

Probing the physics of the early universe

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Thanu Padmanabhan Memorial Lecture, 35th Kerala Science Congress
Mar Baselios Christian College of Engineering and Technology, Kuttikkanam, Idukki
February 10–14, 2023

Plan of the talk

- 1 Thanu Padmanabhan: Brief personal and academic history
- 2 The need for inflation
- 3 Constraints on inflation from Planck
- 4 Enhancing power on small scales
- 5 Implications for formation of PBHs and generation of secondary GWs
- 6 Imprints of reheating on primary GWs
- 7 Summary



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Personal history¹

- ◆ Date of birth: March 10, 1957
- ◆ Place of birth: Thiruvananthapuram
- ◆ Parents: Lakshmi and Thanu Iyer
- ◆ Thanu Iyer worked in the forest department of the Government of Kerala



Paddy's parents.

▶ Back



Paddy in September 2021

¹Photos from [Mathrubhumi \(Malayalam Weekly\)](#), September 19, 2021.



Early education and later academic history²

◆ Early education

- ◆ 1963–1972, I–X Standard, Government Karamana High School, Thiruvananthapuram
- ◆ 1973–1974, Pre-degree, Government Arts College, Thiruvananthapuram
- ◆ 1974–1977, B.Sc. Physics, University College, Thiruvananthapuram
- ◆ 1977–1979, M.Sc. Physics, University College, Thiruvananthapuram

◆ Later academic history

- ◆ 1979–1980, Doctoral student, Tata Institute of Fundamental Research, Mumbai
- ◆ 1980–1983, Research Associate, Tata Institute of Fundamental Research, Mumbai
Thesis supervisor: Jayant Narlikar
- ◆ 1980–1992, Faculty, Tata Institute of Fundamental Research, Mumbai
- ◆ 1992–2021, Faculty, Inter-University Centre for Astronomy and Astrophysics, Pune

²See J. S. Bagla and S. Engineer, *Prof. Padmanabhan: A personal and professional History*, in *Gravity and the Quantum*, Eds. J. S. Bagla and S. Engineer (Springer, Berlin, 2017).



Primary research interests and publications

Primary research interests of Prof. Padmanabhan:

- ◆ Quantum Theory
- ◆ Gravitation
- ◆ Cosmology and Structure Formation in the Universe

Prof. Padmanabhan published nearly 300 papers and reviews³.

³Source <https://www.iucaa.in/~paddy/biodata/mycv.htm>.



Doctoral students⁴

- 1 T. R. Seshadri (University of Delhi)
- 2 T. P. Singh (TIFR, Mumbai)
- 3 J. S. Bagla (IISER, Mohali)
- 4 L. Sriramkumar (IIT Madras)
- 5 K. Srinivasan (CFL, Bengaluru)
- 6 S. Shankaranarayanan (IIT Bombay)
- 7 Sunu Engineer (CEO of Embedded Computing Machines, Pune)
- 8 T. Roy Choudhury (NCRA, Pune)
- 9 Sudipta Sarkar (IIT, Gandhinagar)
- 10 Gaurang Mahajan (NCCS, Pune)

- 11 Dawood Kothawala (IIT Madras)
- 12 Sanved Kolekar (IIA, Bengaluru)
- 13 Suprit Singh (IIT, Delhi)
- 14 Krishna Parattu (IIT Mandi)
- 15 Sumanta Chakraborty (IACS, Kolkata)
- 16 Karthik Rajeev (IIT Bombay)

Joint students

- 17 Nissim Kanekar (NCRA, Pune)
- 18 Ali Nayeri (Chapman University, California, USA).

⁴Source <https://www.iucaa.in/~paddy/biodata/mycv.htm>.



Books I: 1986-2006

- 1 J. V. Narlikar and T. Padmanabhan, *Gravity, Gauge Theories and Quantum Cosmology* (Reidel, Holland, 1986)
- 2 T. Padmanabhan, *Structure Formation in the Universe* (Cambridge University Press, Cambridge, England, 1993)
- 3 T. Padmanabhan, *Cosmology and Astrophysics through Problems* (Cambridge University Press, Cambridge, England, 1996)
- 4 T. Padmanabhan, *After the First Three Minutes—The Story of our Universe* (Cambridge University Press, Cambridge, England, 1998)
- 5 T. Padmanabhan, *Theoretical Astrophysics—Volume I: Astrophysical Processes* (Cambridge University Press, Cambridge, England, 2000)
- 6 T. Padmanabhan, *Theoretical Astrophysics—Volume II: Stars and Stellar Systems* (Cambridge University Press, Cambridge, England, 2001)
- 7 T. Padmanabhan, *Theoretical Astrophysics—Volume III: Galaxies and Cosmology* (Cambridge University Press, Cambridge, England, 2002)
- 8 T. Padmanabhan, *An Invitation to Astrophysics* (World Scientific, Singapore, 2006)



Books II: 2007-2019

- 9 T. Padmanabhan, *Quantum Themes: The Charms of the Microworld* (World Scientific, Singapore, 2009)
- 10 T. Padmanabhan, *Gravitation: Foundations and Frontiers* (Cambridge University Press, Cambridge, England, 2010)
- 11 T. Padmanabhan, *Sleeping Beauties in Theoretical Physics* (Springer, Heidelberg, 2015)
- 12 T. Padmanabhan, *Quantum Field Theory: The Why, What and How*, (Springer, Heidelberg, 2016)
- 13 T. Padmanabhan and V. Padmanabhan, *The Dawn of Science: Glimpses from History for the Curious Mind* (Springer, Heidelberg, 2019)



Awards and distinctions: 1977–2000⁵

- ◆ 1977, Gold medalist, B.Sc., Kerala University
- ◆ 1979, Gold Medalist, M.Sc., Kerala University
- ◆ 1984–1989, Young Associate of Indian Academy of Sciences
- ◆ 1984, **INSA Young Scientist Award**; Fifth Prize in Gravity Essay Contest (awarded by Gravity Research Foundation, USA)
- ◆ 1991, Fellow, Indian Academy of Sciences; **Birla Science Prize**
- ◆ 1993, Fellow, National Academy of Sciences
- ◆ 1995, Fellow, Maharashtra Academy of Sciences,
- ◆ 1996, **Shanti Swarup Bhatnagar Award**
- ◆ 1997, A. C. Banerji Memorial Lecture Award
- ◆ 2000, **Millennium Medal (CSIR)**

⁵Source <https://www.iucaa.in/~paddy/biodata/mycv.htm>.



