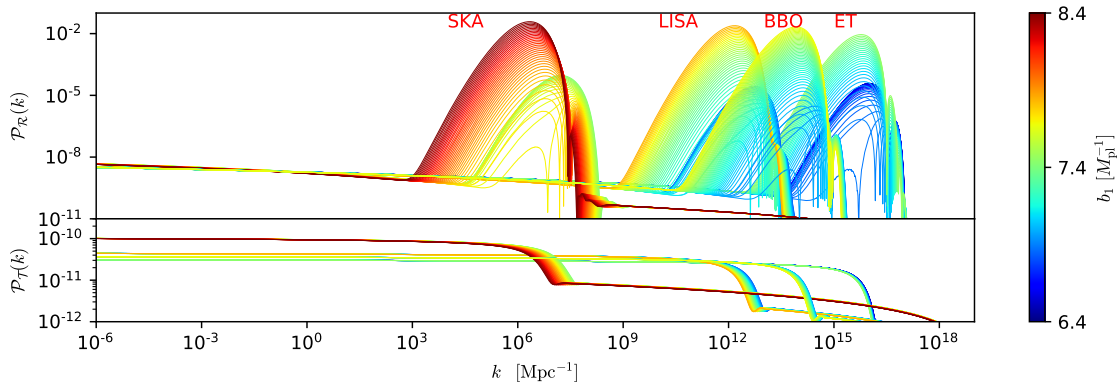


Behavior of the two scalar fields ϕ and χ (in blue and red, on the left) and the first slow roll parameter ϵ_1 (on the right) in the two field model of our interest¹². Note that there arises a turn in the field space around $N = 70$, when the first slow roll parameter begins to decrease before increasing again, leading to the termination of inflation.

¹²M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).



Enhanced power on small scales in two field models



The scalar (on top) and the tensor (at the bottom) power spectra evaluated at the end of inflation have been plotted for a few different sets of initial conditions for the fields and a range of values of the parameter b_1 ¹³.

¹³ M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

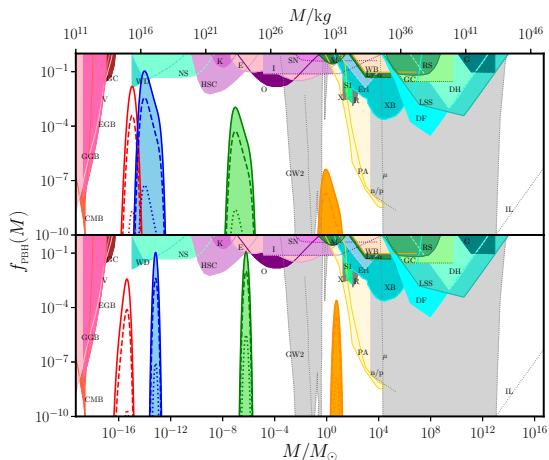


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$f_{\text{PBH}}(M)$ in ultra slow roll and punctuated inflation

