Inflationary magnetogenesis
— Effects due to non-trivial dynamics —

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Plan of the talk

This talk is based on. . .

- S. Tripathy, D. Chowdhury, R. K. Jain and L. Sriramkumar, *Challenges in the choice of the nonconformal coupling function in inflationary magnetogenesis*, Phys. Rev. D **105**, 063519 (2022) [arXiv:2111.01478 [astro-ph.CO]].
- S. Tripathy, D. Chowdhury, H. V. Ragavendra, R. K. Jain and L. Sriramkumar, *Circumventing the challenges in the choice of the non-conformal coupling function in inflationary magnetogenesis*, Phys. Rev. D **107**, 043501 (2023) [arXiv:2211.05834 [astro-ph.CO]].
- S. Tripathy, R. N. Raveendran, K. Parattu and L. Sriramkumar, Amplifying quantum discord during inflationary magnetogenesis through violation of parity, Phys. Rev. D **108**, 123512 (2023) [arXiv:2306.16168 [gr-qc]].

Plan of the talk

Observational evidence for magnetic fields

Magnetic fields are ubiquitous in the universe. They are observed at different strengths over a wide range of scales.

- In galaxies, the strength of the observed magnetic fields is $\mathcal{O}(10^{-6}\,\mathrm{G})$, which is coherent over scales of 1–$10\,\mathrm{Kpc}$[1].
- In clusters of galaxies, the strength of the magnetic fields is $\mathcal{O}(10^{-7}$–$10^{-6}\,\mathrm{G})$ with a coherent length of $10\,\mathrm{Kpc}$–$1\,\mathrm{Mpc}$[2].
- In the intergalactic medium, the strength of the magnetic fields is greater than $10^{-16}\,\mathrm{G}$, which is coherent on scales above $1\,\mathrm{Mpc}$[3].
- The observations of the anisotropies in the cosmic microwave background (CMB) constrain the magnetic fields at the scale of $1\,\mathrm{Mpc}$ to be less than $10^{-9}\,\mathrm{G}$[4].

While astrophysical processes may be sufficient to explain the origin of magnetic fields in galaxies and clusters of galaxies, *one may have to turn to cosmological phenomena to explain the magnetic fields observed in voids*.

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[1] R. Beck, Space Sci. Rev. **99**, 243 (2001).

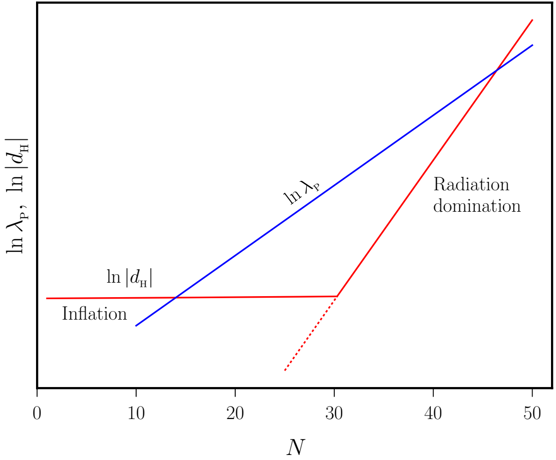
[2] See, for instance, T. E. Clarke, P. P. Kronberg and H. Böhringer, Astrophys. J. **547**, L111 (2001).

[3] A. Neronov and I. Vovk, Science **328**, 73 (2010).

[4] Planck Collaboration (P. A. R. Ade *et al.*), Astron. Astrophys. **594**, A19 (2016).

Plan of the talk

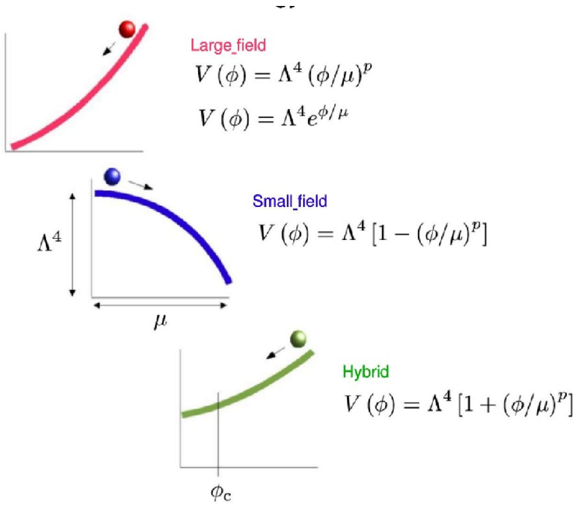
Bringing the modes inside the Hubble radius



The physical wavelength $\lambda_{\rm P} \propto a$ (in blue) and the Hubble radius $d_{\rm H} = H^{-1}$ (in red) in the inflationary scenario[5]. The scale factor is expressed in terms of e-folds N as $a(N) \propto {\rm e}^{N}$.

[5] See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.

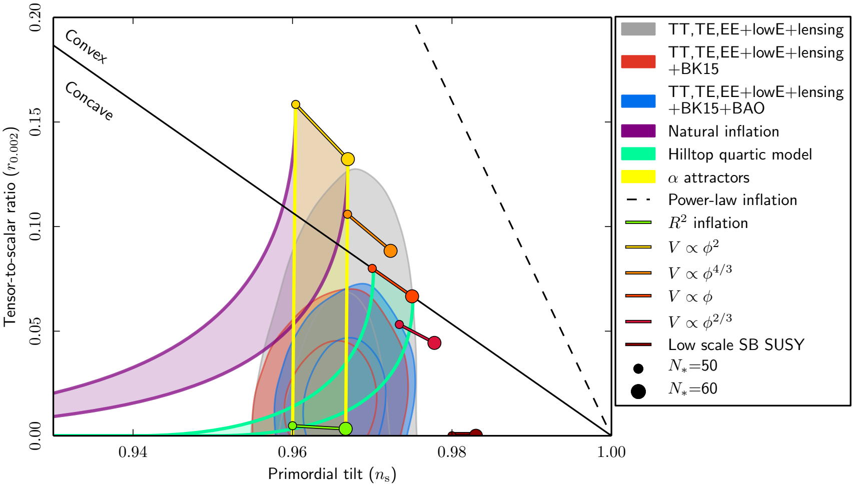
Variety of potentials can drive inflation



A variety of scalar field potentials have been considered to drive inflation[6]. Often, these potentials are classified as small field, large field and hybrid models.

[6] Image from W. Kinney, astro-ph/0301448.

Performance of inflationary models in the n_s-r plane



Joint constraints on n_s and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of some of the popular inflationary models[7].

[7] Planck Collaboration (Y. Akrami *et al.*), Astron. Astrophys. **641**, A10 (2020).

Plan of the talk

Non-conformally coupled electromagnetic fields

We shall assume that the electromagnetic field is described by the action[8]

$$S[A^\mu] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\sqrt{-g}\, J^2(\phi)\, F_{\mu\nu}\, F^{\mu\nu},$$

where $J(\phi)$ denotes the coupling function and the field tensor $F_{\mu\nu}$ is expressed in terms of the vector potential A_μ as $F_{\mu\nu} = (\partial_\mu\, A_\nu - \partial_\nu\, A_\mu)$.

On working in the Coulomb gauge wherein $A_\eta = 0$ and $\partial_i\, A^i = 0$, one finds that the Fourier modes, say, $\bar{A}_k$, describing the vector potential satisfy the differential equation

$$\bar{A}_k'' + 2\,\frac{J'}{J}\,\bar{A}_k' + k^2\,\bar{A}_k = 0.$$

If we write $\bar{A}_k = \mathcal{A}_k/J$, then this equation reduces to

$$\mathcal{A}_k'' + \left(k^2 - \frac{J''}{J}\right)\mathcal{A}_k = 0.$$

[8] See, for instance, J. Martin and J. Yokoyama, JCAP **01**, 025 (2008); K. Subramanian, Astron. Nachr. **331**, 110 (2010).

Quantization and power spectra of the electromagnetic field

For each comoving wave vector $\boldsymbol{k}$, we can define the right-handed orthonormal basis $(\hat{\varepsilon}_1^{\boldsymbol{k}}, \hat{\varepsilon}_2^{\boldsymbol{k}}, \hat{\boldsymbol{k}})$ which satisfy the relations

$$\hat{\varepsilon}_1^{\boldsymbol{k}} \cdot \hat{\varepsilon}_1^{\boldsymbol{k}} = \hat{\varepsilon}_2^{\boldsymbol{k}} \cdot \hat{\varepsilon}_2^{\boldsymbol{k}} = 1,\; \hat{\varepsilon}_1^{\boldsymbol{k}} \cdot \hat{\varepsilon}_2^{\boldsymbol{k}} = 0,\; \hat{\varepsilon}_1^{\boldsymbol{k}} \times \hat{\varepsilon}_2^{\boldsymbol{k}} = \hat{\boldsymbol{k}},\; \hat{\boldsymbol{k}} \cdot \hat{\varepsilon}_1^{\boldsymbol{k}} = \hat{\boldsymbol{k}} \cdot \hat{\varepsilon}_2^{\boldsymbol{k}} = 0,\; \hat{\varepsilon}_1^{-\boldsymbol{k}} = -\hat{\varepsilon}_1^{\boldsymbol{k}},\; \hat{\varepsilon}_2^{-\boldsymbol{k}} = \hat{\varepsilon}_2^{\boldsymbol{k}}.$$

On quantization, the vector potential $\hat{A}_i$ can be decomposed as follows:

$$\hat{A}_i\,(\eta, \boldsymbol{x}) = \sqrt{4\,\pi} \int \frac{\mathrm{d}^3\,\boldsymbol{k}}{(2\,\pi)^{3/2}} \sum_{\lambda=1}^{2} \hat{\varepsilon}_{\lambda i}^{\boldsymbol{k}} \left[\hat{b}_{\boldsymbol{k}}^{\lambda}\, \bar{A}_k^{\lambda}(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{b}_{\boldsymbol{k}}^{\lambda\dagger}\, \bar{A}_k^{\lambda *}(\eta)\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right],$$

where $\hat{\varepsilon}_{\lambda i}^{\boldsymbol{k}}$ represent components of the polarization vectors and the summation is over to the two orthonormal transverse polarizations.

