Inflationary three-point functions

L. Sriramkumar

Department of Physics, Indian Institute of Technology Madras, Chennai

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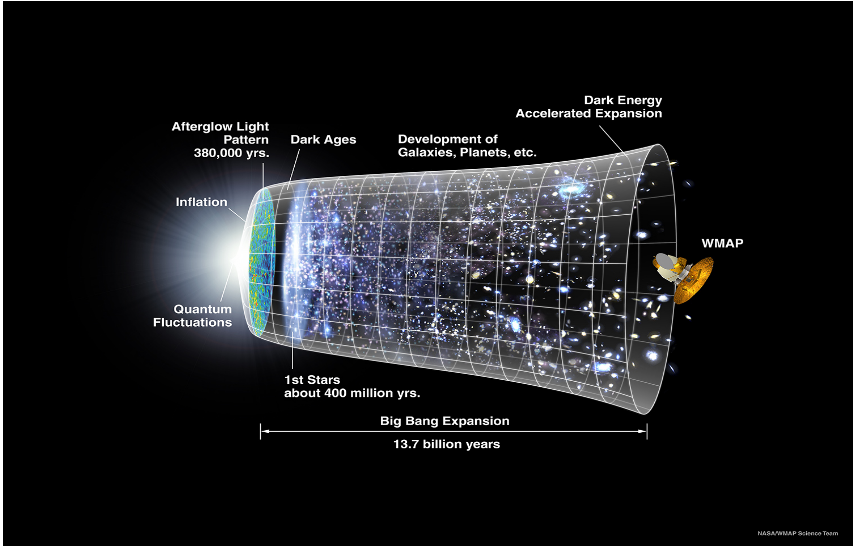
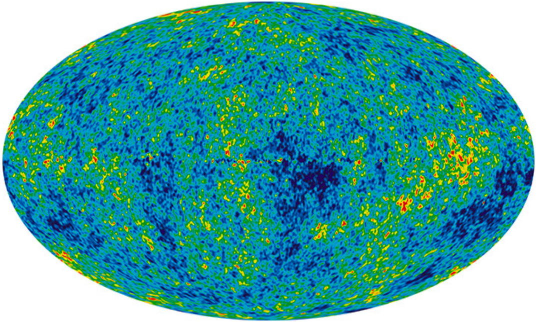
The timeline of the universe[1]

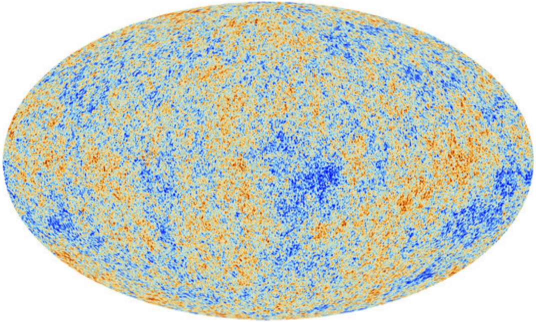


A pictorial timeline of the universe—from the big bang until today.

[1] See http://wmap.gsfc.nasa.gov/media/060915/060915_CMB_Timeline150.jpg.

CMB anisotropies as seen by WMAP and Planck





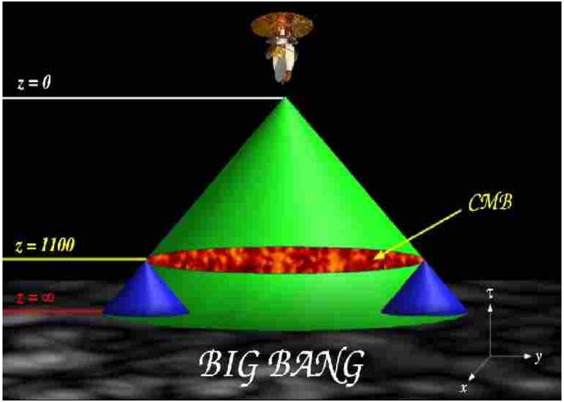
Left: All-sky map of the anisotropies in the Cosmic Microwave Background (CMB) created from nine years of Wilkinson Microwave Anisotropy Probe (WMAP) data[2]. The CMB is a snapshot of the oldest light in our universe, imprinted on the sky when the universe was just 380,000 years old.