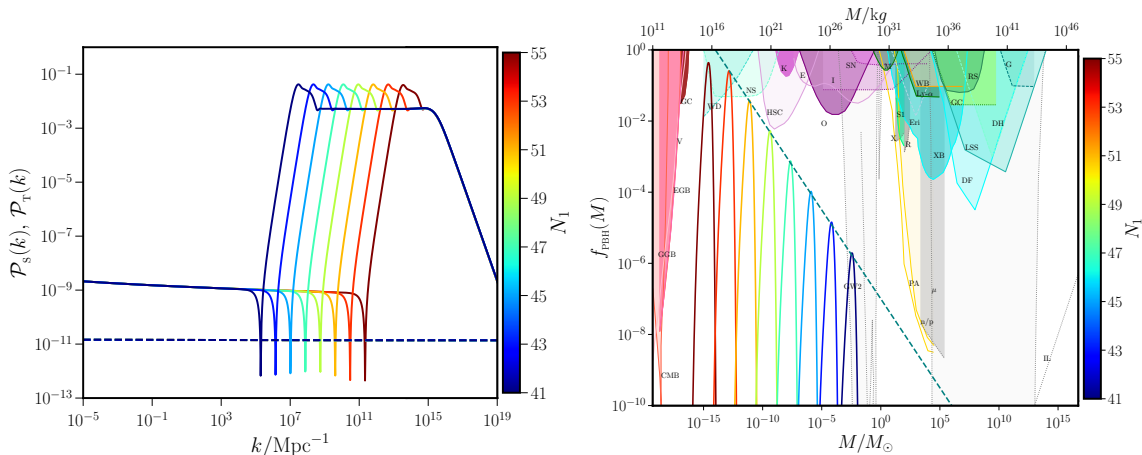


The fraction of PBHs contributing to the dark matter density today $f_{\text{PBH}}(M)$ has been plotted for the various models and scenarios of interest, viz. UR2 and RS1 (on top, in red and blue) and PI3 and RS2 (at the bottom, in red and blue)¹⁴.

¹⁴H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).



Illustrating the $f_{\text{PBH}}(M) \propto M^{-1/2}$ behavior

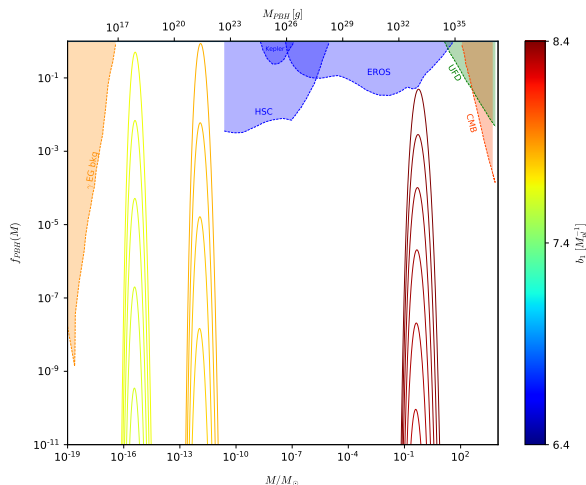


The quantity $f_{\text{PBH}}(M)$ (on the right) corresponding to scalar spectra generated in the reconstructed scenarios (on the left) have been illustrated. We find that $f_{\text{PBH}}(M)$ behaves as $M^{-1/2}$ (in dashed teal, on the right), as expected¹⁵.

¹⁵H. V. Ragavendra and L. Sriramkumar, *Galaxies* **11**, 34 (2023).



$f_{\text{PBH}}(M)$ in the two field model

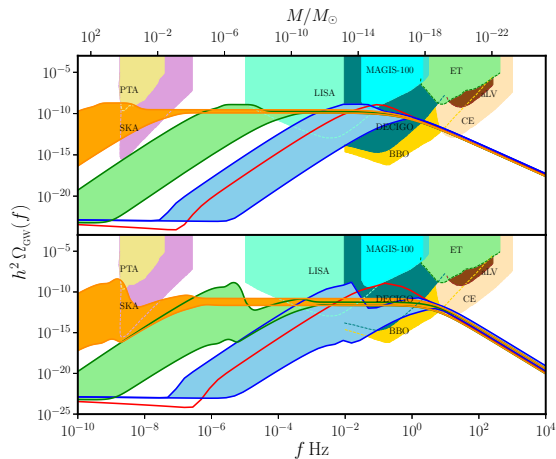


The fraction of PBHs contributing to the dark matter density today $f_{\text{PBH}}(M)$ in the two field model of our interest¹⁶.

¹⁶M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).