The power spectra associated with the magnetic and electric fields are defined to be[9]

$$\mathcal{P}_{_{\mathrm{B}}}(k) = \frac{\mathrm{d}\langle 0|\hat{\rho}_{_{\mathrm{B}}}|0\rangle}{\mathrm{d}\ln k} = \frac{J^2(\eta)}{2\,\pi^2}\,\frac{k^5}{a^4(\eta)}\,|\bar{A}_k(\eta)|^2 = \frac{1}{2\,\pi^2}\,\frac{k^5}{a^4(\eta)}\,|\mathcal{A}_k(\eta)|^2,$$

$$\mathcal{P}_{_{\mathrm{E}}}(k) = \frac{\mathrm{d}\langle 0|\hat{\rho}_{_{\mathrm{E}}}|0\rangle}{\mathrm{d}\ln k} = \frac{J^2(\eta)}{2\,\pi^2}\,\frac{k^3}{a^4(\eta)}\,|\bar{A}_k'(\eta)|^2 = \frac{1}{2\,\pi^2}\,\frac{k^3}{a^4(\eta)}\,\left|\mathcal{A}_k'(\eta) - \frac{J'(\eta)}{J(\eta)}\,\mathcal{A}_k(\eta)\right|^2.$$

[9] J. Martin and J. Yokoyama, JCAP **01**, 025 (2008); K. Subramanian, Astron. Nachr. **331**, 110 (2010).

Initial conditions and normal modes in de Sitter inflation

Let us now consider de Sitter inflation wherein the scale factor is given by $a(\eta) = -1/(H_{\rm I}\,\eta)$, with $H_{\rm I}$ denoting the constant Hubble parameter.

Typically, the non-conformal coupling function J is assumed to depend on the scale factor as follows:

$$J(\eta) = \left[\frac{a(\eta)}{a(\eta_{\rm e})}\right]^{n} = \left(\frac{\eta}{\eta_{\rm e}}\right)^{-n},$$

where $\eta_{\rm e}$ denotes the conformal time at the end of inflation.

The Bunch-Davies initial conditions on the electromagnetic modes $\mathcal{A}_k$ can be imposed in the limit $k \gg \sqrt{J''/J}$, which, for the above coupling function, corresponds to the modes being in the sub-Hubble domain at early times.

For the above coupling function, the solution that satisfies the Bunch-Davies initial conditions is given by

$$\mathcal{A}_k(\eta) = \sqrt{-\frac{\pi\,\eta}{4}}\,{\rm e}^{i\,(n+1)\,\pi/2}\,H_\nu^{(1)}(-k\,\eta),$$

where $\nu = n + (1/2)$ and $H_\nu^{(1)}(z)$ denotes the Hankel function of the first kind.

Power spectra of electromagnetic fields in de Sitter inflation

The spectra of the electromagnetic fields can be evaluated in the limit $k \ll \sqrt{J''/J}$, which corresponds to the super-Hubble limit for our choice of the coupling function.

In the limit $(-k\,\eta_{\rm e}) \ll 1$, the spectra of the magnetic and electric fields $\mathcal{P}_{_{\rm B}}(k)$ and $\mathcal{P}_{_{\rm E}}(k)$ can be obtained to be[10]

$$\mathcal{P}_{_{\rm B}}(k) = \frac{H_{_{\rm I}}^4}{8\,\pi}\,\mathcal{F}(m)\,(-k\,\eta_{\rm e})^{2\,m+6}, \quad \mathcal{P}_{_{\rm E}}(k) = \frac{H_{_{\rm I}}^4}{8\,\pi}\,\mathcal{G}(m)\,(-k\,\eta_{\rm e})^{2\,m+4},$$

where we the quantities $\mathcal{F}(m)$ and $\mathcal{G}(m)$ are given by

$$\mathcal{F}(m) = \frac{1}{2^{2\,m+1}\,\cos^2(m\,\pi)\,\Gamma^2(m+3/2)}, \quad \mathcal{G}(m) = \frac{1}{2^{2\,m-1}\,\cos^2(m\,\pi)\,\Gamma^2(m+1/2)},$$

with

$m=n$ for $n<-1/2$ and $m=-(n+1)$ for $n>-1/2$, in the case of $\mathcal{P}_{_{\rm B}}(k)$,
$m=n$ for $n<1/2$ and $m=-(n-1)$ for $n>1/2$, in the case of $\mathcal{P}_{_{\rm E}}(k)$.

[10] See, for instance, J. Martin and J. Yokoyama, JCAP **01**, 025 (2008); K. Subramanian, Astron. Nachr. **331**, 110 (2010).

Scale invariant spectra of magnetic fields and backreaction

The spectral indices for the magnetic and electric fields, say, $n_{\rm B}$ and $n_{\rm E}$, can be written as

$$n_{\rm B} = (2\,n+6) \text{ for } n < -1/2, \quad n_{\rm B} = (4-2\,n) \text{ for } n > -1/2,$$
$$n_{\rm E} = (2\,n+4) \text{ for } n < 1/2, \quad n_{\rm E} = (6-2\,n) \text{ for } n > 1/2.$$

To be consistent with observations, the spectrum of the magnetic field is expected to be nearly scale invariant and, evidently, this is possible when $n \simeq -3$ or when $n \simeq 2$. In these cases, it is clear that $n_{\rm E} \simeq -2$ and $n_{\rm E} \simeq 2$, respectively.

At late times, $n_{\rm E} \simeq -2$ implies that the energy density in the electric field is significant leading to a large backreaction.

In order to avoid such an issue, one often considers the $n = 2$ case to lead to a scale invariant magnetic field with negligible backreaction due to the electric field. Note that, in this case, the power spectra reduce to the following simple forms (with $k_{\rm e} = -1/\eta_{\rm e}$):

$$\mathcal{P}_{\rm B}(k) = \frac{9\,H_{\rm I}^4}{4\,\pi^2}, \quad \mathcal{P}_{\rm E}(k) = \frac{H_{\rm I}^4}{4\,\pi^2}\,\left(\frac{k}{k_{\rm e}}\right)^2.$$

Helical electromagnetic fields

The action we had considered earlier can be extended to include a parity violating term as follows[11]:

$$S[A^\mu] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\,\sqrt{-g}\,\left[J^2(\phi)\,F_{\mu\nu}\,F^{\mu\nu} - \frac{\gamma}{2}\,I^2(\phi)\,F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right],$$

where $\widetilde{F}^{\mu\nu} = (\epsilon^{\mu\nu\alpha\beta}/\sqrt{-g})\,F_{\alpha\beta}$, with $\epsilon^{\mu\nu\alpha\beta}$ being the completely anti-symmetric Levi-Civita tensor, and γ is a constant.

In such a case, we can define the orthonormal basis of vectors $(\hat{\varepsilon}^{\boldsymbol{k}}_{+}, \hat{\varepsilon}^{\boldsymbol{k}}_{-}, \hat{\boldsymbol{k}})$, where the vectors $\hat{\varepsilon}^{\boldsymbol{k}}_{\pm}$ are defined as

$$\hat{\varepsilon}^{\boldsymbol{k}}_{\pm} = \frac{1}{\sqrt{2}}\,(\hat{\varepsilon}^{\boldsymbol{k}}_{1} \pm i\,\hat{\varepsilon}^{\boldsymbol{k}}_{2}).$$

One can show that these vectors satisfy the following properties:

$$\hat{\varepsilon}^{\boldsymbol{k}}_{+}\cdot\hat{\varepsilon}^{\boldsymbol{k}*}_{+} = 1,\, \hat{\varepsilon}^{\boldsymbol{k}}_{-}\cdot\hat{\varepsilon}^{\boldsymbol{k}*}_{-} = 1,\, \hat{\varepsilon}^{\boldsymbol{k}}_{+}\cdot\hat{\varepsilon}^{\boldsymbol{k}*}_{-} = 0,\, \hat{\varepsilon}^{\boldsymbol{k}*}_{\pm} = \hat{\varepsilon}^{\boldsymbol{k}}_{\mp},\, \hat{\varepsilon}^{-\boldsymbol{k}}_{\pm} = -\hat{\varepsilon}^{\boldsymbol{k}}_{\mp},\, i\,\hat{\boldsymbol{k}}\times\hat{\varepsilon}^{\boldsymbol{k}}_{\pm} = \pm\hat{\varepsilon}^{\boldsymbol{k}}_{\pm}.$$

[11] M. M. Anber and L. Sorbo, JCAP **10**, 018 (2006);
C. Caprini and L. Sorbo, JCAP **10**, 056 (2014);
D. Chowdhury, L. Sriramkumar and M. Kamionkowski, JCAP **10**, 031 (2018).

