Loop contributions to the scalar power spectrum due to quartic order action in ultra slow roll inflation

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Plan of the talk

1. Constraints on inflation from Planck
2. Enhancing power on small scales
3. Formation of PBHs and generation of secondary GWs
4. Non-Gaussianities generated in ultra slow roll and punctuated inflation
5. Loop contributions to the power spectrum
6. Calculating the loop contributions
7. Summary

This talk is based on. . .

- H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, *Primordial black holes and secondary gravitational waves from ultra slow roll and punctuated inflation*, Phys. Rev. D **103**, 083510 (2021) [arXiv:2008.12202 [astro-ph.CO]].
- H. V. Ragavendra and L. Sriramkumar, *Observational imprints of enhanced scalar power on small scales in ultra slow roll inflation and associated non-Gaussianities*, Galaxies **11**, 34 (2023) [arXiv:2301.08887 [astro-ph.CO]].
- S. Maity, H. V. Ragavendra, S. K. Sethi and L. Sriramkumar, *Loop contributions to the scalar power spectrum due to quartic order action in ultra slow roll inflation*, arXiv:2307.13636 [astro-ph.CO].

Plan of the talk

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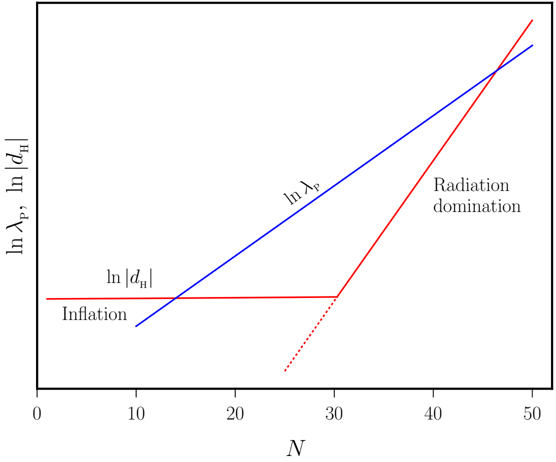
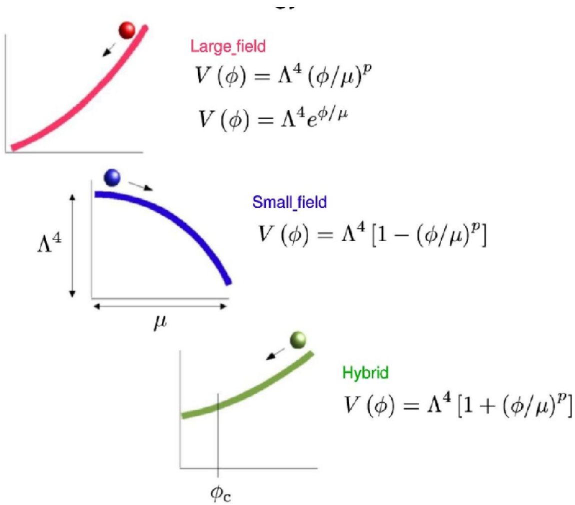
Bringing the modes inside the Hubble radius



The physical wavelength $\lambda_{\rm P} \propto a$ (in blue) and the Hubble radius $d_{\rm H} = H^{-1}$ (in red) in the inflationary scenario[1]. The scale factor is expressed in terms of e-folds N as $a(N) \propto {\rm e}^{N}$.

[1] See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.

A variety of potentials to choose from



A variety of scalar field potentials have been considered to drive inflation[2]. Often, these potentials are classified as small field, large field and hybrid models.

[2] Image from W. Kinney, astro-ph/0301448.

The quadratic action governing the perturbations

One can show that, at the quadratic order, the action governing the curvature perturbation $\mathcal{R}$ and the tensor perturbation γ_{ij} are given by[3]

$$\mathcal{S}_2[\mathcal{R}(\eta,\boldsymbol{x})] = \frac{1}{2}\int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\; z^2\, \left[\mathcal{R}'^2 - (\partial\mathcal{R})^2\right],$$

Back to the cubic scalar action

$$\mathcal{S}_2[\gamma_{ij}(\eta,\boldsymbol{x})] = \frac{M_{_{\mathrm{Pl}}}^2}{8}\int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\; a^2\, \left[\gamma_{ij}'^2 - (\partial\gamma_{ij})^2\right].$$

These actions lead to the following equations of motion governing the scalar and tensor modes, say, f_k and h_k:

Back to the analytical solution

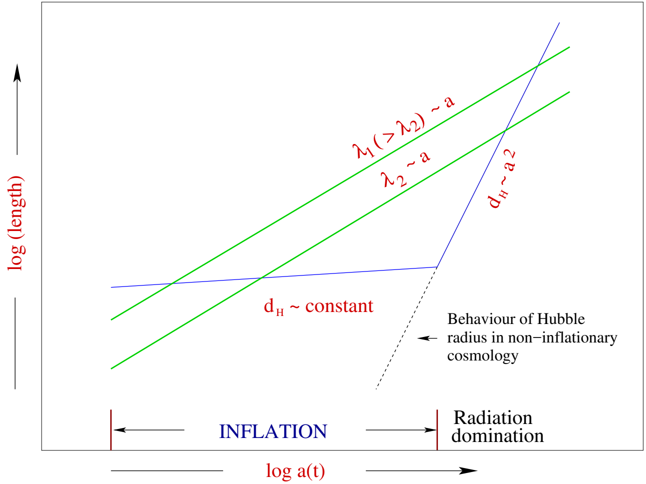
$$f_k'' + 2\,\frac{z'}{z}\, f_k' + k^2\, f_k = 0,$$

$$g_k'' + 2\,\frac{a'}{a}\, g_k' + k^2\, g_k = 0,$$

where $z = a\, M_{_{\mathrm{Pl}}}\, \sqrt{2\,\epsilon_1}$, with $\epsilon_1 = -\mathrm{d}\ln H/\mathrm{d}N$ being the first slow roll parameter.

[3] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).

From inside the Hubble radius to super-Hubble scales



The initial conditions are imposed in the sub-Hubble regime when the modes are well inside the Hubble radius (*viz.* when $k/(a\,H) \gg 1$) and the power spectra are evaluated when they sufficiently outside (*i.e.* as $k/(a\,H) \ll 1$).

Spectral indices and the tensor-to-scalar ratio

The scalar and tensor power spectra, viz. $\mathcal{P}_{\mathrm{S}}(k)$ and $\mathcal{P}_{\mathrm{T}}(k)$, can be expressed in terms of the Fourier modes f_k and g_k as follows:

$$\begin{aligned}\mathcal{P}_{\mathrm{S}}(k) &= \frac{k^3}{2\,\pi^2}\,|f_k(\eta_{\mathrm{e}})|^2,\\ \mathcal{P}_{\mathrm{T}}(k) &= 8\,\frac{k^3}{2\,\pi^2}\,|g_k(\eta_{\mathrm{e}})|^2,\end{aligned}$$

with η_{e} corresponding to suitably late times during inflation.