$\Omega_{\text{GW}}(f)$ in ultra slow roll and punctuated inflation

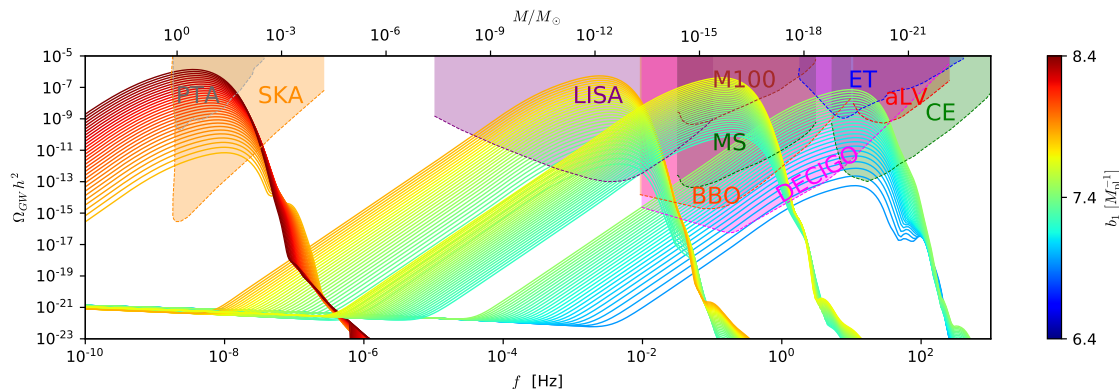


The dimensionless density parameter Ω_{GW} arising in the models and reconstructed scenarios of UR2 and RS1 (in red and blue, on top) as well as PI3 and RS2 (in red and blue, at the bottom) have been plotted as a function of the frequency f ¹⁷.

¹⁷H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).



$\Omega_{\text{GW}}(f)$ in the two field model



The dimensionless density parameter $\Omega_{\text{GW}}(f)$ arising in the two field model has been plotted as function of frequency for a set of initial conditions for the background fields as well as a range of values of the parameter b_1 ¹⁸.

¹⁸ M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).



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Template bispectra

For comparison with the observations, the scalar bispectrum is often expressed in terms of the parameters f_{NL}^{loc} , f_{NL}^{eq} and f_{NL}^{orth} as follows:

$$\mathcal{B}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = f_{NL}^{loc} \mathcal{B}_{loc}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + f_{NL}^{eq} \mathcal{B}_{eq}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + f_{NL}^{orth} \mathcal{B}_{orth}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3).$$

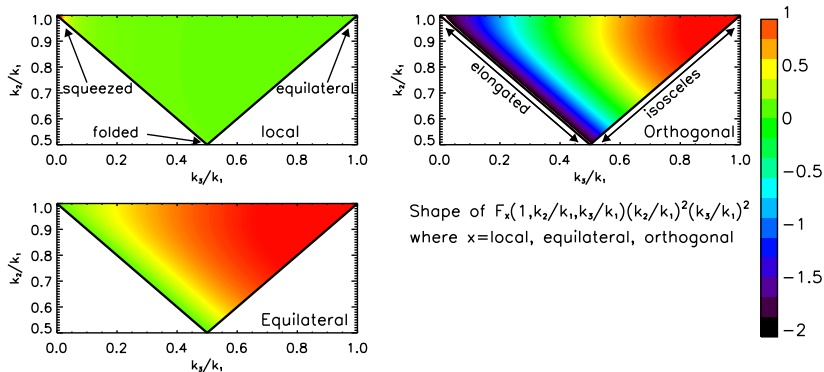


Illustration of the three template basis bispectra¹⁹.

¹⁹E. Komatsu, *Class. Quantum Grav.* **27**, 124010 (2010).



Constraints on the scalar non-Gaussianity parameters

The constraints on the primordial values of the non-Gaussianity parameters from the Planck data are as follows²⁰:

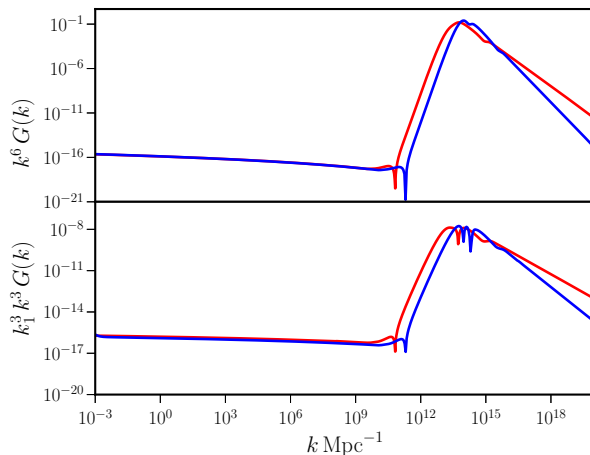
$$\begin{aligned}f_{\text{NL}}^{\text{loc}} &= -0.9 \pm 5.1, \\f_{\text{NL}}^{\text{eq}} &= -26 \pm 47, \\f_{\text{NL}}^{\text{ortho}} &= -38 \pm 24.\end{aligned}$$

These constraints imply that slowly rolling single field models involving the canonical scalar field which are favored by the data at the level of power spectra are also consistent with the data at the level of non-Gaussianities.