Quantization in the helical basis

In the helical basis, on quantization in the Coulomb gauge, the vector potential $\hat{A}_i$ can be Fourier decomposed as follows[12]:

$$\hat{A}_i(\eta, \boldsymbol{x}) = \sqrt{4\,\pi} \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\,\pi)^{3/2}} \sum_{\sigma=\pm} \left[\varepsilon^{\boldsymbol{k}}_{\sigma i}\, \hat{b}^{\sigma}_{\boldsymbol{k}}\, \bar{A}^{\sigma}_{k}(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \varepsilon^{\boldsymbol{k}\,*}_{\sigma i}\, \hat{b}^{\sigma\dagger}_{\boldsymbol{k}}\, \bar{A}^{\sigma *}_{k}(\eta)\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right],$$

where the Fourier modes $\bar{A}^{\sigma}_{k}$ satisfy the differential equation

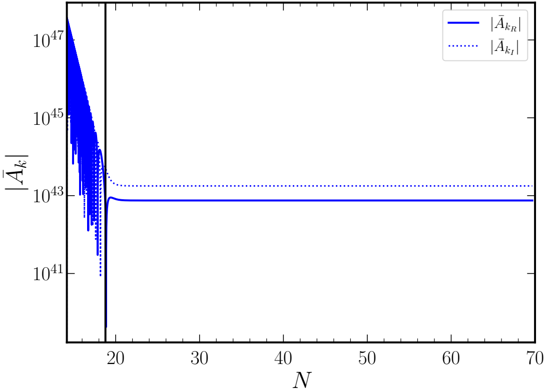
$$\bar{A}^{\sigma\,\prime\prime}_{k} + 2\,\frac{J'}{J}\,\bar{A}^{\sigma\,\prime}_{k} + \left(k^2 + \frac{\sigma\,\gamma\,k}{J^2}\,\frac{\mathrm{d}I^{\,2}}{\mathrm{d}\,\eta}\right)\bar{A}^{\sigma}_{k} = 0.$$

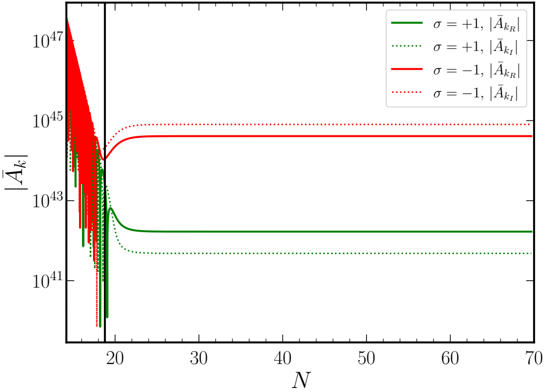
Note that $\sigma = \pm 1$, and this causes difference in the evolution of the modes depending on I and γ. In terms of $\mathcal{A}^{\sigma}_{k} = J\,\bar{A}^{\sigma}_{k}$, the above equation reduces to

$$\mathcal{A}^{\sigma\,\prime\prime}_{k} + \left(k^2 + \frac{2\,\sigma\,\gamma\,k\,I\,I'}{J^2} - \frac{J''}{J}\right)\mathcal{A}^{\sigma}_{k} = 0.$$

[12] See, e.g., C. Caprini and L. Sorbo, JCAP **10**, 056 (2014); D. Chowdhury, L. Sriramkumar and M. Kamionkowski, JCAP **10**, 031 (2018).

Behavior of the non-helical and helical electromagnetic modes





Typical behavior of the real (as solid curves) and imaginary (as dotted curves) parts of the non-helical (in blue, on the left) and helical (in red and green, on the right) electromagnetic modes have been plotted as function of e-folds for a specific wave number. The vertical lines indicates the time when the wave number leaves the Hubble radius[13].

[13] D. Chowdhury, L. Sriramkumar and M. Kamionkowski, JCAP **10**, 031 (2018).

Power spectra of the helical electromagnetic fields

We shall hereafter focus on the case wherein $I = J$.

The power spectra of the magnetic and electric fields can be expressed in terms of the modes $\bar{A}_k^\sigma$ and the coupling function J as follows[14]:

$$\mathcal{P}_{\rm B}(k) = \frac{k^5}{4\,\pi^2}\,\frac{J^2}{a^4}\,\left[\left|\bar{A}_k^+\right|^2 + \left|\bar{A}_k^-\right|^2\right] = \frac{k^5}{4\,\pi^2\,a^4}\,\left[\left|\mathcal{A}_k^+\right|^2 + \left|\mathcal{A}_k^-\right|^2\right],$$

$$\mathcal{P}_{\rm E}(k) = \frac{k^3}{4\,\pi^2}\,\frac{J^2}{a^4}\,\left[\left|\bar{A}_k^{+\prime}\right|^2 + \left|\bar{A}_k^{-\prime}\right|^2\right] = \frac{k^3}{4\,\pi^2\,a^4}\,\left[\left|\mathcal{A}_k^{+\prime} - \frac{J'}{J}\mathcal{A}_k^+\right|^2 + \left|\mathcal{A}_k^{-\prime} - \frac{J'}{J}\mathcal{A}_k^-\right|^2\right].$$

[14] See, for example, C. Caprini and L. Sorbo, JCAP **10**, 056 (2014);
D. Chowdhury, L. Sriramkumar and M. Kamionkowski, JCAP **10**, 031 (2018);
R. Sharma, K. Subramanian and T. R. Seshadri, Phys. Rev. D **97**, 083503 (2018).

Normal modes and power spectra in de Sitter inflation

For the coupling function J in de Sitter inflation we had considered earlier, the solutions to the electromagnetic modes satisfying the Bunch-Davies initial conditions can be written as follows[15]:

$$\mathcal{A}_k^{\sigma}(\eta) = \frac{1}{\sqrt{2\,k}}\,\mathrm{e}^{-\pi\,\sigma\,n\,\gamma/2}\,W_{i\,\sigma\,n\,\gamma,\nu}(2\,i\,k\,\eta),$$

where $\nu = n + (1/2)$, and $W_{\lambda,\mu}(z)$ denotes the Whittaker function.

The resulting power spectra can be expressed as

$$\mathcal{P}_{_{\mathrm{B}}}(k) = \frac{H_{_{\mathrm{I}}}^4}{8\,\pi^2}\,\frac{\Gamma^2(|2\,n+1|)}{|\Gamma(\frac{1}{2}+i\,n\,\gamma+|n+\frac{1}{2}|)|^2}\,\frac{\cosh\,(n\,\pi\,\gamma)}{2^{|2\,n+1|-2}}\,(-k\,\eta_{\mathrm{e}})^{5-|2\,n+1|},$$

$$\mathcal{P}_{_{\mathrm{E}}}(k) = \frac{H_{_{\mathrm{I}}}^4}{4\,\pi^2}\,\frac{\Gamma^2(2\,|n|)}{|\Gamma(|n|+i\,n\,\gamma)|^2}\,\frac{\gamma^2}{1+\gamma^2}\,\frac{\cosh\,(n\,\pi\,\gamma)}{2^{2\,|n|-2}}\,(-k\,\eta_{\mathrm{e}})^{4-2\,|n|},$$

with the factor $\gamma^2/(1+\gamma^2)$ arising *only* for positive values of n. The spectral indices for the magnetic and electric fields are given by $n_{_{\mathrm{B}}} = 5 - |2\,n+1|$ and $n_{_{\mathrm{E}}} = 4 - 2\,|n|$.

[15] See, for instance, R. Sharma, K. Subramanian and T. R. Seshadri, Phys. Rev. D **97**, 083503 (2018).

Scale invariant amplitudes and the non-helical limit

When $n = 2$, we find that the spectra of the helical magnetic and electric fields, evaluated in the limit $(-k\,\eta_{\mathrm{e}}) \ll 1$, can be written as[16]

$$\mathcal{P}_{_{\mathrm{B}}}(k) = \frac{9\,H_{_{\mathrm{I}}}^4}{4\,\pi^2}\,f(\gamma),$$

$$\mathcal{P}_{_{\mathrm{E}}}(k) = \frac{9\,H_{_{\mathrm{I}}}^4}{4\,\pi^2}\,f(\gamma)\left[\gamma^2 - \frac{\sinh^2(2\,\pi\,\gamma)}{3\,\pi\,\left(1+\gamma^2\right)\,f(\gamma)}\,(-k\,\eta_{\mathrm{e}}) + \frac{1}{9}\,\left(1 + 23\,\gamma^2 + 40\,\gamma^4\right)\,(-k\,\eta_{\mathrm{e}})^2\right],$$

where the function $f(\gamma)$ is given by

$$f(\gamma) = \frac{\sinh\,(4\,\pi\,\gamma)}{4\,\pi\,\gamma\,\left(1 + 5\,\gamma^2 + 4\,\gamma^4\right)}.$$

Note that the non-helical case case corresponds to $\gamma = 0$.

Also, note that the power spectra in the helical case are enhanced when compared to the non-helical case.

[16] S. Tripathy, D. Chowdhury, R. K. Jain and L. Sriramkumar, Phys. Rev. D **105**, 063519 (2022).