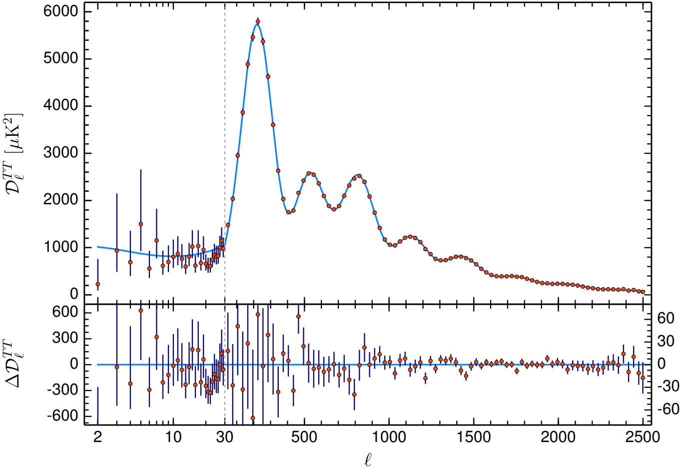
While comparing with the observations, for convenience, one often uses the following power law, template scalar and the tensor spectra:

$$\mathcal{P}_{\mathrm{S}}(k) = A_{\mathrm{S}}\,\left(\frac{k}{k_*}\right)^{n_{\mathrm{S}}-1}, \qquad \mathcal{P}_{\mathrm{T}}(k) = A_{\mathrm{T}}\,\left(\frac{k}{k_*}\right)^{n_{\mathrm{T}}},$$

with the spectral indices n_{S} and n_{T} assumed to be constant. The tensor-to-scalar ratio r is defined as

$$r(k) = \frac{\mathcal{P}_{\mathrm{T}}(k)}{\mathcal{P}_{\mathrm{S}}(k)}.$$

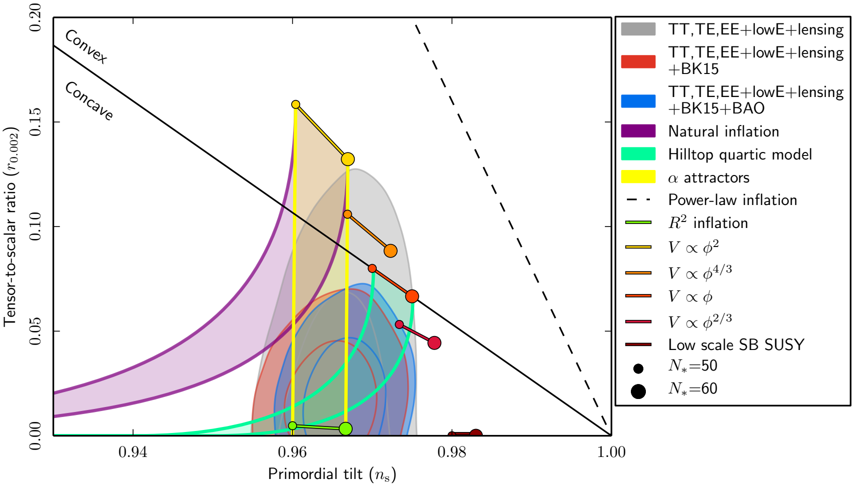
CMB angular power spectrum from Planck



The CMB TT angular power spectrum from the Planck 2018 data (red dots with error bars) and the best fit ΛCDM model with a power law primordial spectrum (solid blue curve)[4].

[4] Planck Collaboration (N. Aghanim *et al.*), Astron. Astrophys. **641**, A6 (2020).

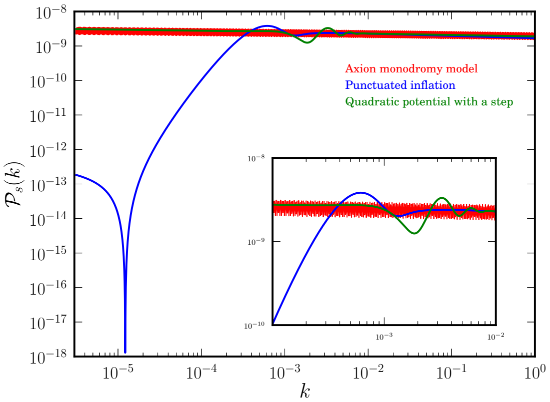
Performance of inflationary models in the n_s-r plane

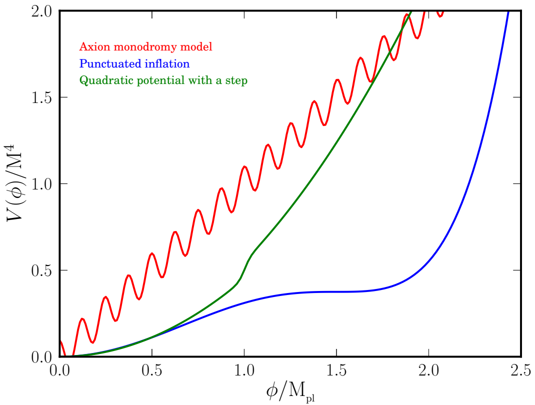


Joint constraints on n_s and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of some of the popular inflationary models[5].

[5] Planck Collaboration (Y. Akrami *et al.*), Astron. Astrophys. **641**, A10 (2020).

Spectra leading to an improved fit to the CMB data





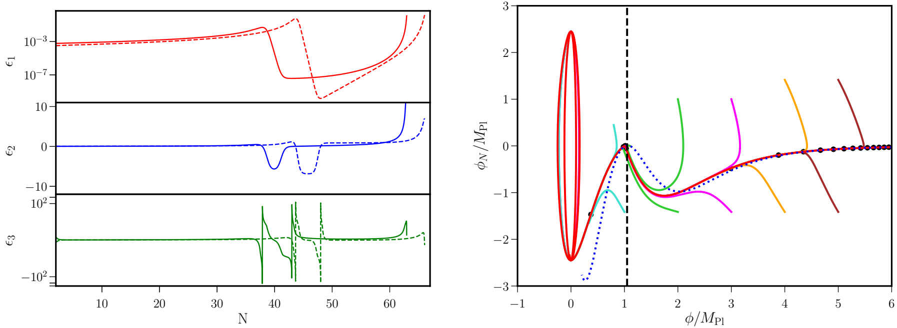
Back to power spectrum in USR

The scalar power spectra (on the left) arising in different inflationary models (on the right) that lead to a better fit to the CMB data than the conventional power law spectrum[6].