Awards and distinctions: 2001–2010⁶

- ◆ 2001, Fellow, Indian National Science Academy
- ◆ 2002, Sackler Distinguished Astronomer, Institute of Astronomy, Cambridge; Al-Khwarizmi International Award; Second Prize in Gravity Essay Contest
- ◆ 2003, Homi Bhabha Fellowship; Fifth Prize in Gravity Essay Contest; G. D. Birla Award for Scientific Research
- ◆ 2004, Miegunah Fellowship Award, University of Melbourne, Australia
- ◆ 2006, Third Prize in Gravity Essay Contest
- ◆ 2007, Padma Shri
- ◆ 2007, INSA Vainu-Bappu Medal
- ◆ 2008, First Prize in Gravity Essay Contest
- ◆ 2008, J. C. Bose Fellowship, Department of Science and Technology
- ◆ 2009, Infosys Science Prize in Physical Sciences, Infosys Science Foundation

⁶Source <https://www.iucaa.in/~paddy/biodata/mycv.htm>.



Awards and distinctions: 2011–2021⁷

- ◆ 2011, Third World Academy of Sciences Prize in Physics
- ◆ 2012, Fifth Prize in Gravity Essay Contest; Fellow, Third World Academy of Sciences
- ◆ 2012–2013, Goyal Prize in Physical Sciences
- ◆ 2014, Third Prize in Gravity Essay Contest; Homi Bhabha Lecturer at UK (IoP-IPA award),
- ◆ 2018, Fourth Prize in Gravity Essay Contest
- ◆ 2019, M. P. Birla Memorial Award
- ◆ 2020, Fifth Prize in Gravity Essay Contest
- ◆ 2021, Fourth Prize in Gravity Essay Contest
- ◆ 2021, Kerala Sasthra Puraskaram (Lifetime Achievement Award of Government of Kerala)

▶ Paddy's photo

⁷Source <https://www.iucaa.in/~paddy/biodata/mycv.htm>.



On the pursuit of physics

From the preface of *Sleeping Beauties in Theoretical Physics*

Theoretical physics is fun. Most of us indulge in it for the same reason a painter paints or a dancer dances—the process itself is so enjoyable! Occasionally, there are additional benefits like fame and glory and even practical uses; but most good theoretical physicists will agree that these are not the primary reasons why they are doing it. The fun in figuring out the solutions to Nature's brain teasers is a reward in itself^a.

^aT. Padmanabhan, *Sleeping Beauties in Theoretical Physics* (Springer, Heidelberg, 2015).



Paddy: A force of nature



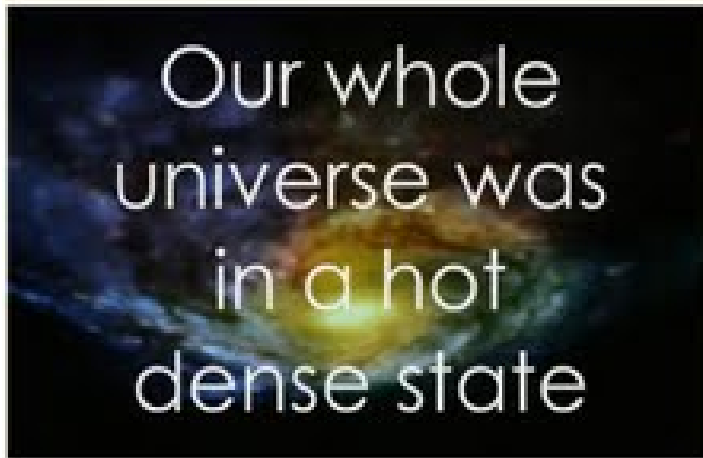
Paddy was a *force of nature*, 'full of energy, unstoppable, unchallengeable, and unforgettable', much like gravitation that he strived all his life to understand.

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The big bang model seems popular!

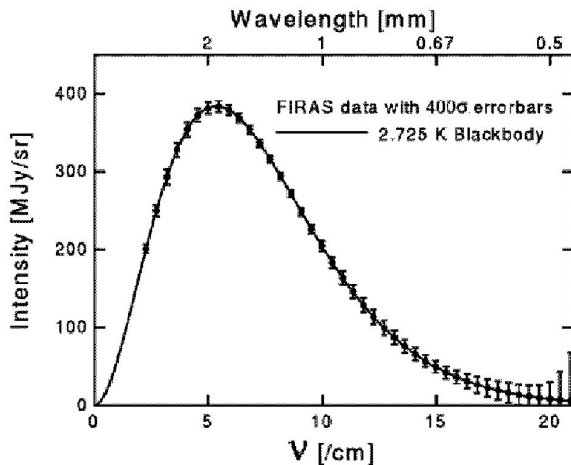


The current view of the universe, encapsulated in the hot big bang model, seems popular. The above image is a screen grab from the theme song of the recent American sitcom 'The Big Bang Theory'⁸!

⁸See http://www.cbs.com/shows/big_bang_theory/.



The cosmic microwave background (CMB)

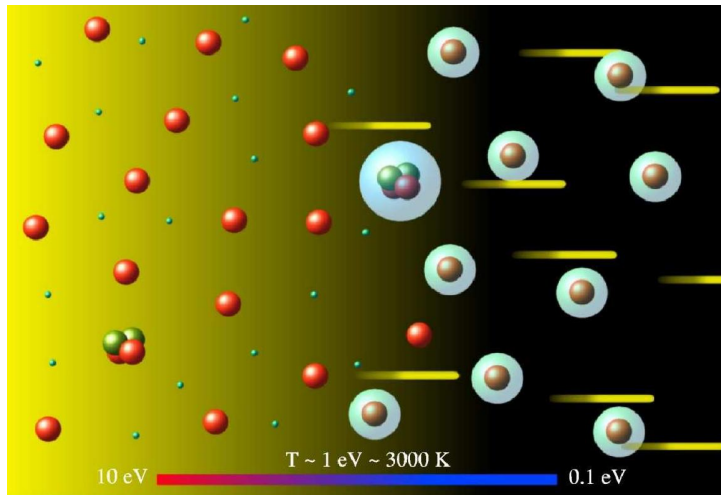


The spectrum of the CMB as measured by the COBE satellite⁹. It is a perfect Planck spectrum, corresponding to a temperature of 2.725° K. The error bars have been amplified **400** times so that they are visible!

