The scalar bi-spectrum in the Starobinsky model

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Proliferation of inflationary models[1]

5-dimensional assisted inflation	extended open inflation	late-time mild inflation	pre-Big-Bang inflation
anisotropic brane inflation	extended warm inflation	low-scale inflation	primary inflation
anomaly-induced inflation	extra dimensional inflation	low-scale supergravity inflation	primordial inflation
assisted inflation	F-term inflation	M-theory inflation	quasi-open inflation
assisted chaotic inflation	F-term hybrid inflation	mass inflation	quintessential inflation
boundary inflation	false vacuum inflation	massive chaotic inflation	R-invariant topological inflation
brane inflation	false vacuum chaotic inflation	moduli inflation	rapid asymmetric inflation
brane-assisted inflation	fast-roll inflation	multi-scalar inflation	running inflation
brane gas inflation	first order inflation	multiple inflation	scalar-tensor gravity inflation
brane-antibrane inflation	gauged inflation	multiple-field slow-roll inflation	scalar-tensor stochastic inflation
braneworld inflation	generalised inflation	multiple-stage inflation	Seiberg-Witten inflation
Brans-Dicke chaotic inflation	generalized assisted inflation	natural inflation	single-bubble open inflation
Brans-Dicke inflation	generalized slow-roll inflation	natural Chaotic inflation	spinodal inflation
bulky brane inflation	gravity driven inflation	natural double inflation	stable starobinsky-type inflation
chaotic hybrid inflation	Hagedorn inflation	natural supergravity inflation	steady-state eternal inflation
chaotic inflation	higher-curvature inflation	new inflation	steep inflation
chaotic new inflation	hybrid inflation	next-to-minimal supersymmetric	stochastic inflation
D-brane inflation	hyperextended inflation	hybrid inflation	string-forming open inflation
D-term inflation	induced gravity inflation	non-commutative inflation	successful D-term inflation
dilaton-driven inflation	induced gravity open inflation	non-slow-roll inflation	supergravity inflation
dilaton-driven brane inflation	intermediate inflation	nonminimal chaotic inflation	supernatural inflation
double inflation	inverted hybrid inflation	old inflation	superstring inflation
double D-term inflation	isocurvature inflation	open hybrid inflation	supersymmetric hybrid inflation
dual inflation	K inflation	open inflation	supersymmetric inflation
dynamical inflation	kinetic inflation	oscillating inflation	supersymmetric topological inflation
dynamical SUSY inflation	lambda inflation	polynomial chaotic inflation	supersymmetric new inflation
eternal inflation	large field inflation	polynomial hybrid inflation	synergistic warm inflation
extended inflation	late D-term inflation	power-law inflation	TeV-scale hybrid inflation

A (partial?) list of ever-increasing number of inflationary models. May be, we should look for models that permit deviations from the standard picture of slow roll inflation.

[1] From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).

Based on

- J. Martin and L. Sriramkumar, *The scalar bi-spectrum in the Starobinsky model I: The equilateral case*, In preparation.

Outline of the talk

1. Features, fits and non-Gaussianities

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3. The non-Gaussianity parameter $f_{_{\rm NL}}$

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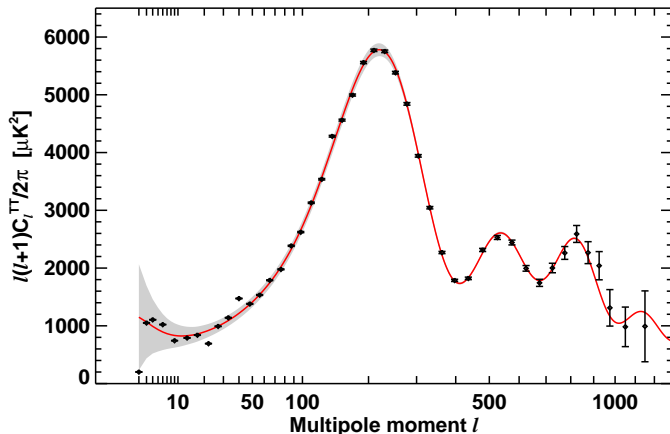
Outline of the talk