[6] R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **01**, 009 (2009); D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **10**, 008 (2010); M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, Phys. Rev. D **87**, 083526 (2013).

Plan of the talk

Potentials admitting ultra slow roll inflation



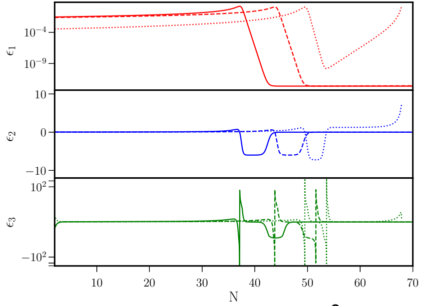
Potentials leading to ultra slow roll inflation (with $x=\phi/v$, v being a constant)[7]:

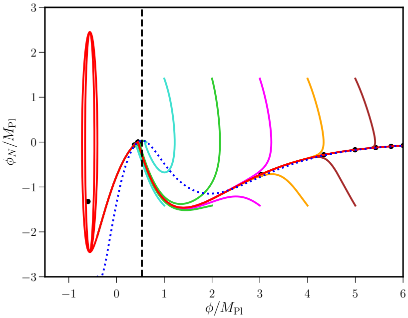
$$\mathrm{USR1}: V(\phi) = V_0\, \frac{6\,x^2-4\,\alpha\,x^3+3\,x^4}{(1+\beta\,x^2)^2},$$

$$\mathrm{USR2}: V(\phi) = V_0\, \left\{\tanh\left(\frac{\phi}{\sqrt{6}\,M_{_{\rm Pl}}}\right) + A\,\sin\left[\frac{\tanh\left[\phi/\left(\sqrt{6}\,M_{_{\rm Pl}}\right)\right]}{f_\phi}\right]\right\}^2.$$

[7] J. Garcia-Bellido and E. R. Morales, Phys. Dark Univ. **18**, 47 (2017);
I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).

Potentials permitting punctuated inflation





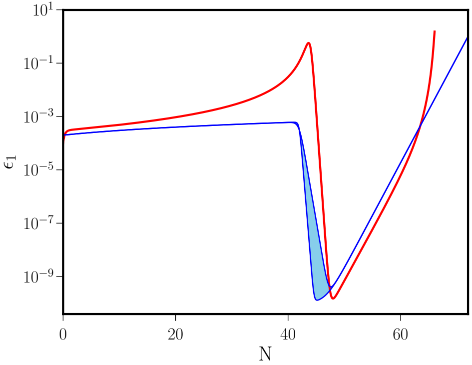
Potentials admitting punctuated inflation[8]:

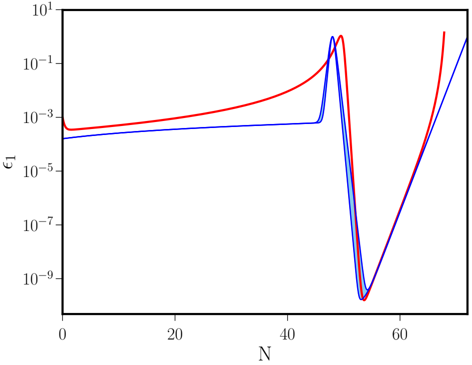
$$\mathrm{PI1}: V(\phi) = V_0\,\left(1 + B\,\phi^4\right), \quad \mathrm{PI2}: V(\phi) = \frac{m^2}{2}\,\phi^2 - \frac{2\,m^2}{3\,\phi_0}\,\phi^3 + \frac{m^2}{4\,\phi_0^2}\,\phi^4,$$

$$\mathrm{PI3}: V(\phi) = V_0\,\left[c_0 + c_1\,\tanh\left(\frac{\phi}{\sqrt{6\,\alpha}\,M_{_{\rm Pl}}}\right) + c_2\,\tanh^2\left(\frac{\phi}{\sqrt{6\,\alpha}\,M_{_{\rm Pl}}}\right) + c_3\,\tanh^3\left(\frac{\phi}{\sqrt{6\,\alpha}\,M_{_{\rm Pl}}}\right)\right]^2.$$

[8] D. Roberts, A. R. Liddle and D. H. Lyth, Phys. Rev. D **51**, 4122 (1995);
R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **01**, 009 (2009);
I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).

Reconstructing scenarios of ultra slow roll and punctuated inflation





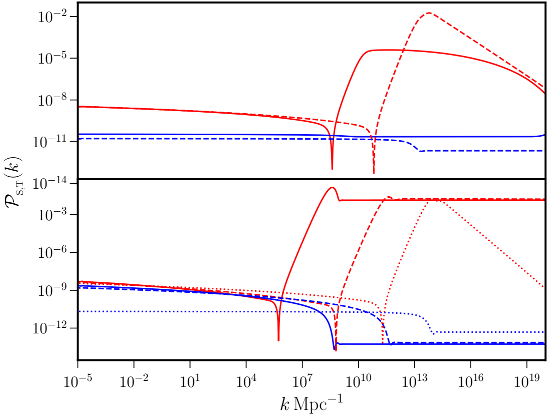
Behavior of the first slow roll parameter $\epsilon_1(N)$ leading to ultra slow roll and punctuated inflation[9]

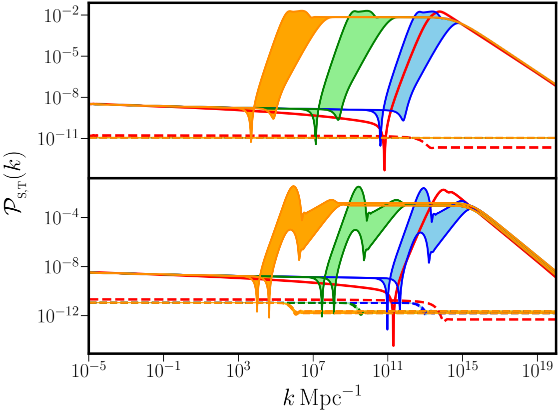
$$\mathrm{RSI}:\ \epsilon_1^{\mathrm{I}}(N) = \left[\epsilon_{1a}\ (1+\epsilon_{2a}\,N)\right]\left[1-\tanh\left(\frac{N-N_1}{\Delta N_1}\right)\right]+\epsilon_{1b}+\exp\left(\frac{N-N_2}{\Delta N_2}\right),$$

$$\mathrm{RSII}:\ \epsilon_1^{\mathrm{II}}(N) = \epsilon_1^{\mathrm{I}}(N)+\cosh^{-2}\left(\frac{N-N_1}{\Delta N_1}\right).$$

[9] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021); H. V. Ragavendra and L. Sriramkumar, Galaxies **11**, 34 (2023).