Estimating the strengths of the magnetic fields at the present epoch I

In the case of *instantaneous* reheating, the spectrum of the magnetic field today, say, $\mathcal{P}_{_{\rm B}}^0(k)$, can be related to the spectrum $\mathcal{P}_{_{\rm B}}(k)$ at the end of inflation as follows:

$$\mathcal{P}_{_{\rm B}}^0(k) \simeq \mathcal{P}_{_{\rm B}}(k)\,\left(\frac{a_{\rm e}}{a_0}\right)^4,$$

where $a_{\rm e}$ is the scale factor at the end of inflation, while a_0 denotes the scale factor today.

Conservation of entropy post inflation allows us to write

$$\frac{a_0}{a_{\rm e}} = \left(\frac{g_{\rm s,e}}{g_{\rm s,0}}\right)^{1/3}\,\frac{T_{\rm e}}{T_0},$$

where $(T_{\rm e}, g_{\rm s,e})$ and $(T_0, g_{\rm s,0})$ denote the temperature and the effective number of relativistic degrees of freedom at the onset of the radiation dominated epoch and today.

Estimating the strengths of the magnetic fields at the present epoch II

If we consider $g_{\mathrm{e}} = 106.75$, since $g_0 = 3.36$ and $T_0 = 2.725\,\mathrm{K}$, we obtain that

$$\frac{a_0}{a_{\mathrm{e}}} \simeq 2.8 \times 10^{28} \left(\frac{H_{_{\mathrm{I}}}}{10^{-5}\,M_{_{\mathrm{Pl}}}}\right)^{1/2}.$$

Given the scale invariant amplitude for the magnetic field at the end of inflation in the $n = 2$, helical case, upon using the above result, we can estimate the present day strength of the magnetic field, say, B_0 (at any scale), to be[17]

$$B_0 \simeq 4.5 \times 10^{-12} \left(\frac{H_{_{\mathrm{I}}}}{10^{-5}\,M_{_{\mathrm{Pl}}}}\right) f^{1/2}(\gamma)\ \mathrm{G}.$$

[17] S. Tripathy, D. Chowdhury, R. K. Jain and L. Sriramkumar, Phys. Rev. D **105**, 063519 (2022).

Plan of the talk

Non-conformal coupling function in the quadratic potential

The first model we shall consider is the popular quadratic potential given by

$$V(\phi) = \frac{m^2}{2}\,\phi^2.$$

In such a potential, it is well known that, under the slow roll approximation, the evolution of the field in terms of the e-folds N can be expressed as

$$\phi^2(N) \simeq \phi_{\rm e}^2 + 4\,(N_{\rm e} - N)\,M_{_{\rm Pl}}^2,$$

where $\phi_{\rm e} \simeq \sqrt{2}\,M_{_{\rm Pl}}$ denotes the value of the field at the end of inflation at the e-fold $N_{\rm e}$.

Clearly, we can arrive at the form of $J(N)$ that we desire if we choose $J(\phi)$ to be[18]

$$J(\phi) = \exp\left[-\frac{n}{4\,M_{_{\rm Pl}}^2}\,(\phi^2 - \phi_{\rm e}^2)\right].$$

[18]See, for example, S. Kanno, J. Soda and M. Watanabe, JCAP **12**, 009 (2009); M. Watanabe, S. Kanno and J. Soda, Phys. Rev. Lett. **102**, 191302 (2009).

Non-conformal coupling function in the Starobinsky model

The other case that we shall discuss is the Starobinsky model described by the potential

$$V(\phi) = V_0\,\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{_{\rm Pl}}}\right)\right]^2.$$

The evolution of the field in the slow roll approximation is described by the expression

$$N - N_{\rm e} \simeq -\frac{3}{4}\left[\exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{_{\rm Pl}}}\right) - \exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi_{\rm e}}{M_{_{\rm Pl}}}\right) - \sqrt{\frac{2}{3}}\,\left(\frac{\phi}{M_{_{\rm Pl}}} - \frac{\phi_{\rm e}}{M_{_{\rm Pl}}}\right)\right],$$

where $\phi_{\rm e}$ is the value of the field at the end of inflation which is determined by the relation $\exp\,[\sqrt{(2/3)}\,\phi_{\rm e}/M_{_{\rm Pl}}] \simeq 1 + 2/\sqrt{3}$.

Therefore, to achieve the desired dependence of the coupling function on the scale factor, we can choose $J(\phi)$ in the model to be

$$J(\phi) = \exp\left\{-\frac{3\,n}{4}\left[\exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{_{\rm Pl}}}\right) - \exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi_{\rm e}}{M_{_{\rm Pl}}}\right) - \sqrt{\frac{2}{3}}\,\left(\frac{\phi}{M_{_{\rm Pl}}} - \frac{\phi_{\rm e}}{M_{_{\rm Pl}}}\right)\right]\right\}.$$

Strengths of helical electromagnetic fields in slow roll inflation

In the helical case, when $n = 2$, the amplitude of the spectra of the magnetic and electric fields can be expressed as

$$\frac{\mathcal{P}_{_{\rm B}}(k)}{M_{_{\rm Pl}}^4} \simeq \frac{9\,\pi^2}{16}\,\left(r\,A_{\rm s}\right)^2\,f(\gamma), \quad \frac{\mathcal{P}_{_{\rm E}}(k)}{M_{_{\rm Pl}}^4} \simeq \frac{\mathcal{P}_{_{\rm B}}(k)}{M_{_{\rm Pl}}^4}\,\gamma^2.$$

In these expressions, $A_{\rm s} = 2.1 \times 10^{-9}$ denotes the observed amplitude of the scalar power spectrum at the pivot scale and r represents the tensor-to-scalar ratio.

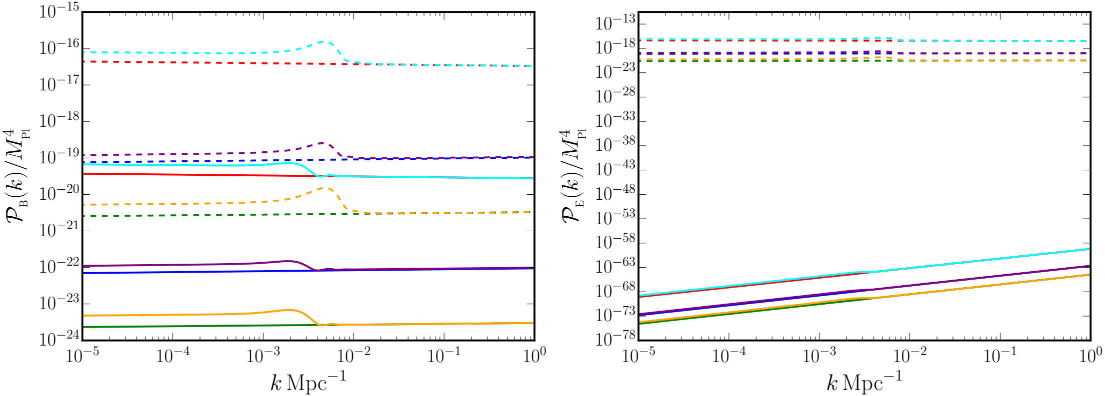
If we need to avoid backreaction due to the helical electromagnetic fields which have been generated, we require that[19]

$$\mathcal{P}_{_{\rm B}}(k) + \mathcal{P}_{_{\rm E}}(k) \ll \rho_{_{\rm I}} = 3\,H_{_{\rm I}}^2\,M_{_{\rm Pl}}^2.$$

Since we are considering inflationary models wherein $H_{_{\rm I}}/M_{_{\rm Pl}} \lesssim 10^{-5}$, the condition for avoiding backreaction leads to $f(\gamma)\,(1+\gamma^2) \lesssim 10^{10}$. This limits the value of γ to be $\gamma \lesssim 2.5$.

[19] See, for instance, T. Markkanen, S. Nurmi, S. Rasanen and V. Vennin, JCAP **06**, 035 (2017).

Spectra of electromagnetic fields in slow roll inflation



The spectra of the magnetic (on the left) and electric (on the right) fields in the non-helical (solid curves) and helical (dashed curves) cases, arising in three slow roll inflationary models, have been plotted (in red, blue and green) over the CMB scales. We have also plotted the corresponding spectra when a step has been introduced in these potentials (in cyan, purple and orange). We have set $\gamma = 1$ for which $f(\gamma) \simeq 10^3$.

Back to scalar power spectra with features
Back to coupling function in the second Starobinsky model

Plan of the talk

Potentials with a step

Given an inflationary model described by the potential $V(\phi)$, we can introduce a step in the potential as follows[20]:

$$V_{\rm step}(\phi) = V(\phi)\,\left[1+\alpha\,\tanh\left(\frac{\phi-\phi_0}{\Delta\phi}\right)\right],$$

where, evidently, ϕ_0, α and $\Delta\phi$ denote the location, the height and the width of the step.

[20] D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **10**, 008 (2010).