1. Features, fits and non-Gaussianities

2. The scalar power spectrum in the Starobinsky model

3. The non-Gaussianity parameter f_{NL}

4. The method for evaluating f_{NL}

5. f_{NL} in the Starobinsky model (in the equilateral limit)

Outline of the talk

Angular power spectrum from the WMAP 7-year data[2]

The WMAP 7-year data for the CMB TT angular power spectrum (the black dots with error bars) and the theoretical, best fit ΛCDM model with a power law primordial spectrum (the solid red curve). Note the outliers near the multipoles $\ell = 2$, 22 and 40.

[2] D. Larson *et al.*, Astrophys. J. Suppl. **192**, 16 (2011).

Reconstructing the primordial spectrum
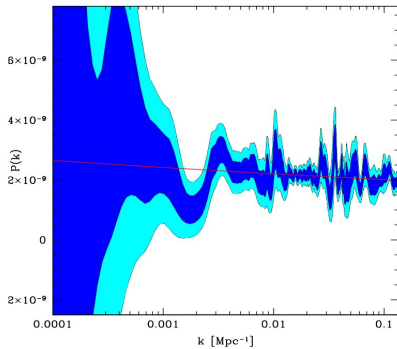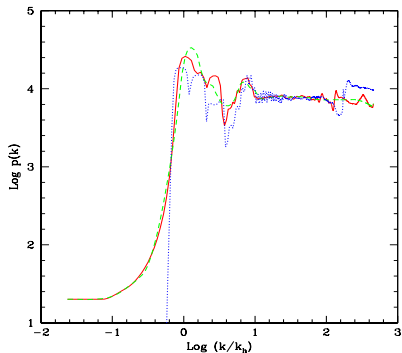
Reconstructed primordial spectra, obtained upon assuming the concordant background ΛCDM model. The recovered spectrum on the left improves the fit to the WMAP 3-year data by $\Delta\chi^2_{\rm eff} \simeq 15$, with respect to the best fit power law spectrum[3]. The spectrum on the right has been recovered from a variety of CMB datasets, including the WMAP 5-year data[4].

[3] A. Shafieloo, T. Souradeep, P. Manimaran, P. K. Panigrahi and R. Rangarajan, Phys. Rev. D **75**, 123502 (2007).

[4] G. Nicholson and C. R. Contaldi, JCAP **0907**, 011 (2009).

Inflationary models leading to features[5]
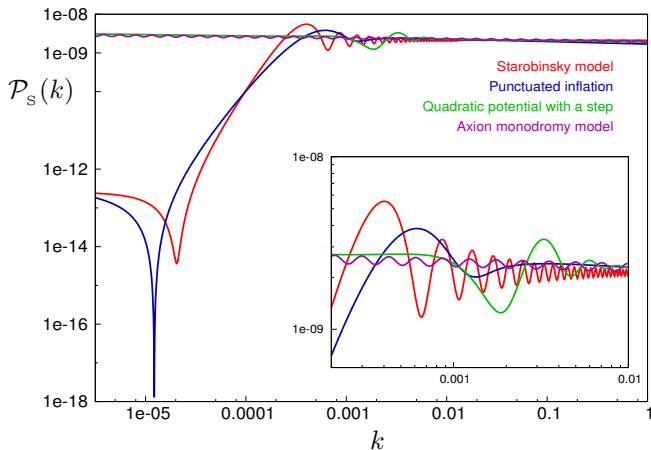
The scalar power spectra in a few different inflationary models that lead to a better fit to the CMB data than the conventional power law spectrum.

[5] R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **0901**, 009 (2009);
D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **1010**, 008 (2010);
M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, arXiv:1106.2798v1 [astro-ph.CO].

'Large' non-Gaussianities and its possible implications

- The WMAP 7-year data constrains the non-Gaussianity parameter f_{NL} to be $f_{\mathrm{NL}} = (26 \pm 140)$ in the equilateral limit, at 68% confidence level[6].