Power spectra in the inflationary models and reconstructed scenarios



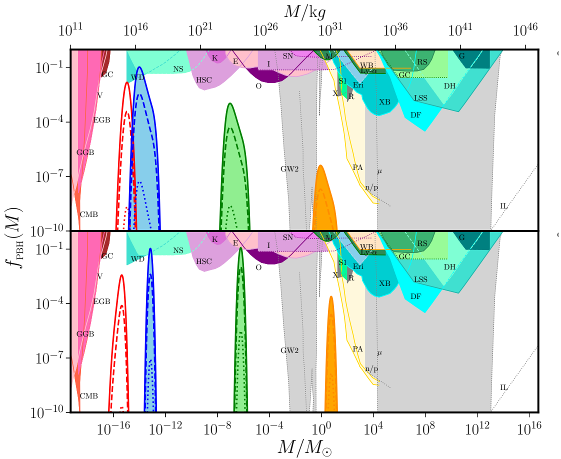


The scalar and the tensor power spectra arising in the various inflationary models (in red and blue on the left) and the reconstructed scenarios (in blue, green and orange, on the right)[10].

[10] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

Plan of the talk

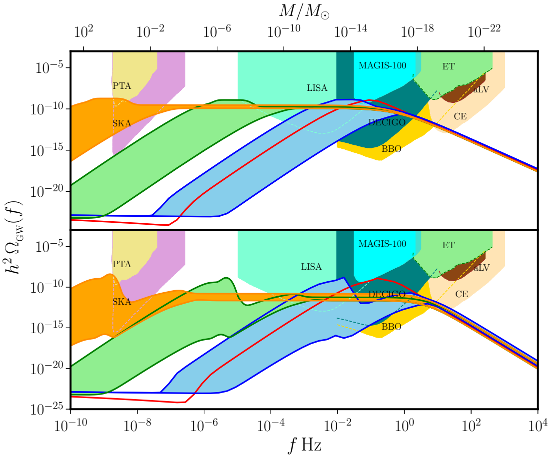
$f_{\rm PBH}(M)$ in ultra slow roll and punctuated inflation



The fraction of PBHs contributing to the dark matter density today $f_{\rm PBH}(M)$ has been plotted for the various models and scenarios of interest, viz. USR2 and RS1 (on top, in red and blue) and PI3 and RS2 (at the bottom, in red and blue)[11].

[11] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

$\Omega_{\rm GW}(f)$ in ultra slow roll and punctuated inflation



The dimensionless density parameter $\Omega_{\rm GW}$ arising in the models and reconstructed scenarios of USR2 and RS1 (in red and blue, on top) as well as PI3 and RS2 (in red and blue, at the bottom) have been plotted as a function of the frequency f[12].

[12] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

Plan of the talk

The scalar bispectrum and the non-Gaussianity parameter $f_{_{\rm NL}}$

The scalar bispectrum $\mathcal{B}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ is related to the three point correlation function of the Fourier modes of the curvature perturbation, evaluated towards the end of inflation, say, at the conformal time $\eta_{\rm e}$, as follows[13]:

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\rm e})\, \hat{\mathcal{R}}_{\boldsymbol{k}_2}(\eta_{\rm e})\, \hat{\mathcal{R}}_{\boldsymbol{k}_3}(\eta_{\rm e})\rangle = (2\,\pi)^3\; \mathcal{B}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\; \delta^{(3)}\left(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3\right).$$

The observationally relevant non-Gaussianity parameter $f_{_{\rm NL}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ is related to the scalar bispectrum through the relation[14]

$$\begin{aligned} f_{_{\rm NL}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) &= -\frac{10}{3}\,(2\,\pi)^{1/2}\;\left(k_1^3\, k_2^3\, k_3^3\right)\, \mathcal{B}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \\ &\quad \times\; \left[k_1^3\; \mathcal{P}_{_{\rm S}}(k_2)\; \mathcal{P}_{_{\rm S}}(k_3) + \text{two permutations}\right]^{-1}. \end{aligned}$$

[13] D. Larson *et al.*, Astrophys. J. Suppl. **192**, 16 (2011);
E. Komatsu *et al.*, Astrophys. J. Suppl. **192**, 18 (2011).

[14] J. Martin and L. Sriramkumar, JCAP **1201**, 008 (2012).

The cubic order action governing the perturbations

At the cubic order, the action describing the curvature perturbation $\mathcal{R}$ can be obtained to be[15]

$$S_3[\mathcal{R}(\eta,\boldsymbol{x})] = M_{\mathrm{Pl}}^2 \int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}} \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x} \biggl[a^2\,\epsilon_1^2\,\mathcal{R}\,\mathcal{R}'^2 + a^2\,\epsilon_1^2\,\mathcal{R}\,(\partial\mathcal{R})^2 - 2\,a\,\epsilon_1\,\mathcal{R}'\,(\partial\mathcal{R})\,(\partial\chi)$$

$$+ \frac{a^2}{2}\,\epsilon_1\,\epsilon_2'\,\mathcal{R}^2\,\mathcal{R}' + \frac{\epsilon_1}{2}\,(\partial\mathcal{R})\,(\partial\chi)\,\partial^2\chi + \frac{\epsilon_1}{4}\,\partial^2\mathcal{R}\,(\partial\chi)^2 + 2\,\mathcal{F}(\mathcal{R})\,\frac{\delta\mathcal{L}_2}{\delta\mathcal{R}}\biggr],$$

where $\mathcal{L}_2$ denotes the Lagrangian density at the second order, while $\partial^2\chi = a\,\epsilon_1\,\mathcal{R}'$, and these bulk terms are supplemented by the following temporal boundary terms[16]:

$$S_3^{\mathrm{B}}[\mathcal{R}(\eta,\boldsymbol{x})] = M_{\mathrm{Pl}}^2 \int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}} \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\, \frac{\mathrm{d}}{\mathrm{d}\eta}\biggl\{-9\,a^3\,H\,\mathcal{R}^3 + \frac{a}{H}\,(1-\epsilon_1)\,\mathcal{R}\,(\partial\mathcal{R})^2 - \frac{1}{4\,a\,H^3}\,(\partial\mathcal{R})^2\,\partial^2\mathcal{R}$$

$$- \frac{a\,\epsilon_1}{H}\,\mathcal{R}\,\mathcal{R}'^2 - \frac{a\,\epsilon_2}{2}\,\mathcal{R}^2\,\partial^2\chi + \frac{1}{2\,a\,H^2}\,\mathcal{R}\,\left(\partial_i\partial_j\mathcal{R}\,\partial_i\partial_j\chi - \partial^2\mathcal{R}\,\partial^2\chi\right)$$