Suppressing power on large scales

In this context, the first example that we shall consider is another model due to Starobinsky, which is governed by the potential[21]

$$V(\phi) = \begin{cases} V_0 + A_+\,(\phi - \phi_0), & \text{for } \phi > \phi_0, \\ V_0 + A_-\,(\phi - \phi_0), & \text{for } \phi < \phi_0. \end{cases}$$

The second model that we shall consider is the so-called punctuated inflationary model described by the potential[22]

$$V(\phi) = \frac{m^2}{2}\,\phi^2 - \frac{2\,m^2}{3\,\phi_0}\,\phi^3 + \frac{m^2}{4\,\phi_0^2}\,\phi^4.$$

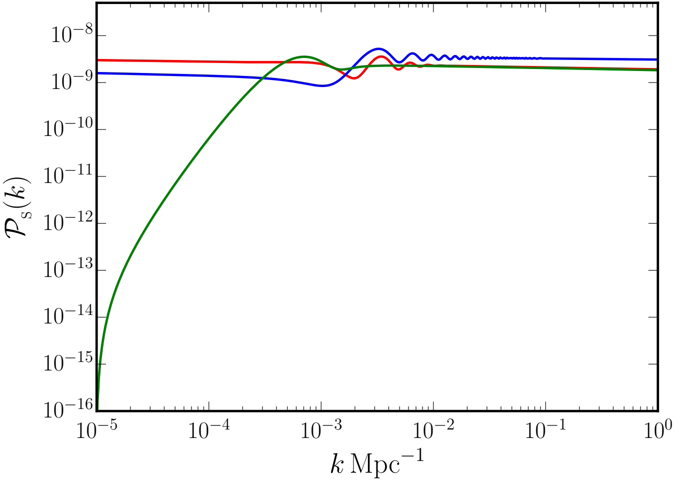
It is easy to show that this potential contains a point of inflection at ϕ_0.

Back to spectra of electromagnetic fields in punctuated inflation

[21] A. A. Starobinsky, JETP Lett. **55**, 489 (1992).

[22] R. K. Jain, P. Chingangbam, J-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **01**, 009 (2009);
R. K. Jain, P. Chingangbam, L. Sriramkumar and T. Souradeep, Phys. Rev. D **82**, 023509 (2010);
H. V. Ragavendra, D. Chowdhury and L. Sriramkumar, Phys. Rev. D **106**, 043535 (2022).

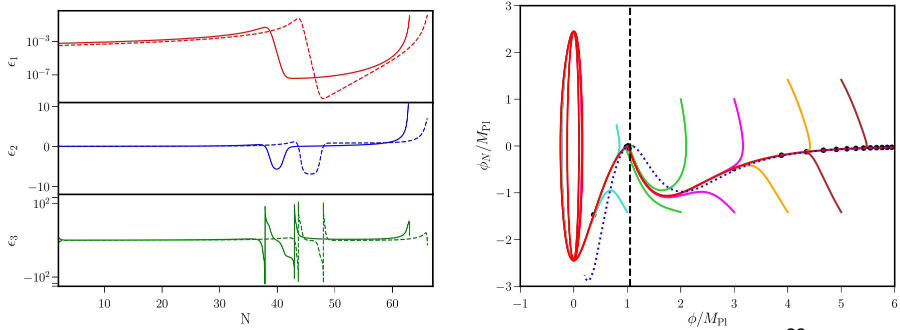
Scalar power spectra with features over the CMB scales



The scalar power spectra with features over the CMB scales in the cases of the quadratic potential with a step (in red), the second Starobinsky model (in blue), and the model of punctuated inflation (in green).

▸ Spectra of electromagnetic fields in slow roll inflation

Potentials admitting ultra slow roll inflation



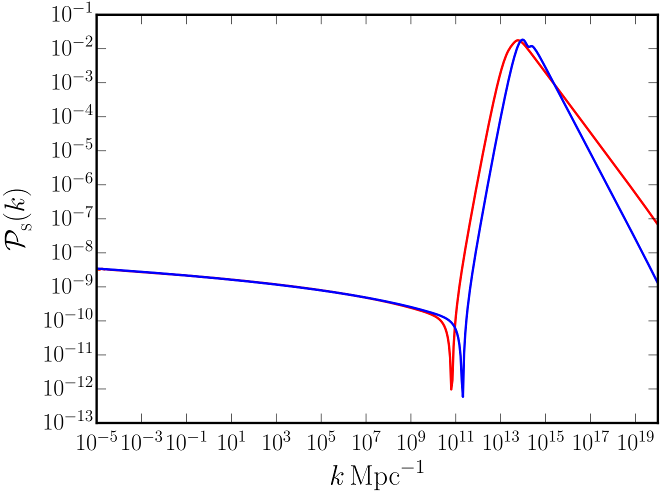
Potentials leading to ultra slow roll inflation (with $x = \phi/v$, v being a constant)[23]:

$$\mathrm{USR1}: V(\phi) = V_0\, \frac{6\,x^2 - 4\,\alpha\,x^3 + 3\,x^4}{(1+\beta\,x^2)^2},$$

$$\mathrm{USR2}: V(\phi) = V_0 \left\{\tanh\left(\frac{\phi}{\sqrt{6}\,M_{_{\rm Pl}}}\right) + A\,\sin\left[\frac{\tanh\left[\phi/\left(\sqrt{6}\,M_{_{\rm Pl}}\right)\right]}{f_\phi}\right]\right\}^2.$$

[23] J. Garcia-Bellido and E. R. Morales, Phys. Dark Univ. **18**, 47 (2017); I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).

Scalar power spectra with enhanced power on small scales



The scalar power spectra with enhanced power on small scales that arise in the inflationary models USR2 (in red) and PI3 (in blue)[24].

[24] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

Non-conformal coupling function(s) in the second Starobinsky model

In the second Starobinsky model, on working in the slow roll approximation, the evolution of the scalar field on either side of the transition can be expressed as

$$\phi_+(N) \simeq -\left(\frac{V_0}{A_+} - \phi_0\right) + \left[\left(\phi_{\rm i} - \phi_0 + \frac{V_0}{A_+}\right)^2 - 2\,M_{_{\rm Pl}}^2\,N\right]^{1/2},$$

$$\phi_-(N) \simeq -\left(\frac{V_0}{A_-} - \phi_0\right) + \left[\left(\frac{V_0}{A_-}\right)^2 - 2\,M_{_{\rm Pl}}^2\,(N - N_0)\right]^{1/2}.$$

These solutions allow us to construct the following coupling functions:

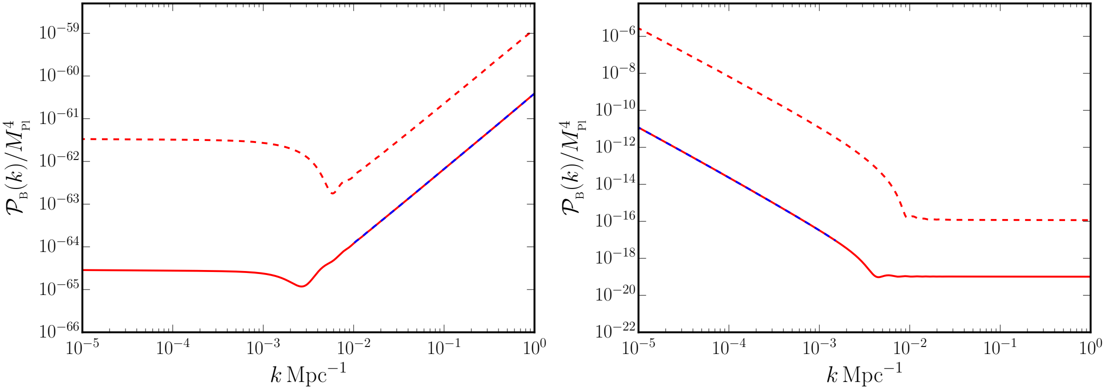
$$J_+(\phi) = J_{0+}\,\exp\left\{-\frac{n}{2\,M_{_{\rm Pl}}^2}\left[\left(\phi_+ - \phi_0 + \frac{V_0}{A_+}\right)^2 - \left(\phi_{\rm i} - \phi_0 + \frac{V_0}{A_+}\right)^2\right]\right\},$$

$$J_-(\phi) = J_{0-}\,\exp\left\{-\frac{n}{2\,M_{_{\rm Pl}}^2}\left[\left(\phi_- - \phi_0 + \frac{V_0}{A_-}\right)^2 - \left(\frac{V_0}{A_-}\right)^2 - 2\,N_0\,M_{_{\rm Pl}}^2\right]\right\},$$

where the constants $J_{0\pm}$ are to be chosen suitably so that $J_\pm(\phi_{\rm e}) = 1$, i.e. the value of J is unity at the end of inflation.

▸ Electromagnetic spectra in potential with a step
▸ Back to ironing out features

Spectra of the magnetic field in the second Starobinsky model



The power spectra of the magnetic field arising in the second Starobinsky model for the two choices of coupling functions $J_{+}(\phi)$ (on the left) and $J_{-}(\phi)$ (on the right) have been plotted for $n = 2$ in the non-helical (in solid red) as well as the helical (in dashed red) cases.

Constructing the coupling function in punctuated inflation

Since the solution for the evolution of the scalar field in the first model of punctuated inflation is not easy to obtain analytically, we need to turn to numerics.