- If forthcoming missions such as Planck detect a large level of non-Gaussianity, as suggested by the above mean value of f_{NL}, then it can result in a substantial tightening in the constraints on the various inflationary models. For example, canonical scalar field models that lead to a nearly scale invariant primordial spectrum contain only a small amount of non-Gaussianity and, hence, will cease to be viable[7] .

- However, it is known that primordial spectra with features can lead to reasonably large non-Gaussianities[8]. Therefore, if the non-Gaussianity parameter f_{NL} indeed proves to be large, then either one has to reconcile with the fact that the primordial spectrum contains features or we have to turn our attention to non-canonical scalar field models such as, say, D brane inflation models[9].

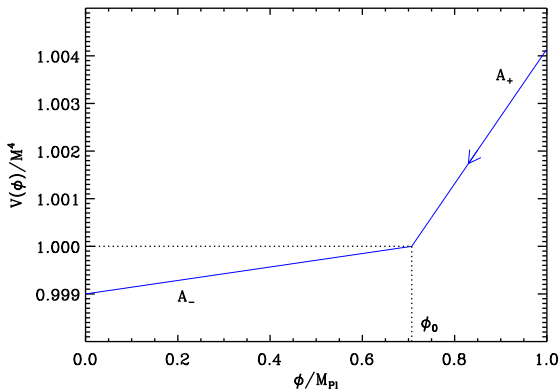[6] E. Komatsu *et al.*, Astrophys. J. Suppl. **192**, 18 (2011).

[7] J. Maldacena, JHEP **05**, 013 (2003).

[8] See, for instance, X. Chen, R. Easther and E. A. Lim, JCAP **0706**, 023 (2007).

[9] See, for example, X. Chen, M.-x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007).

The Starobinsky model[10]

The Starobinsky model involves the canonical scalar field which is described by the potential

$$V(\phi) = \begin{cases} V_0 + A_+ \ (\phi - \phi_0) & \text{for } \phi > \phi_0, \\ V_0 + A_- \ (\phi - \phi_0) & \text{for } \phi < \phi_0. \end{cases}$$

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[10] A. A. Starobinsky, Sov. Phys. JETP Lett. **55**, 489 (1992).

Assumptions and properties

- It is assumed that the constant V_0 is the dominant term in the potential for a range of ϕ near ϕ_0. As a result, over the domain of our interest, the expansion is of the de Sitter form corresponding to a Hubble parameter H_0 determined by V_0.

- The scalar field rolls slowly until it reaches the discontinuity in the potential. It then fast rolls for a brief period as it crosses the discontinuity before slow roll is restored again.

- Since V_0 is dominant, the first slow roll parameter ϵ_1 remains small even during the transition. This property allows the background to be evaluated analytically to a good approximation.

Analytic expressions for the slow roll parameters

Under the assumptions and approximations described above, the slow roll parameters remain small before the transition.

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One can show that, after the transition, the evolution of the first slow roll parameter ϵ_1 can be expressed in terms of the number of e-folds N as follows:

$$\epsilon_{1-} \simeq \frac{A_-^2}{18\,M_{_{\mathrm{Pl}}}^2\,H_0^4}\left[1 - \frac{\Delta A}{A_-}\,\mathrm{e}^{-3\,(N-N_0)}\right]^2,$$

where $\Delta A = (A_- - A_+)$, while N_0 is the e-fold at which the field crosses the discontinuity.

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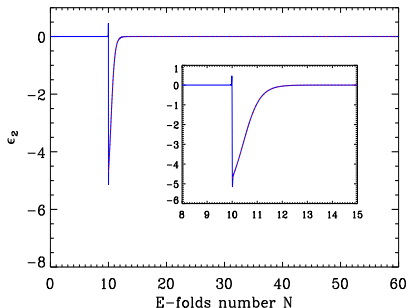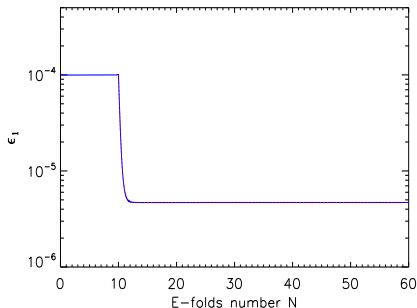
where $\Delta A = (A_- - A_+)$, while N_0 is the e-fold at which the field crosses the discontinuity.