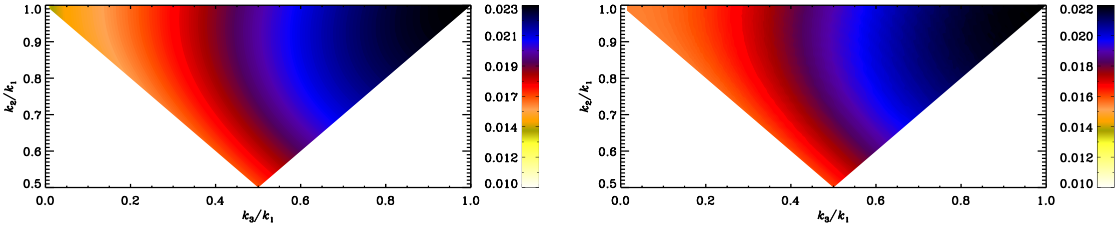
$$- \frac{1}{2\,aH}\,\mathcal{R}\,\left[\partial_i\partial_j\chi\,\partial_i\partial_j\chi - (\partial^2\chi)^2\right]\biggr\}.$$

▸ The quadratic action
▸ Back to the loop contributions

[15] J. Maldacena, JHEP **0305**, 013 (2003);
D. Seery and J. E. Lidsey, JCAP **06**, 003 (2005);
X. Chen, M.-x. Huang, S. Kachru and G. Shiu, JCAP **01**, 002 (2007).

[16] F. Arroja and T. Tanaka, JCAP **05**, 005 (2011).

The shape of the slow roll bispectrum



The non-Gaussianity parameter $f_{\rm NL}$, evaluated in the slow roll approximation (analytically on the left and numerically on the right), has been plotted as a function of k_3/k_1 and k_2/k_1 for the case of the popular quadratic potential[17]. Note that the non-Gaussianity parameter peaks in the equilateral limit wherein $k_1 = k_2 = k_3$. In slow roll scenarios involving the canonical scalar field, the largest value of $f_{\rm NL}$ is found to be of the order of the first slow roll parameter ϵ_1.

[17] D. K. Hazra, L. Sriramkumar and J. Martin, JCAP **05**, 026, (2013).

Template bispectra

For comparison with the observations, the scalar bispectrum is often expressed in terms of the parameters $f_{\rm NL}^{\rm loc}$, $f_{\rm NL}^{\rm eq}$ and $f_{\rm NL}^{\rm orth}$ as follows:

$$\mathcal{B}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = f_{\rm NL}^{\rm loc}\, \mathcal{B}_{\rm loc}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + f_{\rm NL}^{\rm eq}\, \mathcal{B}_{\rm eq}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + f_{\rm NL}^{\rm orth}\, \mathcal{B}_{\rm orth}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3).$$

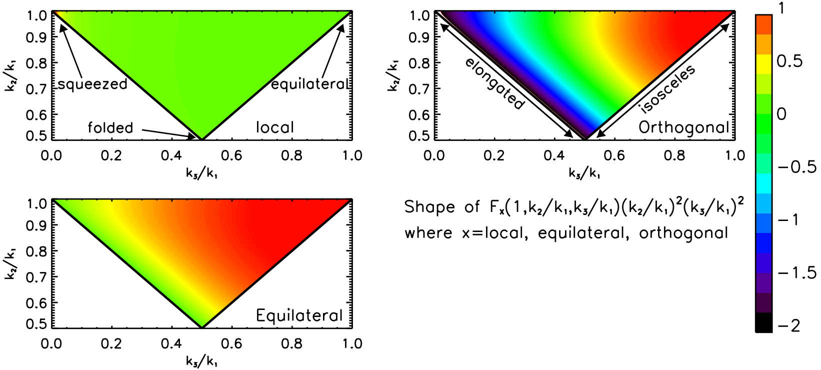


Illustration of the three template basis bispectra[18].

[18] E. Komatsu, Class. Quantum Grav. **27**, 124010 (2010).

Constraints on the scalar non-Gaussianity parameters

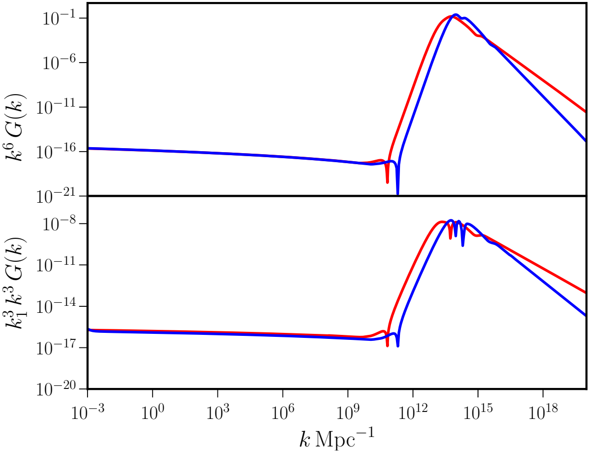
The constraints on the primordial values of the non-Gaussianity parameters from the Planck data are as follows[19]:

$$f_{\mathrm{NL}}^{\mathrm{loc}} = -0.9 \pm 5.1,$$
$$f_{\mathrm{NL}}^{\mathrm{eq}} = -26 \pm 47,$$
$$f_{\mathrm{NL}}^{\mathrm{ortho}} = -38 \pm 24.$$

These constraints imply that slowly rolling single field models involving the canonical scalar field which are favored by the data at the level of power spectra are also consistent with the data at the level of non-Gaussianities.

[19] Planck Collaboration (Y. Akrami *et al.*), Astron. Astrophys. **641**, A9 (2020).

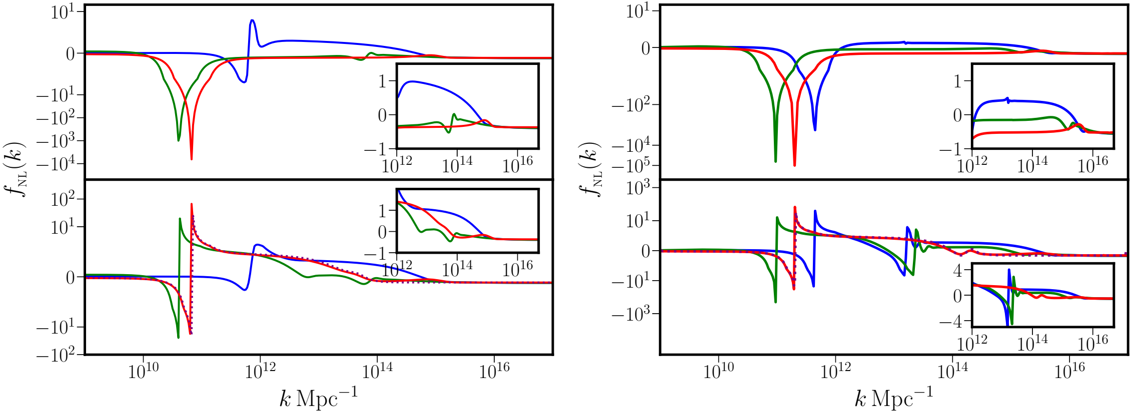
The scalar bispectrum in ultra slow roll and punctuated inflation



The amplitude of the dimensionless scalar bispectra has been plotted in the equilateral (on top) and squeezed limits (at the bottom) for the models USR2 (in red) and PI3 (in blue). The bispectra have approximately the same shape as the corresponding power spectra[20].