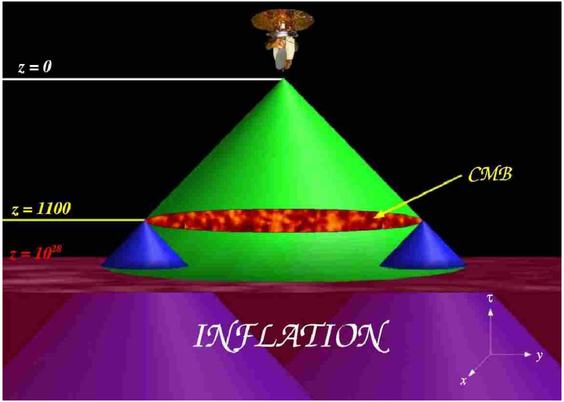
Right: The CMB anisotropies as observed by the more recent Planck mission[3]. The above images show temperature variations (as color differences) of the order of $200^\circ\ \mu\mathrm{K}$. The angular resolution of WMAP was about 1°, while that of Planck was a few arc minutes. These temperature fluctuations correspond to regions of slightly different densities, and they represent the seeds of all the structure around us today.

[2] Image from http://wmap.gsfc.nasa.gov/media/121238/index.html.

[3] Image from http://www.esa.int/Our_Activities/Space_Science/Planck/Planck_reveals_an_almost_perfect_Universe.

Inflation resolves the horizon problem



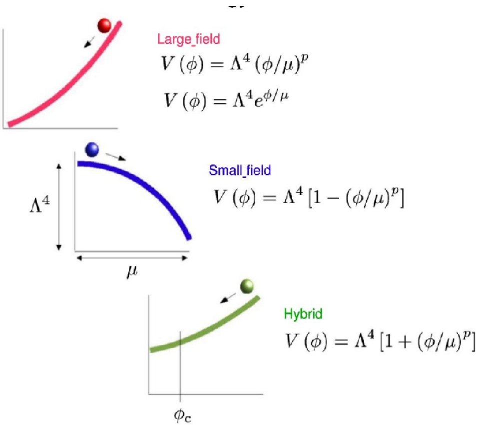


Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

Right: An illustration of how an early and sufficiently long epoch of inflation helps in resolving the horizon problem[4].

[4] Images from W. Kinney, astro-ph/0301448.

Achieving inflation with scalar fields

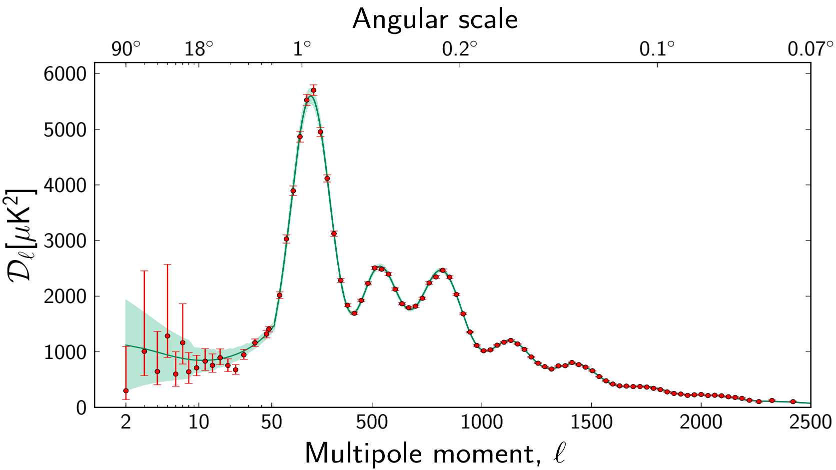


A variety of scalar field potentials have been considered to drive inflation[5]. Often, these potentials are classified as small field, large field and hybrid models.

It is the fluctuations in the inflaton field ϕ that act as the seeds for the scalar perturbations that are primarily responsible for the anisotropies in the CMB and, eventually, the present day inhomogeneities.

[5] Image from W. Kinney, astro-ph/0301448.

Angular power spectrum from the Planck data[6]



The CMB TT angular power spectrum from the Planck data (the red dots with error bars) and the theoretical, best fit ΛCDM model with a power law primordial spectrum (the solid green curve).