It is found that, *immediately after the transition*, the second slow roll parameter ϵ_2 is given by

$$\epsilon_{2-} \simeq \frac{6\,\Delta A}{A_-}\,\frac{\mathrm{e}^{-3\,(N-N_0)}}{1 - (\Delta A/A_-)\,\mathrm{e}^{-3\,(N-N_0)}}.$$

Evolution of the slow roll parameters

The evolution of the first slow roll parameter ϵ_1 on the left, and the second slow roll parameter ϵ_2 on the right in the Starobinsky model. While the blue curves describe the numerical results, the dotted red curves represent the analytical expressions mentioned in the previous slide.

The modes before and after the transition

It can be shown that, under the assumptions that one is working with, the quantity $z = a \, M_{_\mathrm{Pl}} \sqrt{2\,\epsilon_1}$, which determines the evolution of the perturbations, simplifies to

$$z''/z \simeq 2\,\mathcal{H}^2$$

both before *as well as* after the transition with the overprime denoting the derivative with respect to the conformal time, while $\mathcal{H}$ is the conformal Hubble parameter.

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As a result, while the solution to the Mukhanov-Sasaki variable v_k before the transition is given by

$$v_k^+(\eta) = \frac{1}{\sqrt{2\,k}}\,\left(1 - \frac{i}{k\,\eta}\right)\,\mathrm{e}^{-i\,k\,\eta},$$

after the transition, it can be expressed as a linear combination of the positive and the negative frequency modes as follows:

$$v_k^-(\eta) = \frac{\alpha_k}{\sqrt{2\,k}}\,\left(1 - \frac{i}{k\,\eta}\right)\,\mathrm{e}^{-i\,k\,\eta} + \frac{\beta_k}{\sqrt{2\,k}}\,\left(1 + \frac{i}{k\,\eta}\right)\,\mathrm{e}^{i\,k\,\eta},$$

where α_k and β_k are the usual Bogoliubov coefficients.

The scalar power spectrum in the Starobinsky model

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The scalar power spectrum, given by

$$\mathcal{P}_{\mathrm{s}}(k) = (k^3/2\,\pi^2)\,|\mathcal{R}_k|^2 = (k^3/2\,\pi^2)\,(|v_k|/z)^2,$$

where $\mathcal{R}_k$ is the curvature perturbation, can be evaluated at late times to be

$$\mathcal{P}_{\mathrm{s}}(k) = \left(\frac{9\,H_0^6}{4\,\pi^2\,A_-^2}\right)\left\{1 - \frac{3\,\Delta A}{A_+}\frac{k_0}{k}\left[\left(1 - \frac{k_0^2}{k^2}\right)\sin\left(\frac{2\,k}{k_0}\right) + \frac{2\,k_0}{k}\cos\left(\frac{2\,k}{k_0}\right)\right]\right.$$

$$+ \frac{9\,\Delta A^2}{2\,A_+^2}\frac{k_0^2}{k^2}\left(1 + \frac{k_0^2}{k^2}\right)\left[\left(1 + \frac{k_0^2}{k^2}\right) - \frac{2\,k_0}{k}\sin\left(\frac{2\,k}{k_0}\right)\right.$$

$$\left.\left. + \left(1 - \frac{k_0^2}{k^2}\right)\cos\left(\frac{2\,k}{k_0}\right)\right]\right\},$$

where k_0 is the wavenumber of the mode that crosses the Hubble radius when the field crosses the discontinuity. Note that the power spectrum depends on the wavenumber only through the ratio (k/k_0).