⁹Image from http://www.astro.ucla.edu/~wright/cosmo_01.htm.



Decoupling of matter and radiation¹⁰

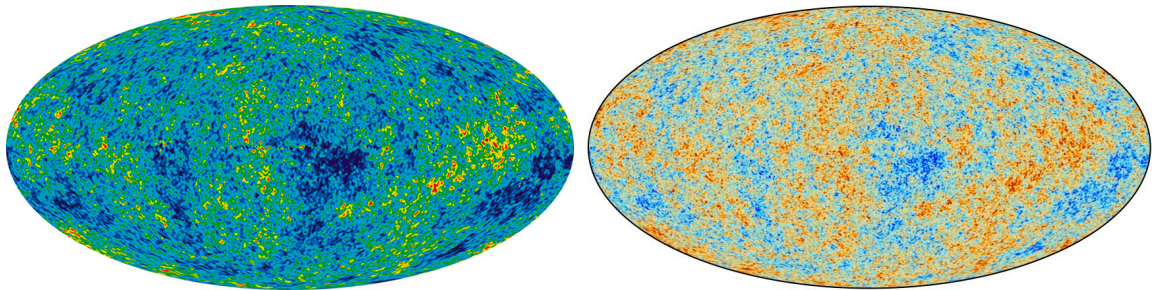


Matter and radiation cease to interact at a temperature of about $T \simeq 3000^\circ \text{ K}$, which corresponds to a redshift of about $z \simeq 1000$.

¹⁰Image from W. H. Kinney, [arXiv:astro-ph/0301448v2](https://arxiv.org/abs/astro-ph/0301448v2).



CMB anisotropies as seen by WMAP and Planck



Left: All-sky map of the anisotropies in the CMB created from nine years of Wilkinson Microwave Anisotropy Probe (WMAP) data¹¹.

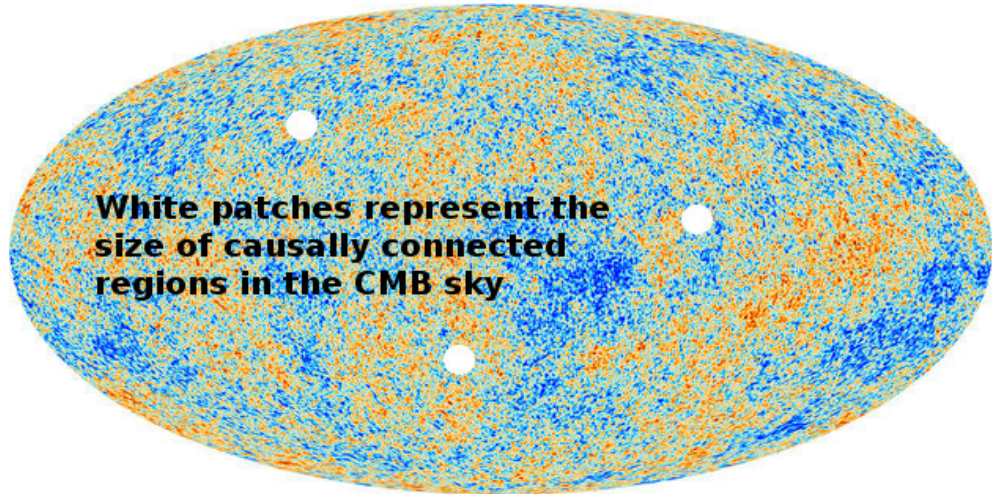
Right: CMB intensity map derived from the joint analysis of Planck, WMAP, and 408 MHz observations¹². The above images show temperature variations (as color differences) of the order of $200^\circ \mu\text{K}$.

¹¹Image from <http://wmap.gsfc.nasa.gov/media/121238/index.html>.

¹²Planck Collaboration (R. Adam *et al.*), *Astron. Astrophys.* **594**, A1 (2016).



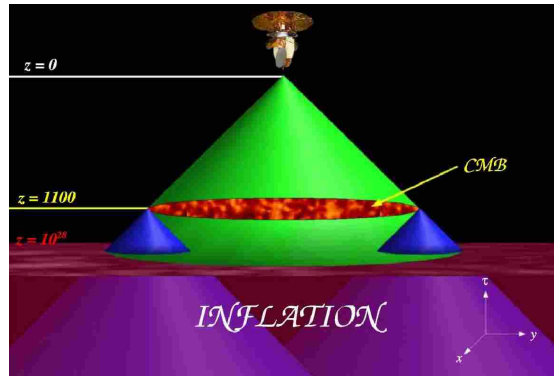
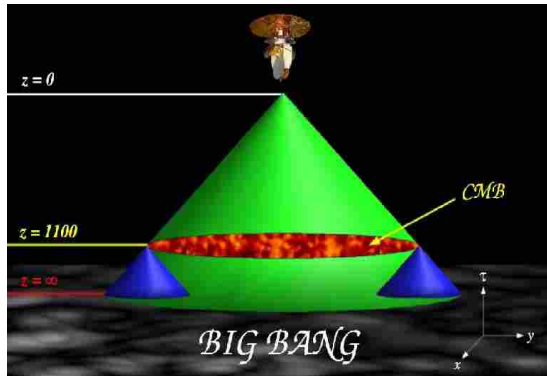
The horizon problem



The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface, which subtends an angle of about 1° today, could not have interacted before decoupling.