[20] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

$f_{\rm NL}$ in ultra slow roll and punctuated inflation



The scalar non-Gaussianity parameter $f_{\rm NL}$ has been plotted in the equilateral (on top) and the squeezed (at the bottom) limits for the models of USR2 and PI3 (in red, on the left and the right) and the reconstructed scenarios RS1 and RS2 (in blue and green, on the left and the right).

Modifications to the scalar power spectrum due to non-Gaussianities

The scalar non-Gaussianity parameter f_{NL} is usually introduced through the relation[21]

$$\mathcal{R}(\eta, \boldsymbol{x}) = \mathcal{R}_{\mathrm{G}}(\eta, \boldsymbol{x}) - \frac{3}{5}\, f_{\mathrm{NL}}\, \mathcal{R}_{\mathrm{G}}^2(\eta, \boldsymbol{x}),$$

where $\mathcal{R}_{\mathrm{G}}$ denotes the Gaussian contribution. Upon using this expression and evaluating the corresponding two-point correlation function in Fourier space, one obtains that

$$\langle \hat{\mathcal{R}}_{\boldsymbol{k}}\, \hat{\mathcal{R}}_{\boldsymbol{k}'} \rangle = \frac{2\,\pi^2}{k^3}\, \delta^{(3)}(\boldsymbol{k}+\boldsymbol{k}') \left[\mathcal{P}_{\mathrm{S}}(k) + \left(\frac{3}{5}\right)^2 \frac{k^3}{2\,\pi}\, f_{\mathrm{NL}}^2 \int \mathrm{d}^3\boldsymbol{p}\, \frac{\mathcal{P}_{\mathrm{S}}(p)}{p^3}\, \frac{\mathcal{P}_{\mathrm{S}}(|\boldsymbol{k}-\boldsymbol{p}|)}{|\boldsymbol{k}-\boldsymbol{p}|^3}\right],$$

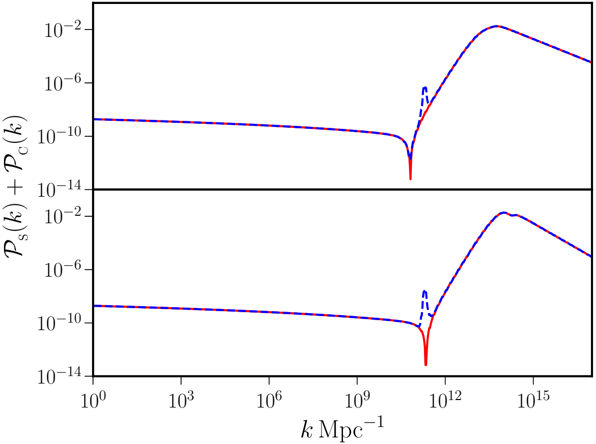
where $\mathcal{P}_{\mathrm{S}}(k)$ is the original scalar power spectrum defined in the Gaussian limit, while the second term represents the leading non-Gaussian correction. The non-Gaussian correction to the scalar power spectrum, say, $\mathcal{P}_{\mathrm{C}}(k)$, can be expressed as follows[22]:

$$\mathcal{P}_{\mathrm{C}}(k) = \left(\frac{12}{5}\right)^2 f_{\mathrm{NL}}^2 \int_0^{\infty} \mathrm{d}s \int_0^1 \frac{\mathrm{d}d}{(s^2-d^2)^2}\, \mathcal{P}_{\mathrm{S}}[k\,(s+d)/2]\, \mathcal{P}_{\mathrm{S}}[k\,(s-d)/2].$$

[21] E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001).

[22] R.-g. Cai, S. Pi and M. Sasaki, Phys. Rev. Lett. **122**, 201101 (2019);
C. Unal, Phys. Rev. D **99**, 041301 (2019).

The modified scalar power spectrum



The original scalar power spectrum $\mathcal{P}_{\rm S}(k)$ (in solid red) and the modified spectrum $\mathcal{P}_{\rm S}(k) + \mathcal{P}_{\rm C}(k)$ (in dashed blue) arrived at upon including the non-Gaussian corrections, have been plotted for the models of USR2 (on top) and PI3 (at the bottom)[23].

[23] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

Plan of the talk

The in-in formalism

In the in-in formalism, given the interaction Hamiltonian, say, H_{int}, the expectation value of the operator $\hat{\mathcal{O}}$ is given by

$$\begin{aligned}\langle\hat{\mathcal{O}}(\eta)\rangle &= \left\langle\mathcal{T}\left\{\mathrm{e}^{i\int \mathrm{d}\eta_1\,\hat{H}_{\mathrm{int}}(\eta_1)}\,\hat{\mathcal{O}}(\eta)\,\mathrm{e}^{-i\int \mathrm{d}\eta_2\,H_{\mathrm{int}}(\eta_2)}\right\}\right\rangle\\ &= \langle\hat{\mathcal{O}}(\eta)\rangle_0 - i\int \mathrm{d}\eta_1\,\langle\mathcal{T}[\hat{\mathcal{O}}(\eta),\hat{H}_{\mathrm{int}}(\eta_1)]\rangle\\ &\quad + \int \mathrm{d}\eta_1\int \mathrm{d}\eta_2\,\langle\mathcal{T}[[\hat{\mathcal{O}}(\eta),\hat{H}_{\mathrm{int}}(\eta_1)],H_{\mathrm{int}}(\eta_2)]\rangle + \cdots,\end{aligned}$$

where $\langle\hat{\mathcal{O}}(\eta)\rangle_0$ indicates the Gaussian contribution, and $\mathcal{T}$ indicates time ordering[24].

Since we are interested in calculating the loop corrections to the power spectrum, we have

$$\hat{\mathcal{O}} = \hat{\zeta}_{\boldsymbol{k}}\,\hat{\zeta}_{\boldsymbol{k}'},$$

where $\zeta_{\boldsymbol{k}}$ is the Fourier mode associated with the curvature perturbation.

[24] S. Weinberg, Phys. Rev. D **72**, 043514 (2005);
S. Weinberg, Phys. Rev. D **74**, 023508 (2006).