Comparison with the numerical result
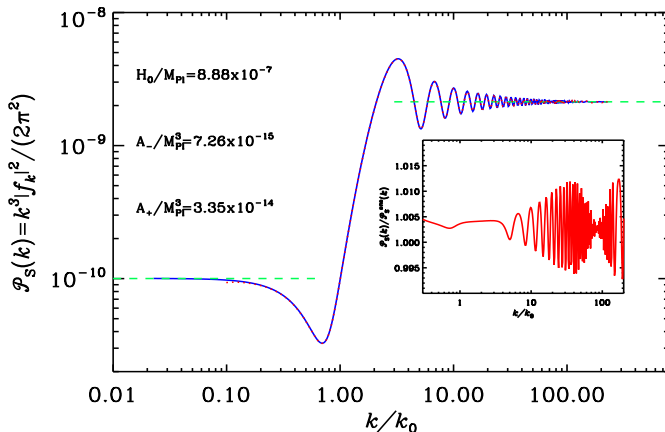
The scalar power spectrum in the Starobinsky model. While the blue solid curve denotes the analytic result, the red dots represent the corresponding numerical scalar power spectrum that has been obtained through an exact integration of the background as well as the perturbations.

The scalar bi-spectrum

The scalar bi-spectrum $\mathcal{B}_{\mathrm{s}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is related to the three point correlation function of the Fourier modes of the curvature perturbation, evaluated towards the end of inflation, say, at the conformal time η_{e}, as follows[11]:

$$\langle \hat{\mathcal{R}}_{\mathbf{k}_1}(\eta_{\mathrm{e}}) \, \hat{\mathcal{R}}_{\mathbf{k}_2}(\eta_{\mathrm{e}}) \, \hat{\mathcal{R}}_{\mathbf{k}_3}(\eta_{\mathrm{e}}) \rangle = (2\,\pi)^3 \ \mathcal{B}_{\mathrm{s}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \, \delta^{(3)}\left(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3\right).$$

For convenience, we shall set

$$\mathcal{B}_{\mathrm{s}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = (2\,\pi)^{-9/2} \ G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3).$$

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[11] D. Larson *et al.*, Astrophys. J. Suppl. **192**, 16 (2011);
E. Komatsu *et al.*, Astrophys. J. Suppl. **192**, 18 (2011).

The introduction of f_{NL}

The observationally relevant non-Gaussianity parameter f_{NL} is introduced through the equation[12]

$$\mathcal{R}(\eta, \mathbf{x}) = \mathcal{R}^{\mathrm{G}}(\eta, \mathbf{x}) - \frac{3\,f_{\mathrm{NL}}}{5} \left[\mathcal{R}^{\mathrm{G}}(\eta, \mathbf{x})\right]^2,$$

where $\mathcal{R}^{\mathrm{G}}$ denotes the Gaussian quantity, and the factor of $(3/5)$ arises due to the relation between the Bardeen potential and the curvature perturbation during the matter dominated epoch.

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[12] J. Maldacena, JHEP **0305**, 013 (2003);
S. Hannestad, T. Haugbolle, P. R. Jarnhus and M. S. Sloth, JCAP **1006**, 001 (2010).

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In Fourier space, the above equation can be written as

$$\mathcal{R}_{\mathbf{k}} = \mathcal{R}^{\mathrm{G}}_{\mathbf{k}} - \frac{3\,f_{\mathrm{NL}}}{5}\,\int \frac{\mathrm{d}^3\mathbf{p}}{(2\,\pi)^{3/2}}\,\mathcal{R}^{\mathrm{G}}_{\mathbf{p}}\,\mathcal{R}^{\mathrm{G}}_{\mathbf{k}-\mathbf{p}}.$$

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[12] J. Maldacena, JHEP **0305**, 013 (2003);
S. Hannestad, T. Haugbolle, P. R. Jarnhus and M. S. Sloth, JCAP **1006**, 001 (2010).