The resolution of the horizon problem in the inflationary scenario

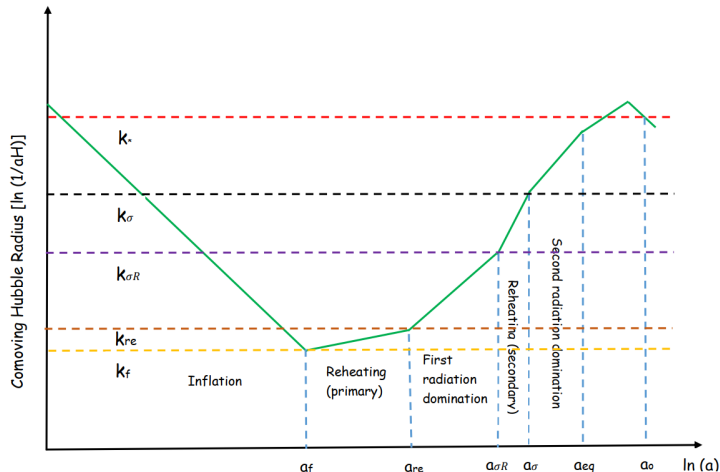


Another illustration of the horizon problem (on the left), and an illustration of its resolution (on the right) through an early and sufficiently long epoch of inflation¹³.

¹³Images from W. Kinney, [astro-ph/0301448](https://arxiv.org/abs/astro-ph/0301448).



Behavior of comoving wave lengths and the Hubble radius

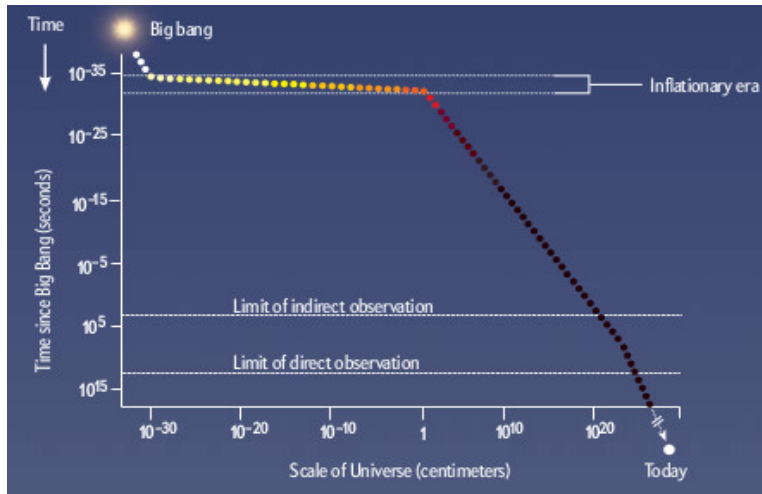


Behavior of the comoving wave lengths (horizontal lines in different colors) and the comoving Hubble radius $d_H/a = (aH)^{-1}$ (in green) across different epochs¹⁴.

¹⁴Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, Phys. Rev. D **104**, 063513 (2021).



The time and duration of inflation

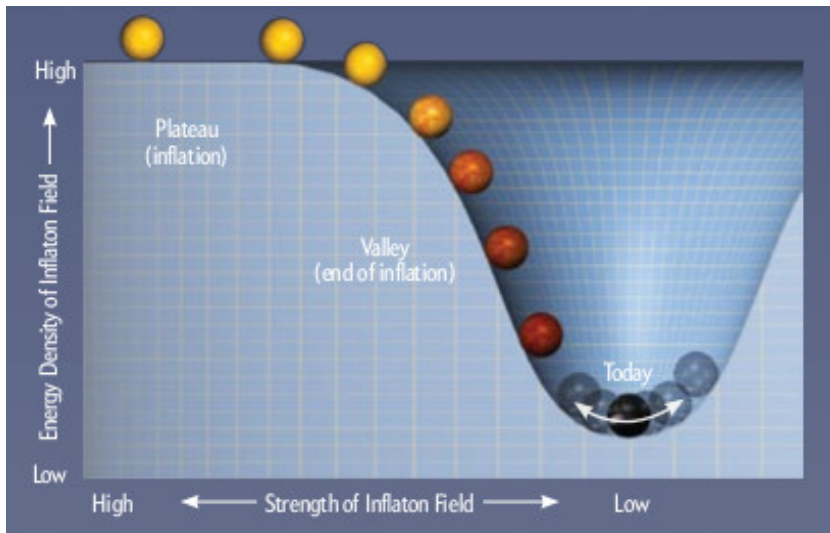


Inflation – a brief period of accelerated expansion – is expected to have taken place during the very stages of the universe¹⁵.

¹⁵Image from P. J. Steinhardt, *Sci. Am.* **304**, 18 (2011).



Driving inflation with scalar fields

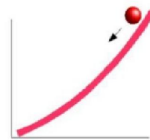


Inflation can be achieved with scalar fields encountered in high energy physics¹⁶.

¹⁶Image from P. J. Steinhardt, *Sci. Am.* **304**, 34 (2011).



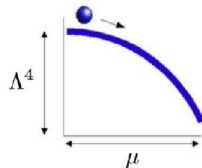
A variety of potentials to choose from



Large field

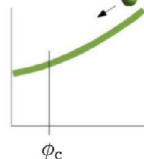
$$V(\phi) = \Lambda^4 (\phi/\mu)^p$$

$$V(\phi) = \Lambda^4 e^{\phi/\mu}$$



Small field

$$V(\phi) = \Lambda^4 [1 - (\phi/\mu)^p]$$



Hybrid

$$V(\phi) = \Lambda^4 [1 + (\phi/\mu)^p]$$

A variety of scalar field potentials have been considered to drive inflation¹⁷. Often, these potentials are classified as small field, large field and hybrid models.

¹⁷Image from [W. Kinney, astro-ph/0301448](https://arxiv.org/abs/astro-ph/0301448).



Spectral indices and the tensor-to-scalar ratio

The scalar and tensor power spectra, viz. $\mathcal{P}_S(k)$ and $\mathcal{P}_T(k)$, can be expressed in terms of the Fourier modes f_k and g_k as follows:

$$\mathcal{P}_S(k) = \frac{k^3}{2\pi^2} |f_k(\eta_e)|^2,$$

$$\mathcal{P}_T(k) = 8 \frac{k^3}{2\pi^2} |g_k(\eta_e)|^2,$$

with η_e corresponding to suitably late times during inflation.