²⁰Planck Collaboration (Y. Akrami *et al.*), *Astron. Astrophys.* **641**, A9 (2020).



The scalar bispectrum in ultra slow roll and punctuated inflation

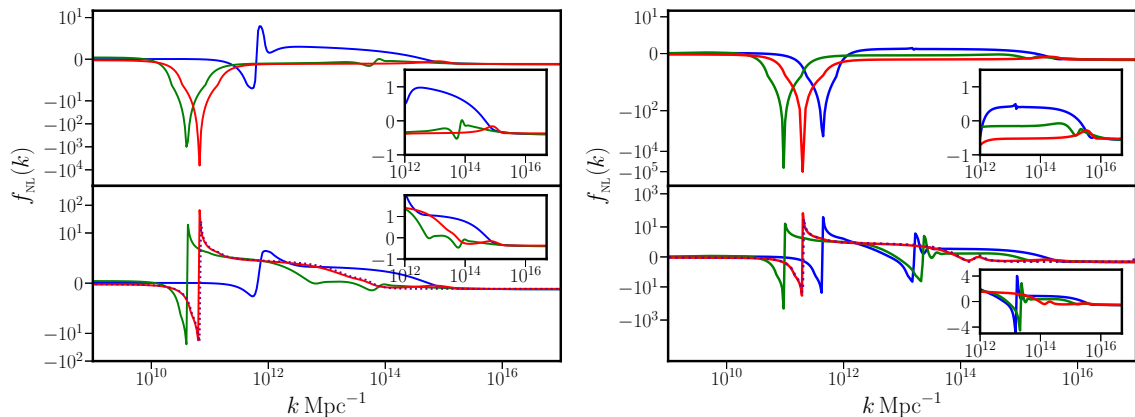


The amplitude of the dimensionless scalar bispectra has been plotted in the equilateral (on top) and squeezed limits (at the bottom) for the models UR2 (in red) and PI3 (in blue). The bispectra have approximately the same shape as the corresponding power spectra²¹

²¹H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, *Phys. Rev. D* **103**, 083510 (2021).



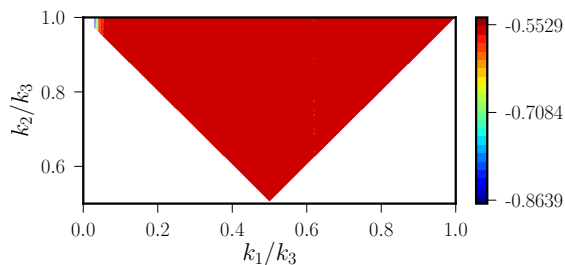
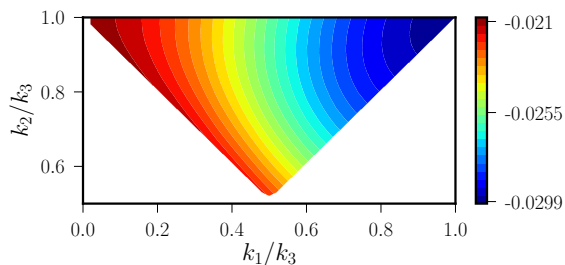
f_{NL} in ultra slow roll and punctuated inflation



The scalar non-Gaussianity parameter f_{NL} has been plotted in the equilateral (on top) and the squeezed (at the bottom) limits for the models of USR2 and PI3 (in red, on the left and the right) and the reconstructed scenarios RS1 and RS2 (in blue and green, on the left and the right).