[6] P. A. R. Ade *et al.*, arXiv:1303.5075 [astro-ph.CO].

The scalar and the tensor perturbation spectra[7]

The dimensionless scalar power spectrum $\mathcal{P}_{\mathrm{S}}(k)$ is defined in terms of the correlation function of the Fourier modes of the so-called curvature perturbation $\hat{\mathcal{R}}_{\boldsymbol{k}}$ as follows:

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}}\, \hat{\mathcal{R}}_{\boldsymbol{k}'} \rangle = \frac{(2\,\pi)^2}{2\,k^3}\, \mathcal{P}_{\mathrm{S}}(k)\ \delta^{(3)}\,(\boldsymbol{k}+\boldsymbol{k}')\,.$$

While comparing with the observations, for convenience, one often uses the following power law, template scalar and the tensor spectra:

$$\mathcal{P}_{\mathrm{S}}(k) = \mathcal{A}_{\mathrm{S}}\,\left(\frac{k}{k_*}\right)^{n_{\mathrm{S}}-1} \quad \text{and} \quad \mathcal{P}_{\mathrm{T}}(k) = \mathcal{A}_{\mathrm{T}}\,\left(\frac{k}{k_*}\right)^{n_{\mathrm{T}}},$$

with the spectral indices n_{S} and n_{T} assumed to be constant.

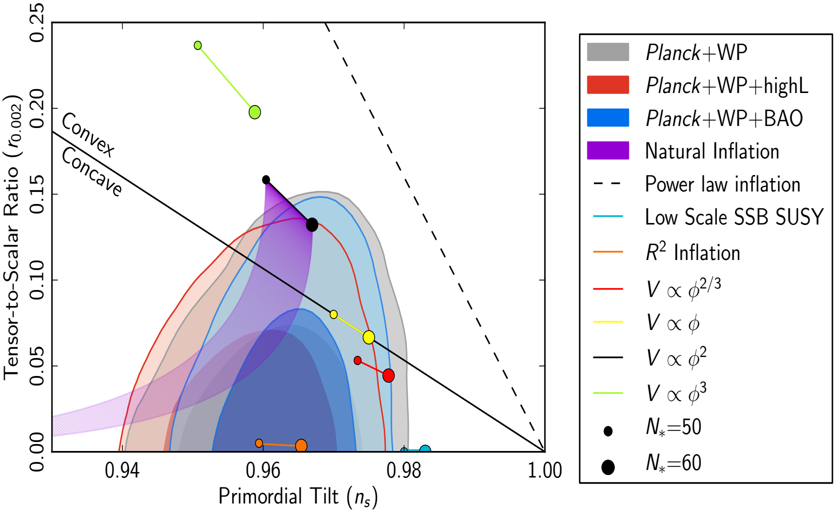
The tensor-to-scalar ratio r is defined as

$$r(k) \equiv \frac{\mathcal{P}_{\mathrm{T}}(k)}{\mathcal{P}_{\mathrm{S}}(k)}$$

and it is usual to further set $r = -8\,n_{\mathrm{T}}$, viz. the so-called consistency relation, which is valid during slow roll inflation.

[7]See, for instance, L. Sriramkumar, Curr. Sci. **97**, 868 (2009).

Constraints from Planck[8]



Joint constraints from Planck and other cosmological data on the inflationary parameters n_s and r. The performance of a few inflationary models against the data have also been indicated.

[8] P. A. R. Ade *et al.*, arXiv:1303.5082 [astro-ph.CO].

The scalar bi-spectrum and the non-Gaussianity parameter $f_{\rm NL}$

The scalar bi-spectrum $\mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ is related to the three-point correlation function of the Fourier modes of the curvature perturbation as follows[9]:

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}_1}\, \hat{\mathcal{R}}_{\boldsymbol{k}_2}\, \hat{\mathcal{R}}_{\boldsymbol{k}_3} \rangle = (2\,\pi)^3\; \mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\; \delta^{(3)}\left(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3\right).$$

For convenience, we shall set

$$\mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = (2\,\pi)^{-9/2}\; G_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3).$$