While comparing with the observations, for convenience, one often uses the following power law, template scalar and the tensor spectra:

$$\mathcal{P}_S(k) = A_S \left(\frac{k}{k_*} \right)^{n_S - 1}, \quad \mathcal{P}_T(k) = A_T \left(\frac{k}{k_*} \right)^{n_T},$$

with the spectral indices n_S and n_T assumed to be constant. The tensor-to-scalar ratio r is defined as

$$r(k) = \frac{\mathcal{P}_T(k)}{\mathcal{P}_S(k)}.$$

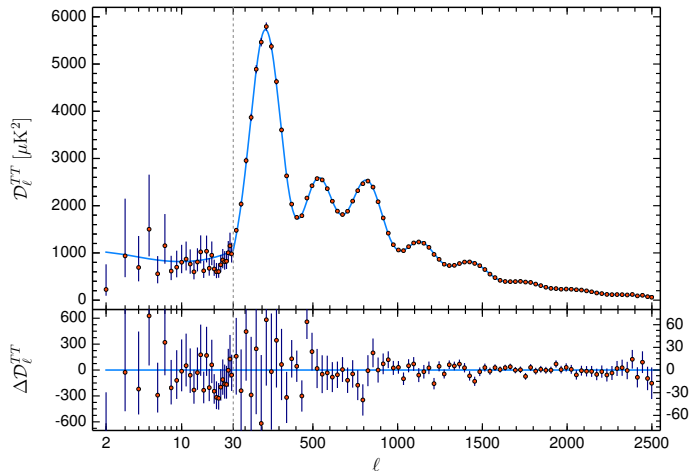


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CMB angular power spectrum from Planck

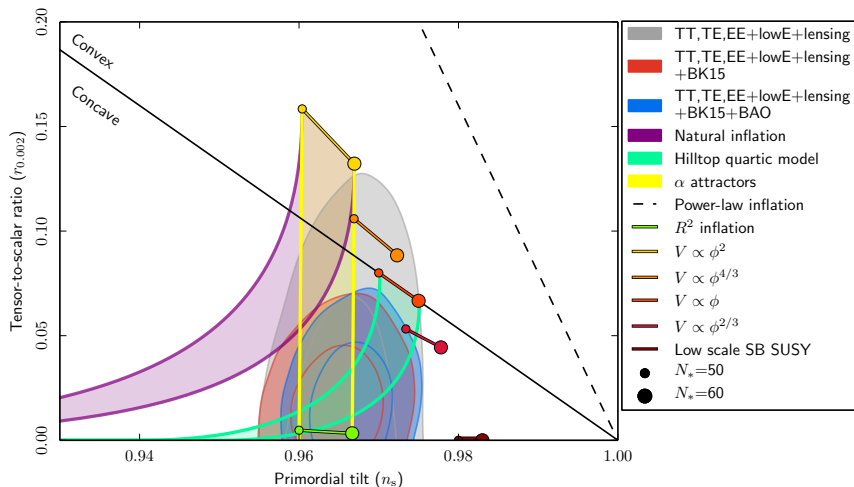


The CMB TT angular power spectrum from the Planck 2018 data (red dots with error bars) and the best fit Λ CDM model with a power law primordial spectrum (solid blue curve)¹⁸

¹⁸Planck Collaboration (N. Aghanim *et al.*), *Astron. Astrophys.* **641**, A6 (2020).



Performance of inflationary models in the n_s - r plane



Joint constraints on n_s and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of some of the popular inflationary models¹⁹.

¹⁹Planck Collaboration (Y. Akrami *et al.*), *Astron. Astrophys.* **641**, A10 (2020).

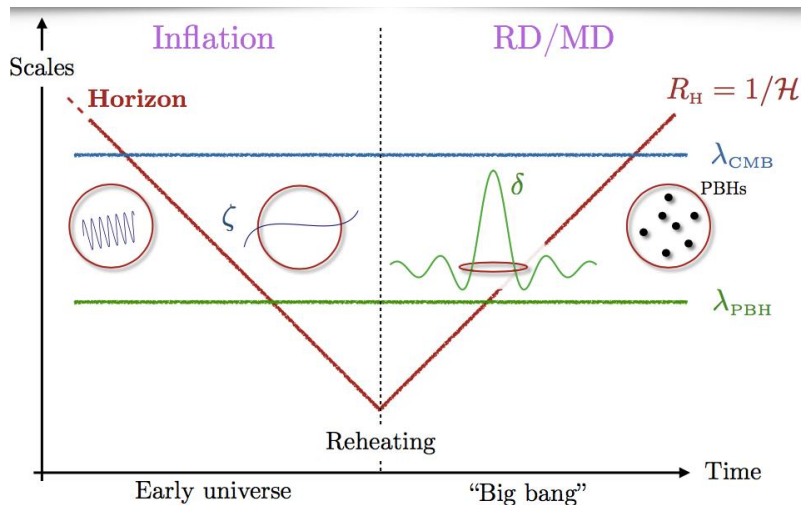


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Formation of black holes in the early universe

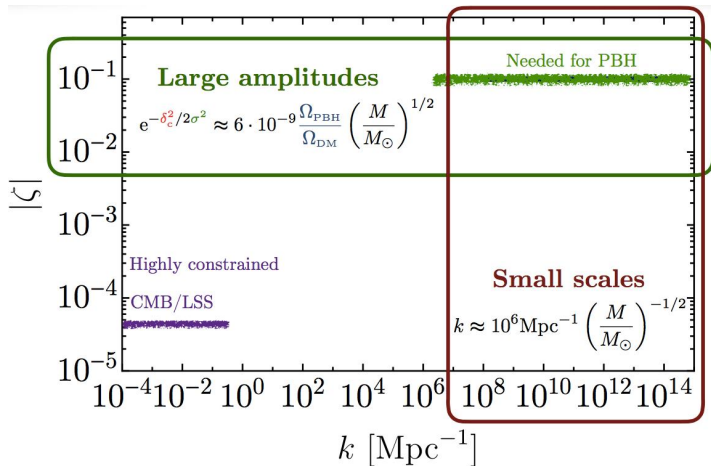


Black holes can form when perturbations with significant amplitudes reenter the Hubble radius during the radiation dominated epoch²⁰.