We choose a quadratic function of the form

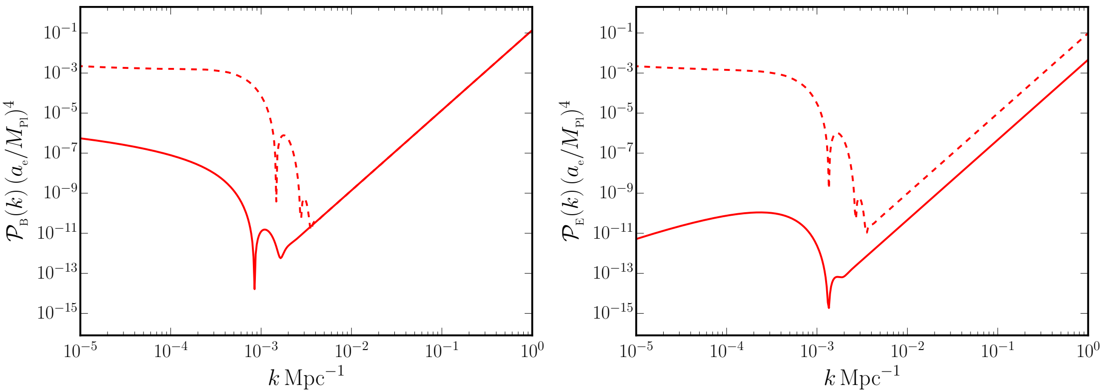
$$N(\phi) = a_1 \left(\frac{\phi^2}{M_{\rm Pl}^2}\right) + b_1 \left(\frac{\phi}{M_{\rm Pl}}\right) + c_1$$

to fit the numerical solution we have obtained in the initial slow roll regime.

To evaluate the spectra of the electromagnetic fields, we shall work with a coupling function of the form

$$J(\phi) = \exp\left\{n\left[a_1 \left(\frac{\phi^2 - \phi_{\rm e}^2}{M_{\rm Pl}^2}\right) + b_1 \left(\frac{\phi - \phi_{\rm e}}{M_{\rm Pl}}\right)\right]\right\}.$$

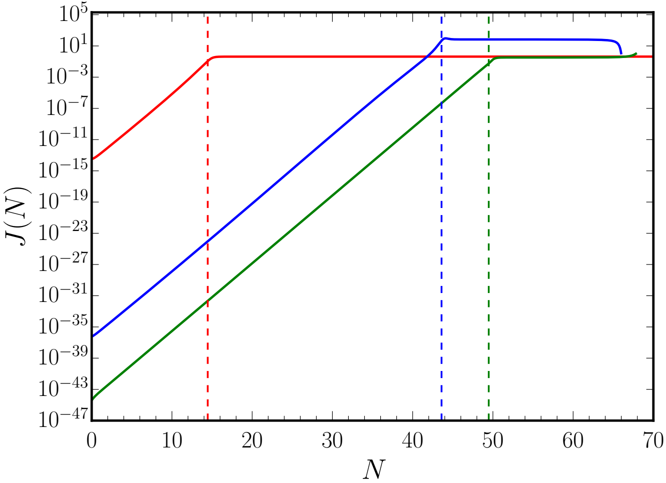
Spectra of electromagnetic fields in punctuation inflation



The spectra of the magnetic (on the left) and electric (on the right) fields arising in the case of the first punctuated inflation model have been plotted for both the non-helical (as solid curves) and helical (as dashed curves) cases.

Models leading to suppression of power on large scales

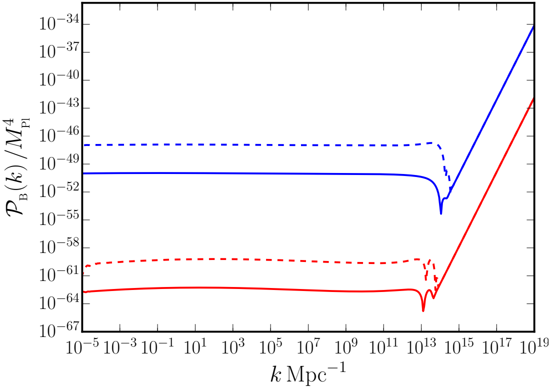
Behavior of the coupling function in models permitting ultra slow roll

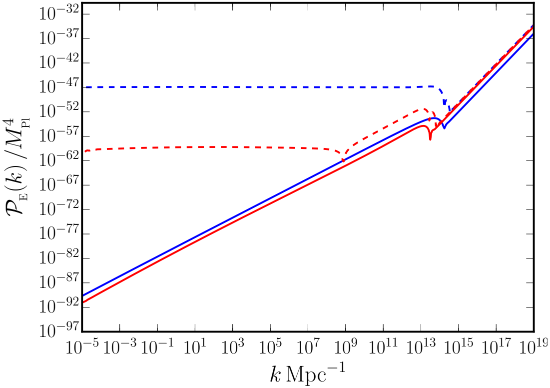


The evolution of the non-conformal coupling function J in inflationary models leading to ultra slow roll. Note that the coupling function does not change appreciably once ultra slow roll sets in (indicated by the vertical lines).

▸ Back to spectra of electromagnetic fields in two field models

Resulting spectra of the electromagnetic fields





The spectra of the magnetic (on the left) and electric (on the right) fields arising in two inflationary models permitting a period of ultra slow roll (at late times) have been plotted in the non-helical (as solid lines) and helical (as dashed lines) cases[25].

[25] S. Tripathy, D. Chowdhury, R. K. Jain and L. Sriramkumar, Phys. Rev. D **105**, 063519 (2022).

Plan of the talk

The two field models of interest

It has been shown that two scalar fields ϕ and χ governed by the action

$$S[\phi,\chi] = \int \mathrm{d}^4x\,\sqrt{-g}\,\left[-\frac{1}{2}\,\partial^\mu\phi\,\partial_\mu\phi - \frac{f(\phi)}{2}\,\partial^\mu\chi\,\partial_\mu\chi - V(\phi,\chi)\right]$$

described by *separable* potentials and the non-canonical functions $f(\phi) = \exp\,(2\,b\,\phi)$ or $f(\phi) = \exp\,(2\,b\,\phi^2)$ can lead to features in the scalar power spectrum.

While the potential[26]

$$V(\phi,\chi) = \frac{1}{2}\,m_\phi^2\,\phi^2 + V_0\,\frac{\chi^2}{\chi_0^2+\chi^2}$$

leads to a suppression in power on large scales, the potential[27]

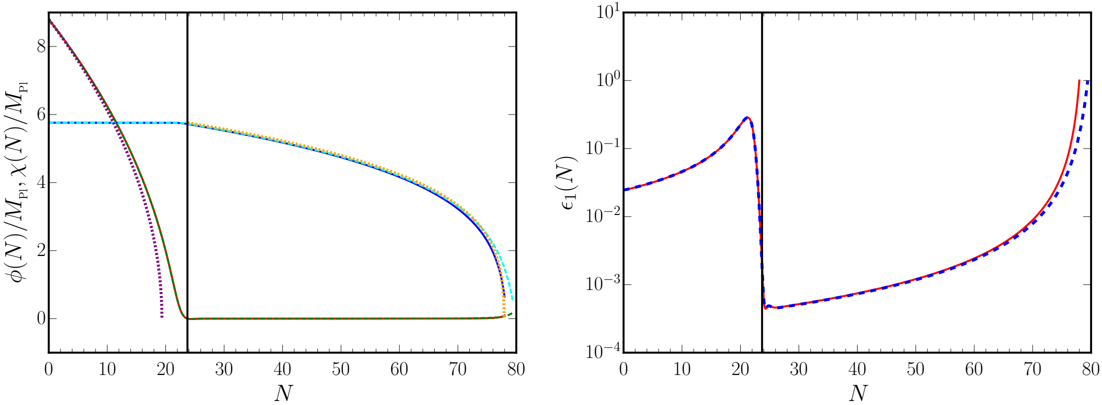
$$V(\phi,\chi) = V_0\,\frac{\phi^2}{\phi_0^2+\phi^2} + \frac{1}{2}\,m_\chi^2\,\chi^2$$

generates enhanced power on small scales.

[26] M. Braglia, D. K. Hazra, L. Sriramkumar and F. Finelli, JCAP **08**, 025 (2020).

[27] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

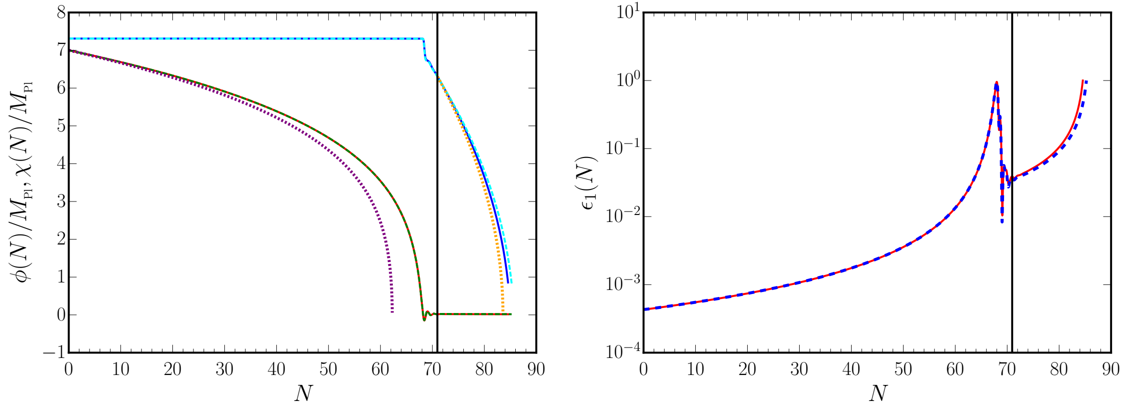
Behavior of the scalar fields and the first slow roll parameter I



Behavior of the two scalar fields ϕ and χ (in blue and red, on the left) and the first slow roll parameter ϵ_1 (on the right) in the first two field model[28]. Note that there arises a turn in the field space around the e-fold $N = 24$, when the first slow roll parameter begins to decrease before increasing again, leading to the termination of inflation.

[28] M. Braglia, D. K. Hazra, L. Sriramkumar and F. Finelli, JCAP **08**, 025 (2020).