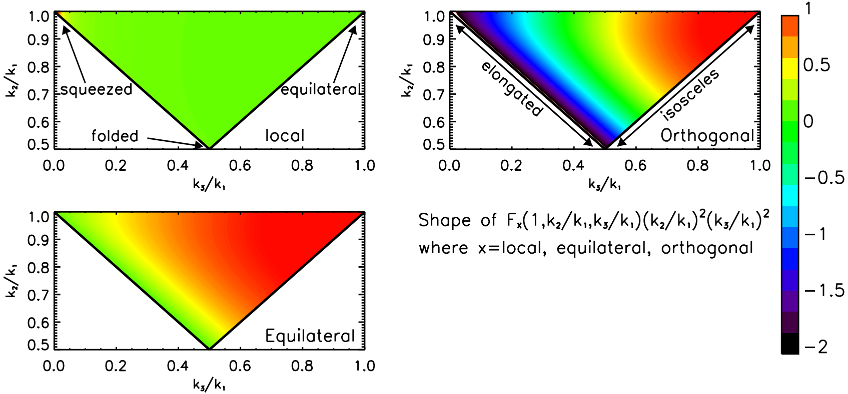
The observationally relevant, dimensionless, non-Gaussianity parameter $f_{\rm NL}$ is related to the scalar bi-spectrum as follows[10]:

$$\begin{aligned} f_{\rm NL}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= -\frac{10}{3}\, (2\,\pi)^{1/2}\; \left(k_1^3\, k_2^3\, k_3^3\right)\, \mathcal{B}_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\ &\quad \times \left[k_1^3\; \mathcal{P}_{_{\rm S}}(k_2)\; \mathcal{P}_{_{\rm S}}(k_3) + \text{two permutations}\right]^{-1} \\ &= -\frac{10}{3}\, \frac{1}{(2\,\pi)^4}\; \left(k_1^3\, k_2^3\, k_3^3\right)\, G_{\mathcal{R}\mathcal{R}\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\ &\quad \times \left[k_1^3\; \mathcal{P}_{_{\rm S}}(k_2)\; \mathcal{P}_{_{\rm S}}(k_3) + \text{two permutations}\right]^{-1}. \end{aligned}$$

[9] D. Larson *et al.*, Astrophys. J. Suppl. **192**, 16 (2011);
E. Komatsu *et al.*, Astrophys. J. Suppl. **192**, 18 (2011).

[10] J. Martin and L. Sriramkumar, JCAP **1201**, 008 (2012).

Template bi-spectra



An illustration of the three template basis bi-spectra, *viz.* the local (top left), the equilateral (bottom) and the orthogonal (top right) forms for a generic triangular configuration of the wavevectors[11].

[11] E. Komatsu, Class. Quantum Grav. **27**, 124010 (2010).

Constraints from Planck on $f_{_{\rm NL}}$

The constraints on the non-Gaussianity parameters from the recent Planck data are as follows[12]:

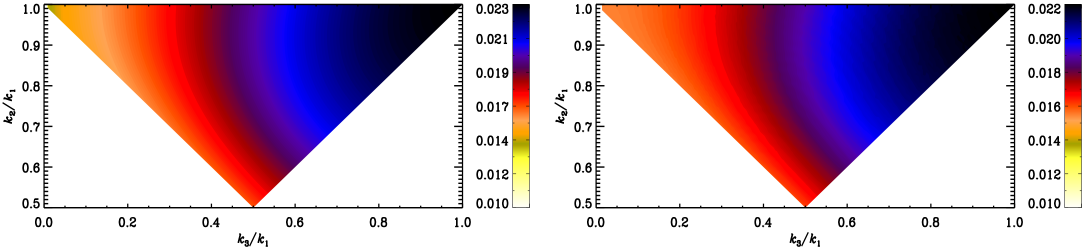
$$\begin{aligned} f_{_{\rm NL}}^{\rm loc} &= 2.7 \pm 5.8, \\ f_{_{\rm NL}}^{\rm eq} &= -42 \pm 75, \\ f_{_{\rm NL}}^{\rm orth} &= -25 \pm 39. \end{aligned}$$

It should be stressed here that these are constraints on the primordial values.

Also, the constraints on each of the $f_{_{\rm NL}}$ parameter have been arrived at assuming that the other two parameters are zero.