Complete shape of f_{NL} in ultra slow roll inflation



The shape of the scalar non-Gaussianity parameter f_{NL} in a ultra slow roll scenario around the pivot scale (on the left) and around the peak in the scalar power spectrum (on the right)²².

²²H. V. Ragavendra and L. Sriramkumar, *Galaxies* **11**, 34 (2023).

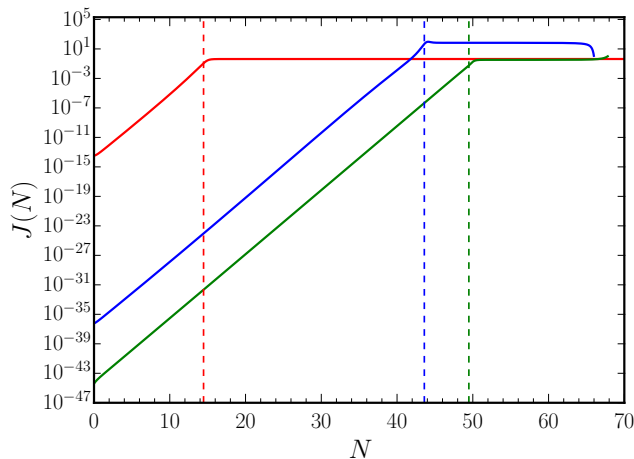


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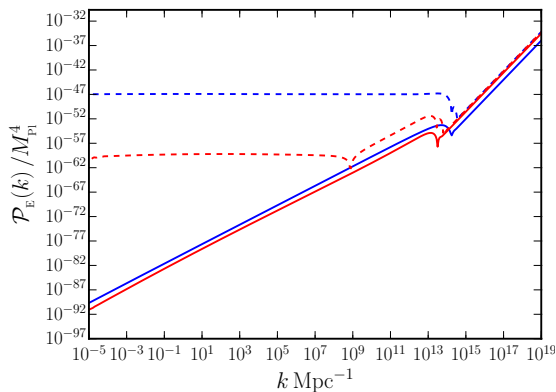
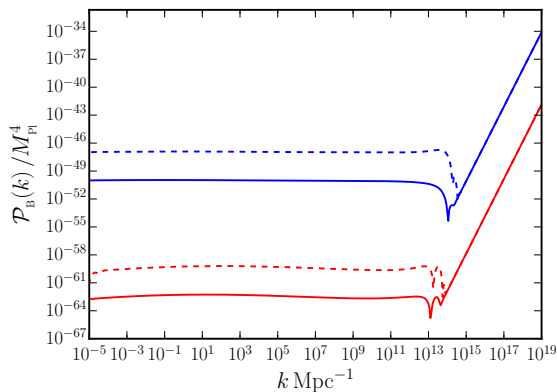
Behavior of the coupling function in models permitting ultra slow roll



The evolution of the non-minimal coupling function J in the Lagrangian density $\mathcal{L} \propto J^2(\phi) F_{\mu\nu} F^{\mu\nu}$ has been illustrated in inflationary models leading to an epoch of ultra slow roll. Note that the coupling function does not change appreciably once ultra slow roll sets in (indicated by the vertical lines).



Resulting spectra of the electromagnetic fields

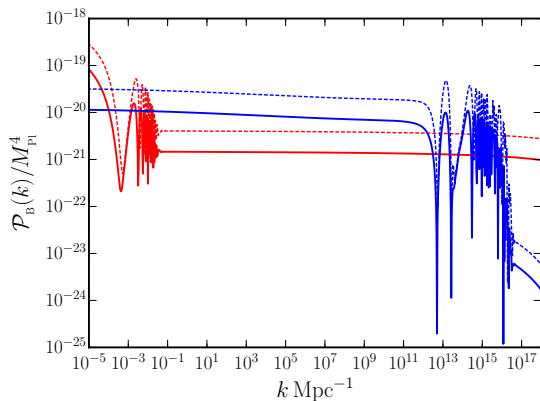
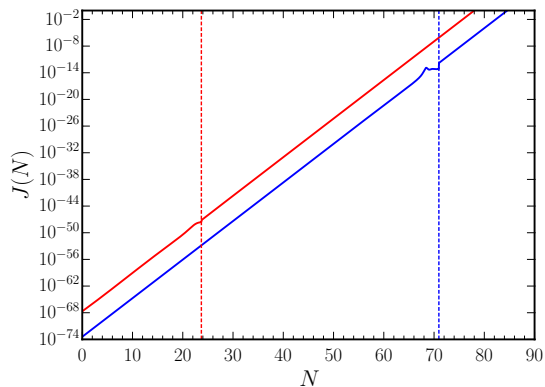