²⁰Figure from G. Franciolini, [arXiv:2110.06815 \[astro-ph.CO\]](https://arxiv.org/abs/2110.06815).



Amplitude required to form significant number of PBHs

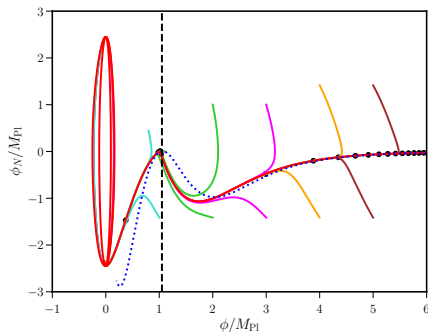
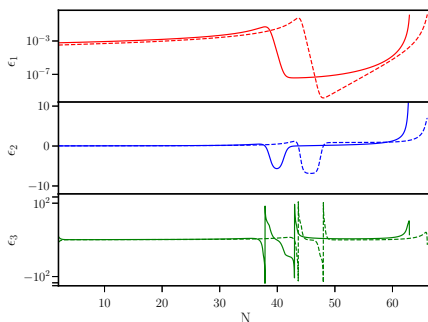


In order to form significant number of primordial black holes (PBHs), the amplitude of the perturbations on small scales has to be large enough such that the dimensionless amplitude of the scalar perturbation is close to unity²¹.

²¹ Figure credit G. Franciolini.



Potentials admitting ultra slow roll inflation



Potentials leading to ultra slow roll inflation (with $x = \phi/v$, v being a constant)²²:

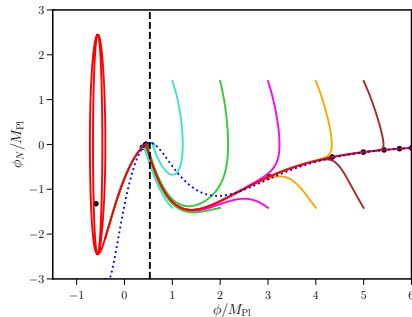
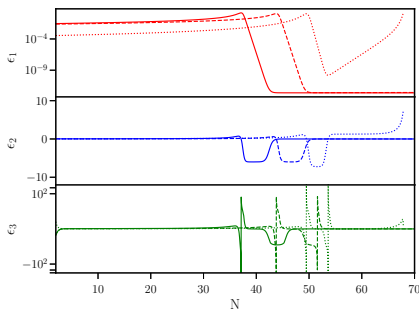
$$\text{USR1} : V(\phi) = V_0 \frac{6x^2 - 4\alpha x^3 + 3x^4}{(1 + \beta x^2)^2},$$

$$\text{USR2} : V(\phi) = V_0 \left\{ \tanh\left(\frac{\phi}{\sqrt{6} M_{\text{Pl}}}\right) + A \sin\left[\frac{\tanh\left[\phi/(\sqrt{6} M_{\text{Pl}})\right]}{f_\phi}\right] \right\}^2.$$

²² J. Garcia-Bellido and E. R. Morales, Phys. Dark Univ. **18**, 47 (2017);
I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).



Potentials permitting punctuated inflation



Potentials admitting punctuated inflation²³:

$$\text{PI1} : V(\phi) = V_0 (1 + B \phi^4), \quad \text{PI2} : V(\phi) = \frac{m^2}{2} \phi^2 - \frac{2m^2}{3\phi_0} \phi^3 + \frac{m^2}{4\phi_0^2} \phi^4,$$

$$\text{PI3} : V(\phi) = V_0 \left[c_0 + c_1 \tanh \left(\frac{\phi}{\sqrt{6\alpha} M_{\text{Pl}}} \right) + c_2 \tanh^2 \left(\frac{\phi}{\sqrt{6\alpha} M_{\text{Pl}}} \right) + c_3 \tanh^3 \left(\frac{\phi}{\sqrt{6\alpha} M_{\text{Pl}}} \right) \right]^2.$$

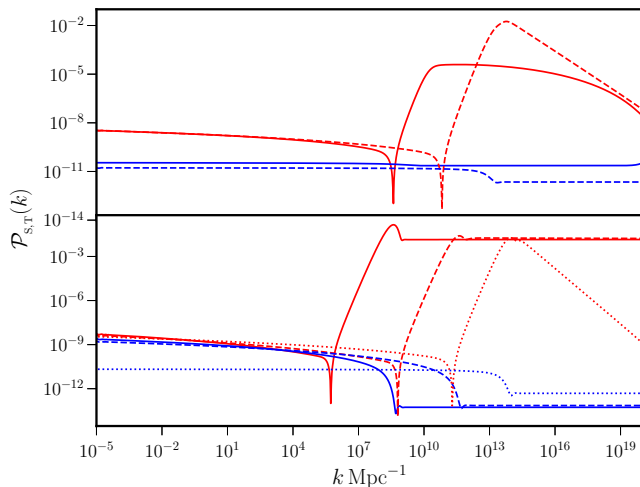
²³D. Roberts, A. R. Liddle and D. H. Lyth, Phys. Rev. D **51**, 4122 (1995);

R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **01**, 009 (2009);

I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).



Power spectra in ultra slow roll and punctuated inflation



Scalar and tensor power spectra arising in the ultra slow roll and punctuated inflationary models (in red and blue)²⁴.

²⁴H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, *Phys. Rev. D* **103**, 083510 (2021).