The introduction of $f_{\rm NL}$ ... continued

Using this relation and Wick's theorem, one can arrive at the three point correlation of the curvature perturbation in Fourier space in terms of the parameter $f_{\rm NL}$. It is found to be

$$\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle = -\left(\frac{3\,f_{\rm NL}}{10}\right)\,(2\,\pi)^4\,(2\,\pi)^{-3/2}\,\left(\frac{1}{k_1^3\,k_2^3\,k_3^3}\right)\,\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$
$$\times \left[k_1^3\,\mathcal{P}_{\rm S}(k_2)\,\mathcal{P}_{\rm S}(k_3) + \text{two permutations}\right].$$

The relation between $f_{\rm NL}$ and the bi-spectrum

Using the above expression for the three point function of the curvature perturbation and the definition of the bi-spectrum, we can then arrive at the following relation:

$$
\begin{aligned}
f_{\rm NL} &= -\left(\frac{10}{3}\right)(2\,\pi)^{-4}\,(2\,\pi)^{9/2}\,\left(k_1^3\,k_2^3\,k_3^3\right)\,\mathcal{B}_{\rm S}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \\
&\qquad\qquad \times \left[k_1^3\,\mathcal{P}_{\rm S}(k_2)\,\mathcal{P}_{\rm S}(k_3) + \text{two permutations}\right]^{-1} \\
&= -\left(\frac{10}{3}\right)(2\,\pi)^{-4}\,\left(k_1^3\,k_2^3\,k_3^3\right)\,G(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \\
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&\qquad\qquad \times \left[k_1^3\,\mathcal{P}_{\mathrm{S}}(k_2)\,\mathcal{P}_{\mathrm{S}}(k_3) + \text{two permutations}\right]^{-1}.
\end{aligned}
$$

In the equilateral limit (i.e. when $\mathbf{k}_1 = \mathbf{k}_2 = \mathbf{k}_3$), this expression for f_{NL} simplifies to

$$
f_{\mathrm{NL}} = -\left(\frac{10}{9}\right)(2\,\pi)^{-4}\,\left(\frac{k^6\,G(k)}{\mathcal{P}_{\mathrm{S}}^2(k)}\right).
$$

The action at the cubic order[13]

It can be shown that the third order term in the action describing the curvature perturbations is given by

$$\mathcal{S}_3[\mathcal{R}] = M_{\mathrm{Pl}}^2 \int \mathrm{d}\eta \int \mathrm{d}^3\mathbf{x} \left[a^2\,\epsilon_1^2\,\mathcal{R}\,\mathcal{R}'^2 + a^2\,\epsilon_1^2\,\mathcal{R}\,(\partial\mathcal{R})^2 - 2\,a\,\epsilon_1\,\mathcal{R}'\,(\partial^i\mathcal{R})\,(\partial_i\chi) \right.$$

$$\left. + \frac{a^2}{2}\,\epsilon_1\,\epsilon_2'\,\mathcal{R}^2\,\mathcal{R}' + \frac{\epsilon_1}{2}\,(\partial^i\mathcal{R})\,(\partial_i\chi)\,(\partial^2\chi) + \frac{\epsilon_1}{4}\,(\partial^2\mathcal{R})\,(\partial\chi)^2 + \mathcal{F}\left(\frac{\delta\mathcal{L}_2}{\delta\mathcal{R}}\right) \right],$$

where $\mathcal{F}(\delta\mathcal{L}_2/\delta\mathcal{R})$ denotes terms involving the variation of the second order action with respect to $\mathcal{R}$, while χ is related to the curvature perturbation $\mathcal{R}$ through the relations

$$\Lambda = a\,\epsilon_1\,\mathcal{R}' \quad \text{and} \quad \partial^2\chi = \Lambda.$$

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[13] J. Maldacena, JHEP **0305**, 013 (2003);
D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005);
X. Chen, M.-x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007).