The contributions due to the cubic order Hamiltonian

Given the Hamiltonian at the third order, say, $H_{\mathrm{int}}^{(3)}$, the leading order correction is given by

$$\langle\hat{\mathcal{O}}(\eta)\rangle_1 = -i\int \mathrm{d}\eta_1 \langle\mathcal{T}[\hat{\mathcal{O}}(\eta), \hat{H}_{\mathrm{int}}^{(3)}(\eta_1)]\rangle,$$

which vanishes for $\hat{\mathcal{O}} = \hat{\zeta}_{\boldsymbol{k}}\,\hat{\zeta}_{\boldsymbol{k}'}$.

Therefore, the dominant loop contributions due to $H_{\mathrm{int}}^{(3)}$ can be written as[25]

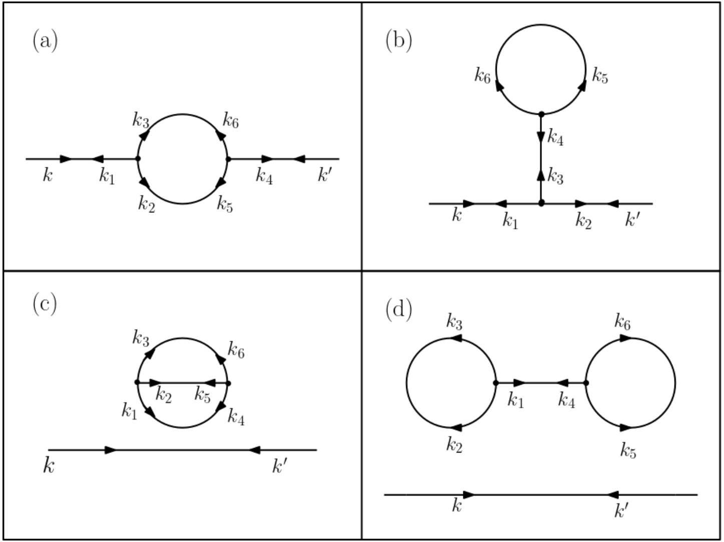
$$\langle\hat{\mathcal{O}}(\eta)\rangle_{\mathrm{C}} = \langle\hat{\mathcal{O}}(\eta)\rangle_{(0,2)} + \langle\hat{\mathcal{O}}(\eta)\rangle_{(2,0)} + \langle\hat{\mathcal{O}}(\eta)\rangle_{(1,1)},$$

where

$$\begin{aligned}
\langle\hat{\mathcal{O}}(\eta)\rangle_{(2,0)} &= \langle\hat{\mathcal{O}}^{\dagger}(\eta)\rangle_{(0,2)} = -\int_{-\infty}^{\eta}\mathrm{d}\eta_1\int_{-\infty}^{\eta}\mathrm{d}\eta_2\,\langle\hat{H}_{\mathrm{int}}^{(3)}(\eta_1)\,\hat{H}_{\mathrm{int}}^{(3)}(\eta_2)\,\hat{\mathcal{O}}(\eta)\rangle,\\
\langle\hat{\mathcal{O}}(\eta)\rangle_{(1,1)} &= \int_{-\infty}^{\eta}\mathrm{d}\eta_1\int_{-\infty}^{\eta}\mathrm{d}\eta_2\,\langle\hat{H}_{\mathrm{int}}^{(3)}(\eta_1)\,\hat{\mathcal{O}}(\eta)\,\hat{H}_{\mathrm{int}}^{(3)}(\eta_2)\rangle.
\end{aligned}$$

[25] J. Kristiano and J. Yokoyama, arXiv:2211.03395 [hep-th].

Feynman diagrams corresponding to the third order Hamiltonian



The Feynman diagrams that correspond to the third order Hamiltonian.

Plan of the talk

Modeling the background

We shall consider a scenario wherein a phase of ultra slow roll (USR) is sandwiched between two domains of slow roll (SR) inflation.

We shall assume that the first slow roll parameter ϵ_1 during the three phases is given by

$$\epsilon_1(\eta) = \begin{cases} \epsilon_{1\mathrm{i}} & \text{in phase I during } \eta < \eta_1, \\ \epsilon_{1\mathrm{i}}\,(\eta/\eta_1)^6 & \text{in phase II during } \eta_1 < \eta < \eta_2, \\ \epsilon_{1\mathrm{f}} & \text{in phase III during } \eta > \eta_2. \end{cases}$$

In such a case, the second slow roll parameter behaves as

$$\epsilon_2(\eta) = \begin{cases} 0 & \text{in phase I during } \eta < \eta_1, \\ -6 & \text{in phase II during } \eta_1 < \eta < \eta_2, \\ 0 & \text{in phase III during } \eta > \eta_2. \end{cases}$$

so that, at the transitions, at η_1 and η_2, we have

Back to smoothing the transitions | Back to loop contributions

$$\epsilon_2' = -6\,\delta^{(1)}(\eta - \eta_1), \quad \epsilon_2' = 6\,\delta^{(1)}(\eta - \eta_2).$$

Analytical solution for the Mukhanov-Sasaki variable I

The Mukhanov-Sasaki variable $v_k = z\, f_k$ satisfies the differential equation Equation of motion

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0.$$

The quantity z''/z can be expressed as[26]

$$\frac{z''}{z} = a^2\, H^2\, \left(2 - \epsilon_1 + \frac{3\,\epsilon_2}{2} + \frac{\epsilon_2^2}{4} - \frac{\epsilon_1\,\epsilon_2}{2} + \frac{\epsilon_2\,\epsilon_3}{2}\right).$$

During SR [$(\epsilon_1, \epsilon_2, \epsilon_3) \ll 1$] *as well as USR* ($\epsilon_1 \ll 1, \epsilon_2 \simeq -6, \epsilon_3 \simeq 0$), we have $z''/z \simeq 2/\eta^2$.

The solution to v_k in the region I (i.e. during first SR phase), satisfying the Bunch-Davies initial condition, is given by

$$v_k^{\mathrm{I}}(\eta) = \frac{1}{\sqrt{2\,k}}\, \left(1 - \frac{i}{k\,\eta}\right)\, \mathrm{e}^{-i\,k\,\eta}.$$

[26] See, for example, J. Martin and L. Sriramkumar, JCAP **01**, 008 (2012).