The two field model of interest

It has been noticed that two scalar fields ϕ and χ governed by the following action:

$$S[\phi, \chi] = \int d^4x \sqrt{-g} \left[-\frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{f(\phi)}{2} \partial^\mu \chi \partial_\mu \chi - V(\phi, \chi) \right]$$

described by a potential such as

$$V(\phi, \chi) = V_0 \frac{\phi^2}{\phi_0^2 + \phi^2} + \frac{m_\chi^2}{2} \chi^2$$

and the non-canonical coupling functions

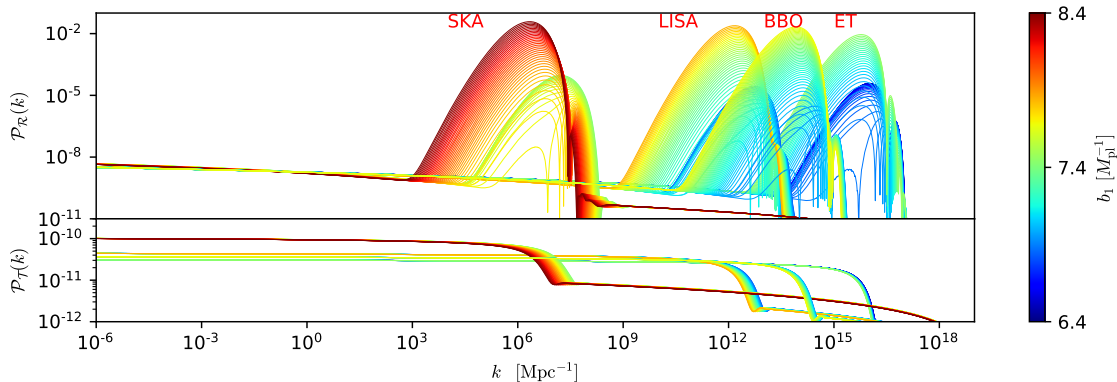
$$f_1(\phi) = e^{2b_1 \phi} \quad \text{or} \quad f_2(\phi) = e^{2b_2 \phi^2}$$

can lead to features in the scalar power spectrum²⁵.

²⁵M. Braglia, D. K. Hazra, L. Sriramkumar and F. Finelli, JCAP **08** 025 (2020).



Enhanced power on small scales in two field models



The scalar (on top) and the tensor (at the bottom) power spectra evaluated at the end of inflation have been plotted for a few different sets of initial conditions for the fields and a range of values of the parameter b_1 ²⁶.

²⁶ M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

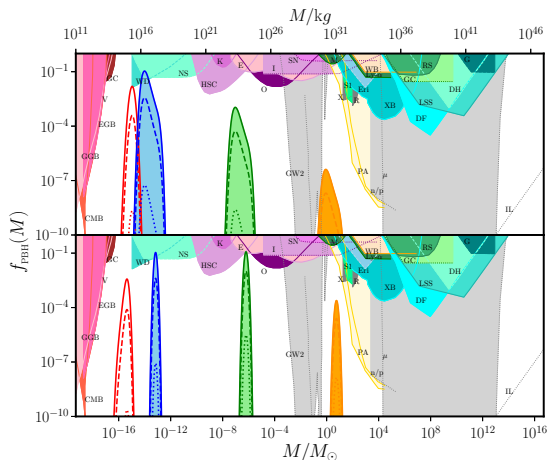


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$f_{\text{PBH}}(M)$ in ultra slow roll and punctuated inflation

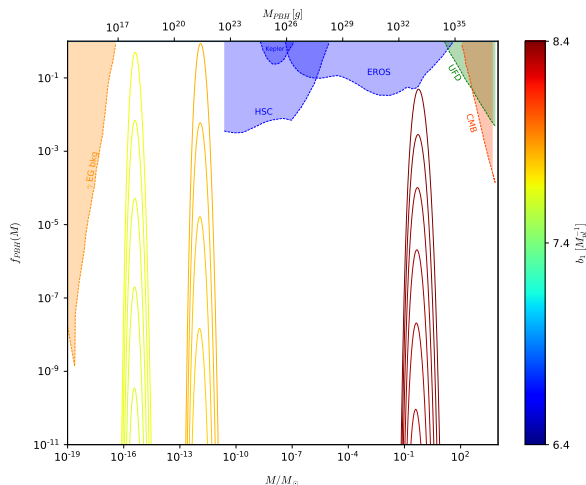


The fraction of PBHs contributing to the dark matter density today f_{PBH} has been plotted as a function of the mass M of the PBHs for the various models of interest, viz. USR2 and PI3 (in red, on top and at the bottom)²⁷.

²⁷H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, *Phys. Rev. D* **103**, 083510 (2021).



$f_{\text{PBH}}(M)$ in the two field model

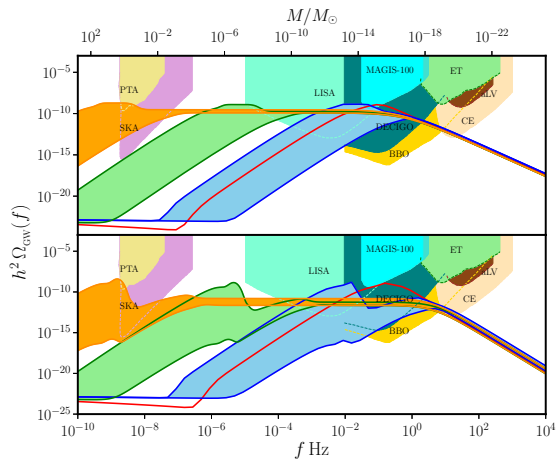


The fraction of PBHs contributing to the dark matter density today $f_{\text{PBH}}(M)$ in the two field model of our interest²⁸.

²⁸ M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).



$\Omega_{\text{GW}}(f)$ in ultra slow roll and punctuated inflation

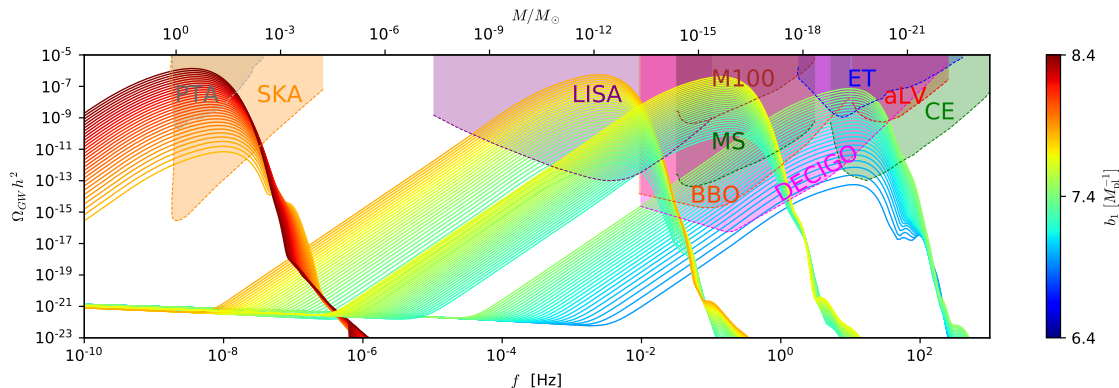


The dimensionless density parameter Ω_{GW} of secondary gravitational waves (GWs) arising in the models of USR2 and PI3 (in red, on top and at the bottom) have been plotted as a function of the frequency f ²⁹.