Evaluating the bi-spectrum

At the leading order in the perturbations, one then finds that the three point correlation in Fourier space is described by the integral

$$\langle \hat{\mathcal{R}}_{\mathbf{k}_1}(\eta_e)\, \hat{\mathcal{R}}_{\mathbf{k}_2}(\eta_e)\, \hat{\mathcal{R}}_{\mathbf{k}_3}(\eta_e) \rangle = -i \int_{\eta_i}^{\eta_e} \mathrm{d}\eta\, a(\eta) \left\langle \left[\hat{\mathcal{R}}_{\mathbf{k}_1}(\eta_e)\, \hat{\mathcal{R}}_{\mathbf{k}_2}(\eta_e)\, \hat{\mathcal{R}}_{\mathbf{k}_3}(\eta_e), \hat{H}_{\rm I}(\eta) \right] \right\rangle,$$

where $\hat{H}_{\rm I}$ is the operator corresponding to the above third order action, while η_i is the time at which the initial conditions are imposed on the modes when they are well inside the Hubble radius, and η_e denotes a very late time, say, close to when inflation ends.

In the equilateral limit, the quantity $G(k)$, evaluated towards the end of inflation at the conformal time $\eta = \eta_e$, can be written as

$$G(k) \equiv \sum_{C=1}^{6} G_{_C}(k) = M_{_{\rm Pl}}^2 \sum_{C=1}^{6} \left[f_k^3(\eta_e)\, \mathcal{G}_{_C}(k) + f_k^{*\,3}(\eta_e)\, \mathcal{G}_{_C}^*(k) \right],$$

where the quantities $\mathcal{G}_{_C}(k)$ are integrals that correspond to six terms that arise in the action at the third order in the perturbations, while f_k are the modes associated with the curvature perturbation $\mathcal{R}_k$.

Evaluating $f_{\rm NL}$ in the Starobinsky model

When there exist deviations from slow roll, it is found that the fourth term $\mathcal{G}_4$ provides the dominant contribution to $f_{\rm NL}$.

It is described by the following integral

$$\mathcal{G}_4(k) = 3\,i \int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta \; a^2 \, \epsilon_1 \, \epsilon_2' \, f_k^{*\,2} \, f_k'^{*}.$$

In the case of the Starobinsky model, as ϵ_2 is a constant before the transition, ϵ_2' vanishes, and hence the above integral $\mathcal{G}_4$ is non-zero only post-transition.

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In fact, with some effort, analytic expressions can be arrived at for all the $\mathcal{G}_n$.

The dominant contribution in the Starobinsky model
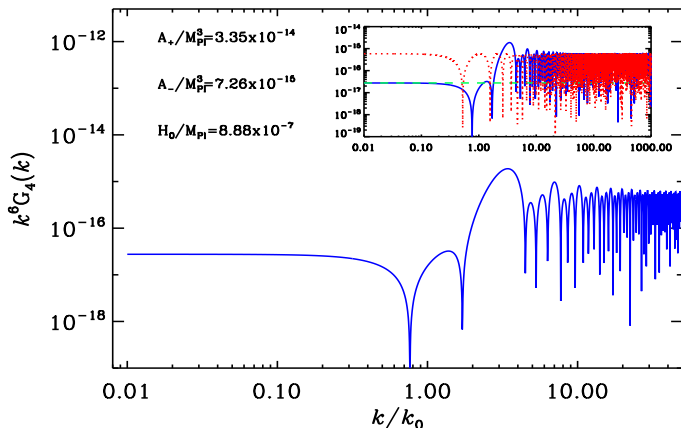
The absolute value of the quantity $[k^6\, G_4]$ has been plotted as a function of (k/k_0) (the blue curve). We have worked with the same of values of A_+, A_- and V_0 as in the earlier figure wherein we had plotted the power spectrum. The green and the red curves in the inset represent the limiting values for $k \ll k_0$ and $k \gg k_0$, respectively.
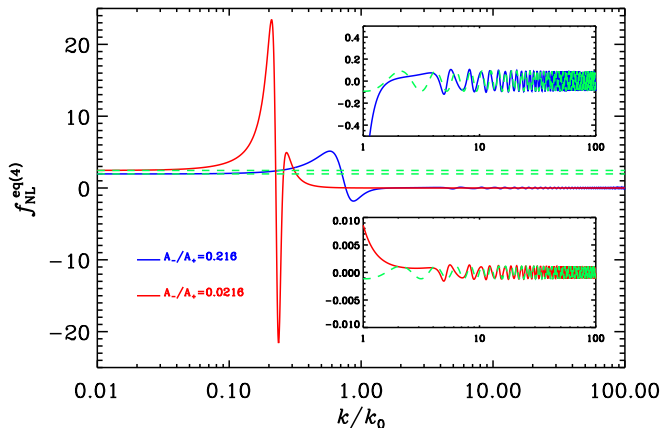
The different contributions
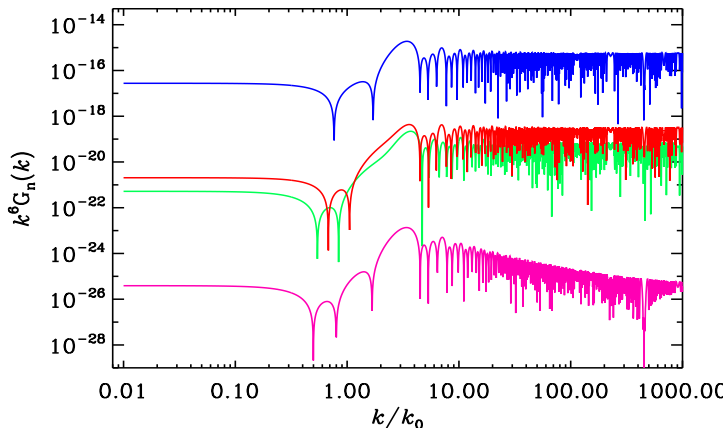
The quantities k^6 times the absolute values of $(G_1 + G_3)$ (in green), G_2 (in red), G_4 (in blue) and $(G_5 + G_6)$ (in purple) have been plotted as a function of (k/k_0) for the Starobinsky model.