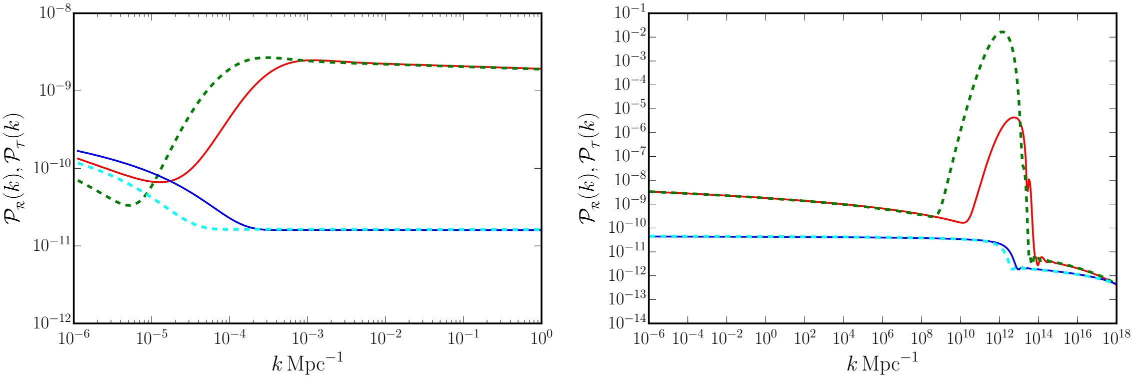
Behavior of the scalar fields and the first slow roll parameter II



Behavior of the two scalar fields and the first slow roll parameter in the second two field model has been plotted as in the case of the first model[29]. Evidently, barring the time of the turn in field space, the fields broadly behave in the same manner as in the previous example.

[29] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

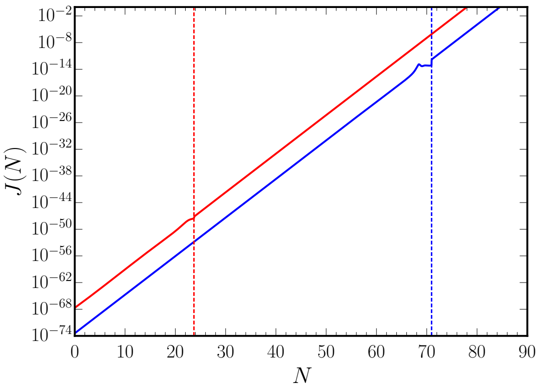
Scalar and tensor power spectra in the two field models

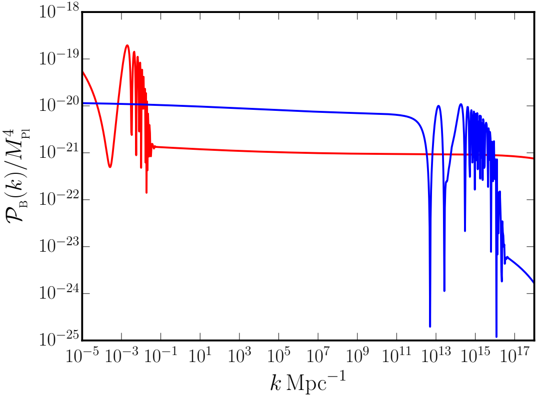


The spectra of curvature (in solid red and dashed green) and tensor (in solid blue and dashed cyan) perturbations, viz. $\mathcal{P}_{\mathcal{R}}(k)$ and $\mathcal{P}_{\mathcal{T}}(k)$, have been plotted for the two field inflationary models that we have considered.[30].

[30] S. Tripathy, D. Chowdhury, H. V. Ragavendra, R. K. Jain and L. Sriramkumar, Phys. Rev. D **107**, 043501 (2023).

Circumventing the challenge in two field models



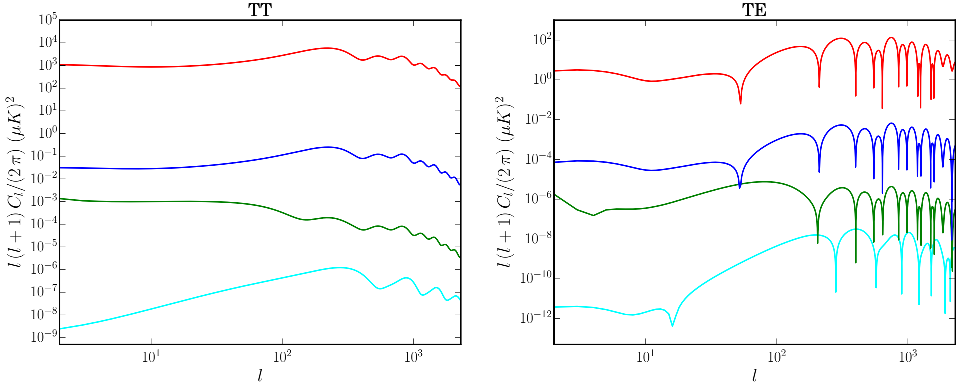


The evolution of the non-conformal coupling function J (on the left) and the corresponding spectrum of magnetic field (on the right) arising in the two field inflationary models leading to features on large (in red) and on small (in blue) scales[31]. The vertical lines (on the left) indicate the time when the turn in the field space takes place.

Evolution of J in ultra slow roll inflation

[31] S. Tripathy, D. Chowdhury, H. V. Ragavendra, R. K. Jain and L. Sriramkumar, Phys. Rev. D **107**, 043501 (2023).

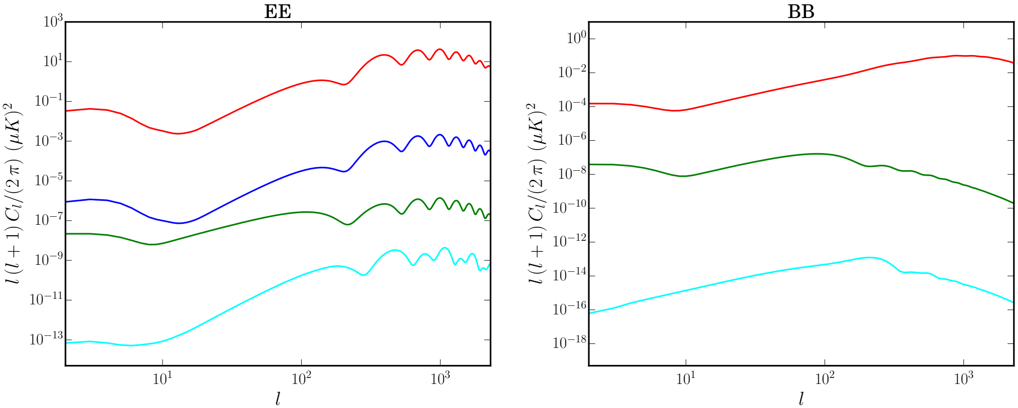
Effects of magnetic fields on the CMB anisotropies I



Contributions of the magnetic modes to the temperature and polarization angular power spectra of the CMB due to the total (i.e. scalar plus tensor) passive (in green) and the total compensated (in cyan) modes in the second two field model. We have arrived at these quantities using MagCAMB corresponding to a magnetic field with smoothed strength of $B^0_{1\,\mathrm{Mpc}} = 2.05\times 10^{-1}\,\mathrm{nG}$ *today* and a spectral index of $n_{\mathrm{B}} = -0.0112$[32].

[32] A. Zucca, Y. Li and L. Pogosian, Phys. Rev. D **95**, 063506 (2017).

Effects of magnetic fields on the CMB anisotropies II



In addition, using CAMB, we have plotted the standard angular power spectra of the CMB (in red) induced by the primary scalar and tensor perturbations. Moreover, we have also presented the contribution due to the curvature perturbation induced by the magnetic field during inflation (in blue), which we have computed using CAMB[33].

[33] S. Tripathy, D. Chowdhury, H. V. Ragavendra, R. K. Jain and L. Sriramkumar, Phys. Rev. D **107**, 043501 (2023).

Plan of the talk

Action describing the Fourier modes of the electromagnetic field

The helical electromagnetic modes $\mathcal{A}_{\boldsymbol{k}}^{\sigma}$ are described by the action

$$S[\mathcal{A}_{\boldsymbol{k}}^{\sigma}] = \int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{k} \sum_{\sigma=\pm} \left[\frac{1}{2}\,|\mathcal{A}_{\boldsymbol{k}}^{\sigma\prime}|^2 - \frac{\kappa}{2}\,\left(\mathcal{A}_{\boldsymbol{k}}^{\sigma\prime}\,\mathcal{A}_{\boldsymbol{k}}^{\sigma*} + \mathcal{A}_{\boldsymbol{k}}^{\sigma\prime*}\,\mathcal{A}_{\boldsymbol{k}}^{\sigma}\right) - \frac{\mu^2}{2}\,|\mathcal{A}_{\boldsymbol{k}}^{\sigma}|^2\right],$$

where the quantities μ^2 and κ are given by

$$\mu^2 = k^2 - \left(\frac{J'}{J}\right)^2 + \frac{2\,\sigma\,\gamma\,k\,I^2\,J'}{J^3}, \quad \kappa = \frac{J'}{J} - \frac{\sigma\,\gamma\,k\,I^2}{J^2}.$$