The spectra of the magnetic (on the left) and electric (on the right) fields arising in two inflationary models permitting a period of ultra slow roll (at late times) have been plotted in the non-helical (as solid lines) and helical (as dashed lines) cases²³.

²³S. Tripathy, D. Chowdhury, R. K. Jain and L. Sriramkumar, *Phys. Rev. D* **105**, 063519 (2022).



Circumventing the challenge in two field models



The evolution of the non-conformal coupling J (on the left) and the corresponding spectrum of magnetic field (on the right) arising in the two field inflationary model leading to features on large (in red) and on small (in blue) scales²⁴. The vertical lines indicate the time when the turn in the field space takes place.

► The two field model



²⁴S. Tripathy, D. Chowdhury, H. V. Ragavendra, R. K. Jain and L. Sriramkumar, Phys. Rev. D **107**, 043501 (2023).

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Outlook

There are many related effects, with strong observational implications, that are being examined currently. These include

- ◆ Imprints on the power and bi-spectra of neutral hydrogen²⁵,
- ◆ Effects of non-Gaussianities on the formation of PBHs²⁶,
- ◆ Effects of non-Gaussianities on secondary GWs²⁷,
- ◆ Loop corrections to the primordial power spectrum²⁸.

²⁵ S. Balaji, H. V. Ragavendra, S. K. Sethi, J. Silk and L. Sriramkumar, *Phys. Rev. Lett.* **129**, 261301 (2022).

²⁶ M. Taoso and A. Urbano, *JCAP* **08**, 016 (2021);
T. Matsubara and M. Sasaki, *JCAP* **10**, 094 (2022).

²⁷ P. Adshead, K. D. Lozanov, Z. J. Weiner, *JCAP* **10**, 080 (2021);
H. V. Ragavendra, *Phys. Rev. D* **105**, 063533 (2022).

²⁸ J. Kristiano and J. Yokoyama, [arXiv:2211.03395 \[hep-th\]](https://arxiv.org/abs/2211.03395);
K. Inomata, M. Braglia and X. Chen, [arXiv:2211.02586 \[astro-ph.CO\]](https://arxiv.org/abs/2211.02586);
A. Riotto, [arXiv:2301.00599 \[astro-ph.CO\]](https://arxiv.org/abs/2301.00599).



Talk based on

- ◆ M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, *Generating PBHs and small-scale GWs in two-field models of inflation*, JCAP **08**, 001 (2020) [arXiv:2005.02895 [astro-ph.CO]].
- ◆ H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, *PBHs and secondary GWs from ultra slow roll and punctuated inflation*, Phys. Rev. D **103**, 083510 (2021) [arXiv:2008.12202 [astro-ph.CO]].
- ◆ S. Tripathy, D. Chowdhury, R. K. Jain and L. Sriramkumar, *Challenges in the choice of the nonconformal coupling function in inflationary magnetogenesis*, Phys. Rev. D **105**, 063519 (2022) [arXiv:2111.01478 [astro-ph.CO]].
- ◆ S. Tripathy, D. Chowdhury, H. V. Ragavendra, R. K. Jain and L. Sriramkumar, *Circumventing the challenges in the choice of the non-conformal coupling function in inflationary magnetogenesis*, Phys. Rev. D **107**, 043501 (2023) [arXiv:2211.05834 [astro-ph.CO]].
- ◆ H. V. Ragavendra and L. Sriramkumar, *Observational imprints of enhanced scalar power on small scales in ultra slow roll inflation and associated non-Gaussianities*, Galaxies, **11**, 34 (2023) [arXiv:2301.08887 [astro-ph.CO]].



Thank you for your attention