[12] P. A. R. Ade *et al.*, arXiv:1303.5084 [astro-ph.CO].

BINGO: A code to numerically compute the scalar bi-spectrum



A comparison of the analytical results (on the left) for the non-Gaussianity parameter $f_{_{\rm NL}}$ with the numerical results (on the right) from the BI-spectra and Non-Gaussianity Operator (BINGO) code for a generic triangular configuration of the wavevectors in the case of the standard quadratic potential[13]. The maximum difference between the numerical and the analytic results is found to be about 5%.

[13] D. K. Hazra, L. Sriramkumar and J. Martin, JCAP **05**, 026 (2013).

The cross-correlations and the tensor bi-spectrum

The cross-correlations involving two scalars and a tensor and a scalar and two tensors are defined as

$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\rm e})\,\hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_{\rm e})\,\hat{\gamma}^{\boldsymbol{k}_3}_{m_3 n_3}(\eta_{\rm e})\,\rangle = (2\,\pi)^3\,\mathcal{B}^{m_3 n_3}_{\mathcal{R}\mathcal{R}\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\;\delta^{(3)}\left(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3\right),$$

$$\begin{aligned}\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\rm e})\,\hat{\gamma}^{\boldsymbol{k}_2}_{m_2 n_2}(\eta_{\rm e})\,\hat{\gamma}^{\boldsymbol{k}_3}_{m_3 n_3}(\eta_{\rm e})\rangle &= (2\,\pi)^3\,\mathcal{B}^{m_2 n_2 m_3 n_3}_{\mathcal{R}\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\\ &\quad\times\,\delta^{(3)}\left(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3\right),\end{aligned}$$

while the tensor bi-spectrum is given by

$$\begin{aligned}\langle\hat{\gamma}^{\boldsymbol{k}_1}_{m_1 n_1}(\eta_{\rm e})\,\hat{\gamma}^{\boldsymbol{k}_2}_{m_2 n_2}(\eta_{\rm e})\,\hat{\gamma}^{\boldsymbol{k}_3}_{m_3 n_3}(\eta_{\rm e})\rangle &= (2\,\pi)^3\,\mathcal{B}^{m_1 n_1 m_2 n_2 m_3 n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\\ &\quad\times\,\delta^{(3)}\left(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3\right).\end{aligned}$$

As in the pure scalar case, we shall set

$$\mathcal{B}_{\rm ABC}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = (2\,\pi)^{-9/2}\;G_{\rm ABC}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3),$$

where each of $(\mathrm{A},\mathrm{B},\mathrm{C})$ can be either a $\mathcal{R}$ or a γ.

New non-Gaussianity parameters for cross-correlations

As in the scalar case, one can define dimensionless non-Gaussianity parameters to characterize the scalar-scalar-tensor and the scalar-tensor-tensor cross-correlations and the tensor bi-spectrum, respectively, as follows:

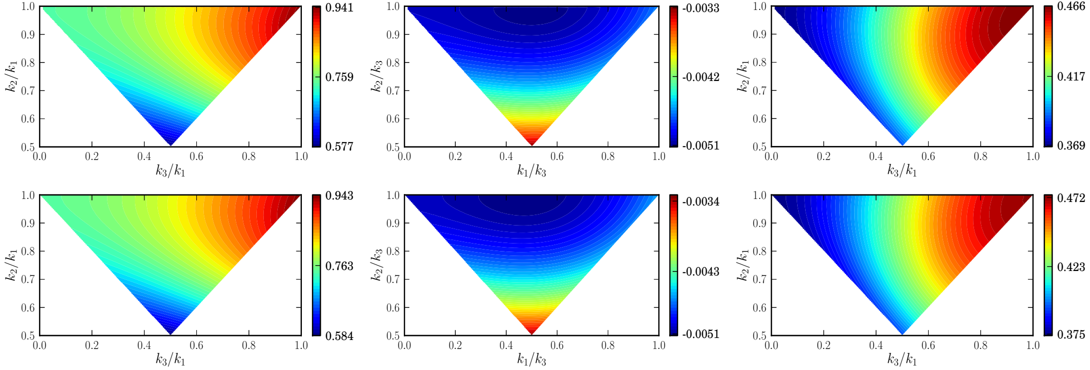
$$
\begin{aligned}
C_{\rm NL}^{\mathcal{R}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= -\frac{4}{(2\,\pi^2)^2}\, \left[k_1^3\, k_2^3\, k_3^3\, G_{\mathcal{R}\mathcal{R}\gamma}^{m_3 n_3}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\right] \\
&\quad \times \left(\Pi_{m_3 n_3, \bar{m}\bar{n}}^{\boldsymbol{k}_3}\right)^{-1} \left\{\left[k_1^3\, \mathcal{P}_{_{\rm S}}(k_2) + k_2^3\, \mathcal{P}_{_{\rm S}}(k_1)\right]\, \mathcal{P}_{_{\rm T}}(k_3)\right\}^{-1}, \\
C_{\rm NL}^{\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= -\frac{4}{(2\,\pi^2)^2}\, \left[k_1^3\, k_2^3\, k_3^3\, G_{\mathcal{R}\gamma\gamma}^{m_2 n_2 m_3 n_3}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\right] \\
&\quad \times \left\{\mathcal{P}_{_{\rm S}}(k_1)\, \left[\Pi_{m_2 n_2, m_3 n_3}^{\boldsymbol{k}_2}\, k_3^3\, \mathcal{P}_{_{\rm T}}(k_2) + \Pi_{m_3 n_3, m_2 n_2}^{\boldsymbol{k}_3}\, k_2^3\, \mathcal{P}_{_{\rm T}}(k_3)\right]\right\}^{-1}, \\
h_{\rm NL}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= -\left(\frac{4}{2\,\pi^2}\right)^2\, \left[k_1^3\, k_2^3\, k_3^3\, G_{\gamma\gamma\gamma}^{m_1 n_1 m_2 n_2 m_3 n_3}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\right] \\
&\quad \times \left[\Pi_{m_1 n_1, m_2 n_2}^{\boldsymbol{k}_1}\, \Pi_{m_3 n_3, \bar{m}\bar{n}}^{\boldsymbol{k}_2}\, k_3^3\, \mathcal{P}_{_{\rm T}}(k_1)\, \mathcal{P}_{_{\rm T}}(k_2) + \text{five permutations}\right]^{-1},
\end{aligned}
$$

where the quantity $\Pi_{m_1 n_1, m_2 n_2}^{\boldsymbol{k}}$ is defined as

$$
\Pi_{m_1 n_1, m_2 n_2}^{\boldsymbol{k}} = \sum_s \varepsilon_{m_1 n_1}^{s}(\boldsymbol{k})\, \varepsilon_{m_2 n_2}^{s\,*}(\boldsymbol{k}),
$$

with $\varepsilon_{ij}^{s}(\boldsymbol{k})$ denoting the polarization tensor associated with the gravitational waves.

Comparison between the analytical and numerical results



A comparison of the analytical results (at the bottom) for the non-Gaussianity parameters $C_{\rm NL}^{\mathcal{R}}$ (on the left), $C_{\rm NL}^{\gamma}$ (in the middle) and $h_{\rm NL}$ (on the right) with the numerical results (on top) for a generic triangular configuration of the wavevectors in the case of the standard quadratic potential[14]. As in the case of the scalar bi-spectrum, the maximum difference between the numerical and the analytic results is about 5%.

[14] V. Sreenath, R. Tibrewala and L. Sriramkumar, JCAP **1312**, 037 (2013).

Outlook

- The strong constraints on the non-Gaussianity parameter $f_{\rm NL}$ from Planck suggests that additional inputs beyond the power spectrum can aid us considerably in arriving at smaller and smaller classes of viable inflationary models[15].
- The new non-Gaussianity parameters $C_{\rm NL}^{\mathcal{R}}$ and $C_{\rm NL}^{\gamma}$ that we have introduced can play a vital role towards characterizing as well as constraining inflationary models further[16].

[15] In this context, see J. Martin, C. Ringeval and V. Vennin, arXiv:1303.3787 [astro-ph.CO].

[16] V. Sreenath, R. Tibrewala and L. Sriramkumar, JCAP **1312**, 037 (2013).

Thank you for your attention