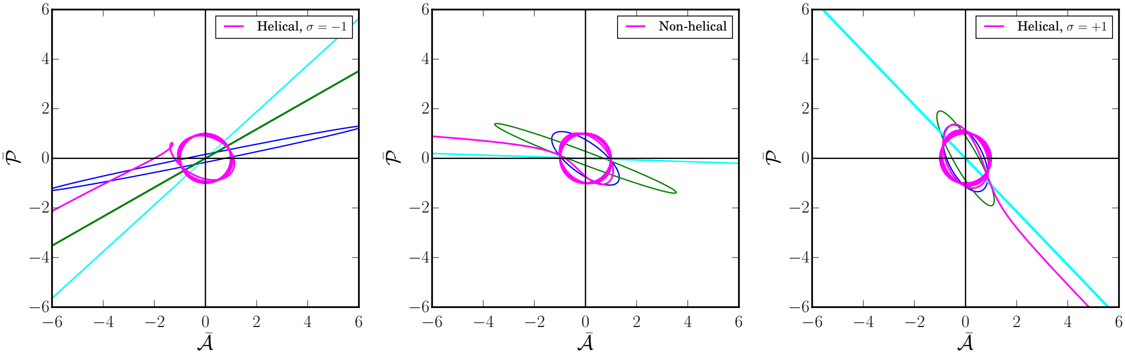
Upon adding the total time derivative $-[(\sigma\,\gamma\,k\,I^2/J^2)\,|\mathcal{A}_{\boldsymbol{k}}^{\sigma}|^2]'$, the action reduces to

$$S[\mathcal{A}_{\boldsymbol{k}}^{\sigma}] = \int \mathrm{d}\eta \int_{\mathbb{R}^3/2} \mathrm{d}^3\boldsymbol{k} \sum_{\sigma=\pm} \left[|\mathcal{A}_{\sigma}'|^2 - \frac{J'}{J}\,\left(\mathcal{A}_{\boldsymbol{k}}^{\sigma\prime}\,\mathcal{A}_{\boldsymbol{k}}^{\sigma*} + \mathcal{A}_{\boldsymbol{k}}^{\sigma\prime*}\,\mathcal{A}_{\boldsymbol{k}}^{\sigma}\right) - \bar{\mu}^2\,|\mathcal{A}_{\boldsymbol{k}}^{\sigma}|^2\right],$$

where $\bar{\mu}^2$ is given by

$$\bar{\mu}^2 = k^2 + \frac{2\,\sigma\,\gamma\,k\,I\,I'}{J^2} - \left(\frac{J'}{J}\right)^2.$$

Behavior of the Wigner ellipse



Evolution of the Wigner ellipse (in red, blue, green and cyan) and the classical trajectory (in magenta) in the phase space $\bar{\mathcal{A}}$-$\bar{\mathcal{P}}$ has been plotted for the electromagnetic mode in slow roll inflation with the wave number corresponding to the CMB pivot scale[34].

[34] S. Tripathy, R. N. Raveendran, K. Parattu and L. Sriramkumar, arXiv:2306.16168 [gr-qc].

Hamiltonian governing modes with wavevectors $\boldsymbol{k}$ and $-\boldsymbol{k}$

Since $\mathcal{A}^\sigma_{-\boldsymbol{k}} = \mathcal{A}^{\sigma\,*}_{\boldsymbol{k}}$, the original action above can be expressed as

$$S[\mathcal{A}^\sigma_{\boldsymbol{k}}, \mathcal{A}^\sigma_{-\boldsymbol{k}}] = \int \mathrm{d}\eta \int_{\mathbb{R}^3/2} \mathrm{d}^3\boldsymbol{k} \sum_{\sigma=\pm} \left[\mathcal{A}^{\sigma\prime}_{\boldsymbol{k}}\, \mathcal{A}^{\sigma\prime}_{-\boldsymbol{k}} - \kappa\, \left(\mathcal{A}^{\sigma\prime}_{\boldsymbol{k}}\, \mathcal{A}^{\sigma}_{-\boldsymbol{k}} + \mathcal{A}^{\sigma\prime}_{-\boldsymbol{k}}\, \mathcal{A}^{\sigma}_{\boldsymbol{k}}\right) - \mu^2\, \mathcal{A}^{\sigma}_{\boldsymbol{k}}\, \mathcal{A}^{\sigma}_{-\boldsymbol{k}}\right].$$

Motivated by the approach adopted in the case of the scalar perturbations, we can define the new variables $(x^\sigma_{\boldsymbol{k}}, p^\sigma_{\boldsymbol{k}})$ in terms of $(\mathcal{A}^\sigma_{\boldsymbol{k}}, \mathcal{P}^\sigma_{\boldsymbol{k}})$ and $(\mathcal{A}^\sigma_{-\boldsymbol{k}}, \mathcal{P}^\sigma_{-\boldsymbol{k}})$ as follows[35]:

$$x^\sigma_{\boldsymbol{k}} = \frac{1}{2}\,\left(\mathcal{A}^\sigma_{\boldsymbol{k}} + \mathcal{A}^\sigma_{-\boldsymbol{k}}\right) + \frac{i}{2\,\widetilde{\omega}}\,\left(\mathcal{P}^\sigma_{\boldsymbol{k}} - \mathcal{P}^\sigma_{-\boldsymbol{k}}\right), \quad p^\sigma_{\boldsymbol{k}} = \frac{1}{2}\,\left(\mathcal{P}^\sigma_{\boldsymbol{k}} + \mathcal{P}^\sigma_{-\boldsymbol{k}}\right) - \frac{i\,\widetilde{\omega}}{2}\,\left(\mathcal{A}^\sigma_{\boldsymbol{k}} - \mathcal{A}^\sigma_{-\boldsymbol{k}}\right),$$

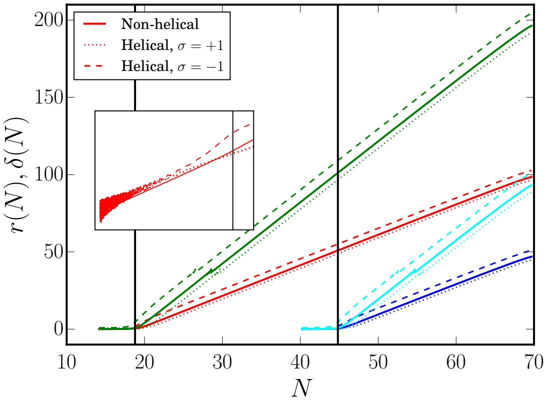
where $\widetilde{\omega}^2 = k^2\,(1 + \gamma^2\, I^4/J^4)$.

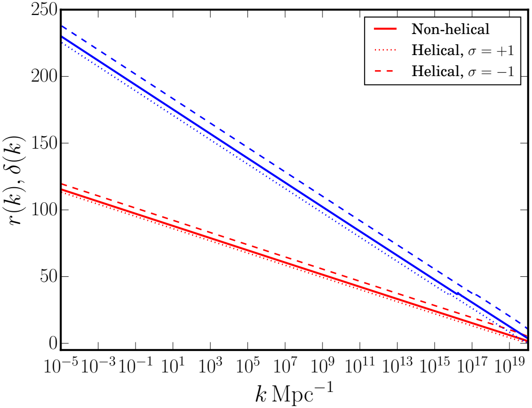
The Hamiltonian density in Fourier space is then given by

$$\mathcal{H} = \frac{1}{2}\,\left(p^\sigma_{\boldsymbol{k}}\, p^\sigma_{\boldsymbol{k}} + p^\sigma_{-\boldsymbol{k}}\, p^\sigma_{-\boldsymbol{k}}\right) + \kappa\,\left(x^\sigma_{\boldsymbol{k}}\, p^\sigma_{-\boldsymbol{k}} + x^\sigma_{-\boldsymbol{k}}\, p^\sigma_{\boldsymbol{k}}\right) + \frac{\widetilde{\omega}^2}{2}\,\left(x^\sigma_{\boldsymbol{k}}\, x^\sigma_{\boldsymbol{k}} + x^\sigma_{-\boldsymbol{k}}\, x^\sigma_{-\boldsymbol{k}}\right).$$

[35] J. Martin and V. Vennin, Phys. Rev. D **93**, (2016) 2, 023505 (2016); J. Martin, Universe **5**, 92 (2019).

'Spectra' of the squeezing amplitude and entanglement entropy





Evolution of the squeezing amplitude $r(N)$ (in red and blue) and the entanglement entropy $\delta(N)$ (in green and cyan) have been plotted (on the left) for electromagnetic modes with two different wave numbers in slow roll inflation. We have also plotted (on the right) the 'spectra' of the squeezing amplitude $r(k)$ and the entanglement entropy $\delta(k)$[36].

[36] S. Tripathy, R. N. Raveendran, K. Parattu and L. Sriramkumar, arXiv:2306.16168 [gr-qc].

Plan of the talk

Summary

- In single field models of inflation, substantial departures from slow roll inflation can lead to strong features in the spectra of magnetic fields.
- Strong departures from slow roll inflation can also suppress the strength of the magnetic field on large scales.
- Some of these challenges can be overcome in two field models of inflation. However, there always seems to be an element of fine tuning involved.
- It seems necessary to examine the behavior of additional quantities such as the three-point functions and the corresponding observables to arrive at constraints on the nature and form of the non-conformal coupling function.
- On a different note, we find that violation of parity leads to an enhancement of the squeezing amplitude and the entanglement entropy associated with one of the two states of polarization of the electromagnetic field.

Collaborators

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Krishnamohan Parattu

Rajeev Kumar Jain

Thank you for your attention