$f_{\rm NL}$ due to the dominant contribution

The non-Gaussianity parameter $f_{\rm NL}$ due to the dominant term in the Starobinsky model, plotted as a function of (k/k_0) for $(A_-/A_+) = 0.216$ and $(A_-/A_+) = 0.0216$. Larger the difference between A_- and A_+, larger is the corresponding $f_{\rm NL}$.

f_{NL} for a range of values of the parameters
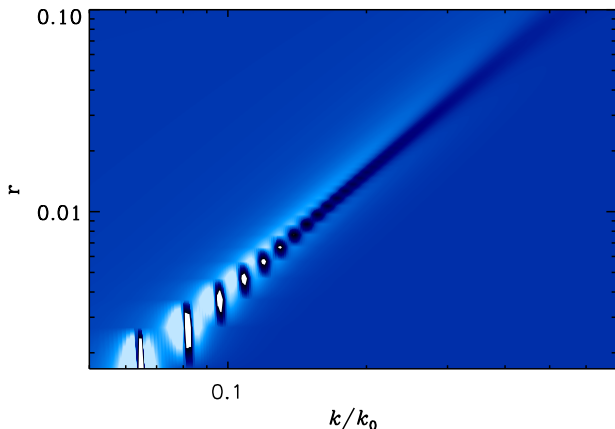
The non-Gaussianity parameter f_{NL} due to the dominant term in the Starobinsky model, plotted as a function of (k/k_0) and the ratio $r = (A_-/A_+)$. The white contours indicate regions wherein f_{NL} can be as large as 50. Note that, provided r is reasonably small, f_{NL} can be of the order of 20 or so, as is indicated by the currently observed mean value.

Summary

- Amazingly, we find that, for a certain range of values of the parameters involved, the non-Gaussianity parameter f_{NL} can be evaluated analytically, to a good accuracy (as is confirmed by comparison with numerical computations) in the Starobinsky model.

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- Amazingly, we find that, for a certain range of values of the parameters involved, the non-Gaussianity parameter $f_{\rm NL}$ can be evaluated analytically, to a good accuracy (as is confirmed by comparison with numerical computations) in the Starobinsky model.

- Interestingly, for suitably small values of $r = (A_-/A_+)$, $f_{\rm NL}$ in the Starobinsky model can be as large as indicated by the currently observed mean values.

Thank you for your attention