²⁹H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).



$\Omega_{\text{GW}}(f)$ in the two field model



The dimensionless spectral density $\Omega_{\text{GW}}(f)$ in the two field model has been plotted for a set of initial conditions of the background fields as well as a range of values of the parameter b_1 ³⁰.

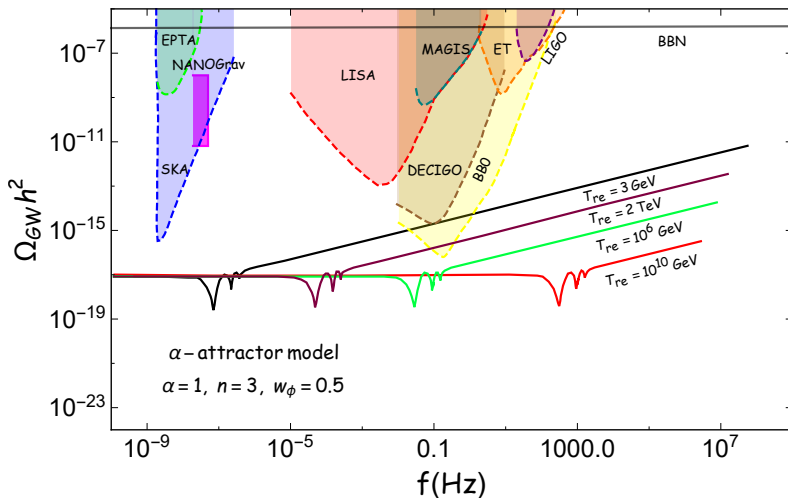
³⁰ M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).



Plan of the talk

- 1 Thanu Padmanabhan: Brief personal and academic history
- 2 The need for inflation
- 3 Constraints on inflation from Planck
- 4 Enhancing power on small scales
- 5 Implications for formation of PBHs and generation of secondary GWs
- 6 Imprints of reheating on primary GWs**
- 7 Summary



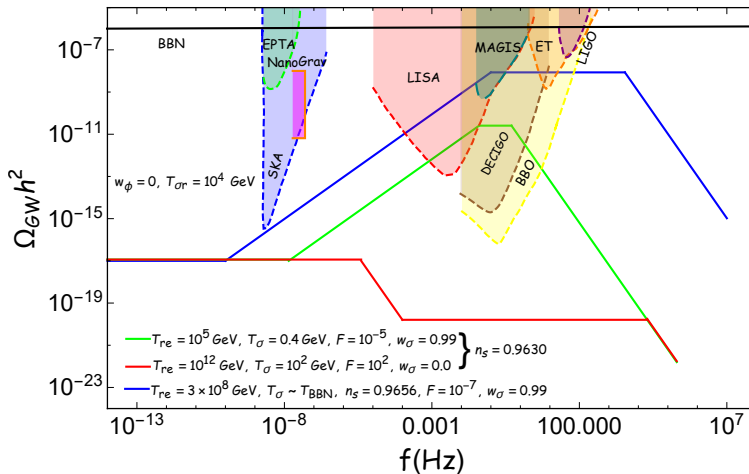
Effects on $\Omega_{\text{GW}}(f)$ due to reheating

► Behavior of the scalar field

The dimensionless spectral energy density of *primary* GWs today, viz. $\Omega_{\text{GW}}(f)$, has been plotted for different reheating temperatures (in red, green, brown and black)³¹.

³¹Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, Phys. Rev. D **104**, 063513 (2021).



Effects on $\Omega_{\text{GW}}(f)$ due to late time entropy production

The dimensionless spectral energy density of primary GWs observed today $\Omega_{\text{GW}}(f)$ has been plotted in a scenario involving late time production of entropy.



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Summary

- ◆ Inflationary models permitting an epoch of ultra slow roll lead to enhanced power on small scales, resulting in significant production of PBHs and increased strengths of secondary GWs.
- ◆ The two field models require less amount of fine tuning to generate features in the primordial spectrum³².
- ◆ A secondary phase of reheating with a suitable equation of state parameter leads to primary GWs with significantly high amplitudes³³.

³²G. A. Palma, S. Sypsas, C. Zenteno, *Phys. Rev. Lett.* **125**, 121301, (2020);
J. Fumagalli, S. Renaux-Petel, J. W. Ronayne, L. T. Witkowski, arXiv:2004.08369 [hep-th].

³³Y. Gouttenoire, G. Servant, and P. Simakachorn, arXiv:2108.10328 [hep-ph];
R. T. Co, D. Dunskey, N. Fernandez, A. Ghalsasi, L. J. Hall, K. Harigaya and J. Shelton, *JHEP* **09**, 116 (2022).



Talk based on

- ◆ M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, *Generating PBHs and small-scale GWs in two-field models of inflation*, JCAP **08**, 001 (2020) [arXiv:2005.02895 [astro-ph.CO]].
- ◆ H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, *PBHs and secondary GWs from ultra slow roll and punctuated inflation*, Phys. Rev. D **103**, 083510 (2021) [arXiv:2008.12202 [astro-ph.CO]].
- ◆ Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, *Decoding the phases of early and late time reheating through imprints on primordial gravitational waves*, Phys. Rev. D **104**, 063513 (2021) [arXiv:2105.09242 [astro-ph.CO]].
- ◆ H. V. Ragavendra and L. Sriramkumar, *Observational imprints of enhanced scalar power on small scales in ultra slow roll inflation and associated non-Gaussianities*, arXiv:2301.08887 [astro-ph.CO], invited review, to appear in Galaxies.



Thank you for your attention