Analytical solution for the Mukhanov-Sasaki variable II

In regions II and III (i.e. during the USR and SR phases), we can write

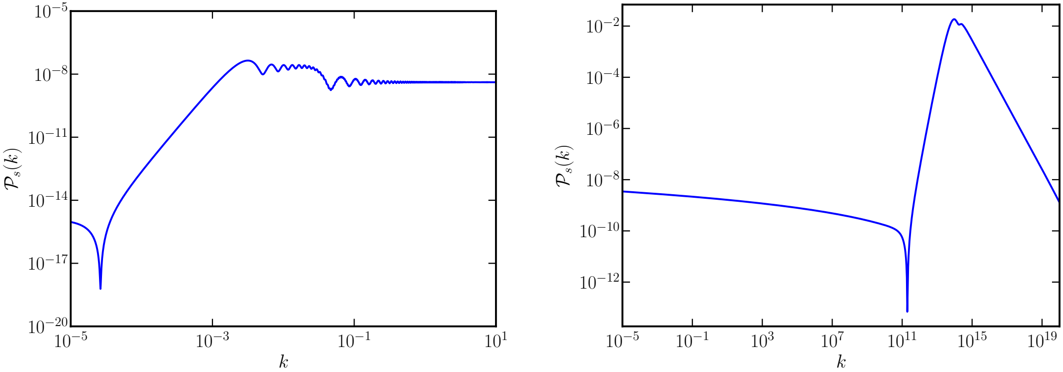
$$v_k^{\rm II}(\eta) = \frac{\gamma_k}{\sqrt{2\,k}}\,\left(1-\frac{i}{k\,\eta}\right)\,{\rm e}^{-i\,k\,\eta} + \frac{\delta_k}{\sqrt{2\,k}}\,\left(1+\frac{i}{k\,\eta}\right)\,{\rm e}^{i\,k\,\eta},$$
$$v_k^{\rm III}(\eta) = \frac{\alpha_k}{\sqrt{2\,k}}\,\left(1-\frac{i}{k\,\eta}\right)\,{\rm e}^{-i\,k\,\eta} + \frac{\beta_k}{\sqrt{2\,k}}\,\left(1+\frac{i}{k\,\eta}\right)\,{\rm e}^{i\,k\,\eta},$$

where the quantities $(\gamma_k, \delta_k, \alpha_k, \beta_k)$ are given by

$$\gamma_k = 1 + \frac{3\,i}{2\,k\,\eta_1} + \frac{3\,i}{2\,k^3\,\eta_1^3}, \qquad \delta_k = \left(-\frac{3\,i}{2\,k\,\eta_1} - \frac{3}{k^2\,\eta_1^2} + \frac{3\,i}{2\,k^3\,\eta_1^3}\right)\,{\rm e}^{-2\,i\,k\,\eta_1},$$
$$\alpha_k = \left(1 - \frac{3\,i}{2\,k\,\eta_2} - \frac{3\,i}{2\,k^3\,\eta_2^3}\right)\,\gamma_k - \left(\frac{3\,i}{2\,k\,\eta_2} - \frac{3}{k^2\,\eta_2^2} - \frac{3\,i}{2\,k^3\,\eta_2^3}\right)\,\delta_k\,{\rm e}^{2\,i\,k\,\eta_2},$$
$$\beta_k = \left(1 + \frac{3\,i}{2\,k\,\eta_2} + \frac{3\,i}{2\,k^3\,\eta_2^3}\right)\,\delta_k + \left(\frac{3\,i}{2\,k\,\eta_2} + \frac{3}{k^2\,\eta_2^2} - \frac{3\,i}{2\,k^3\,\eta_2^3}\right)\,\gamma_k\,{\rm e}^{-2\,i\,k\,\eta_2}.$$

which are obtained by matching the solutions at η_1 and η_2.

Power spectra in USR inflation



The scalar power spectrum arising due to an early (on the left) and late (on the right) onset of ultra slow roll inflation[27].

▸ Power spectrum in punctuated inflation

[27] R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **01**, 009 (2009); H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

Loop contributions due to the Hamiltonian at the cubic order

When there arise deviations from slow roll, the dominant term in the Hamiltonian at the cubic order in the perturbations is given by

Action at the cubic order

$$H_{\mathrm{int}}^{(3)} = -\frac{M_{\mathrm{Pl}}^2}{2} \int \mathrm{d}^3x\, a^2\, \epsilon_1\, \epsilon_2'\, \zeta'\zeta^2.$$

It has been argued that instantaneous transitions across the three phases leads to large loop contributions due to above the third order term, indicating the breakdown of perturbation theory[28].

It is expected that, if the transitions are not sharp, the loop contributions may not be large enough to cause a breakdown of perturbation theory.

We should mention that, in these cases, the corrections to the power spectrum turn out to be positive.

[28] J. Kristiano and J. Yokoyama, arXiv:2211.03395 [hep-th].

Dominant term in the action at the quartic order

At the quartic order, the dominant term in the action describing the scalar perturbations is given by[29]

$$S_4[\delta\phi(t,\boldsymbol{x})] = -\frac{1}{24}\int \mathrm{d}t \int \mathrm{d}^3\boldsymbol{x}\, a^3\, V_{\phi\phi\phi\phi}\, \delta\phi^4,$$

where $\delta\phi$ denotes the perturbations in the scalar field, and $V_{\phi\phi\phi\phi} = \mathrm{d}^4 V/\mathrm{d}\phi^4$.

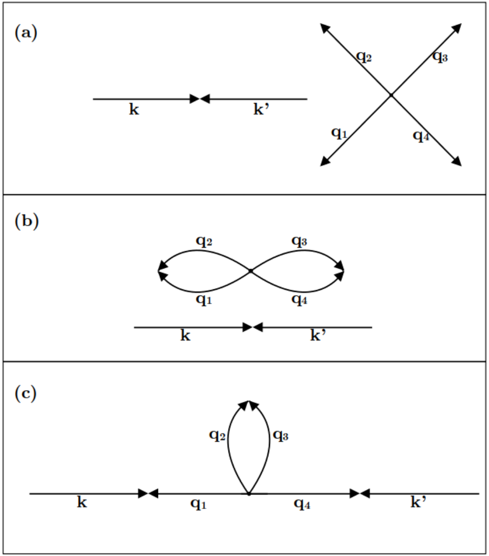
We can express the above action in terms of SR parameters as follows:

$$\begin{aligned} S_4[\delta\phi(\eta,\boldsymbol{x})] = \frac{1}{288\, M_{\mathrm{Pl}}^4} \int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\, \frac{a^4}{\epsilon_1} \Big[& -48\,\epsilon_1^3 + 16\,\epsilon_1^4 + 72\,\epsilon_1^2\,\epsilon_2 - 56\,\epsilon_1^3\,\epsilon_2 - 9\,\epsilon_1\,\epsilon_2^2 \\ & + 39\,\epsilon_1^2\,\epsilon_2^2 - 3\,\epsilon_1\,\epsilon_2^3 - 24\,\epsilon_1\,\epsilon_2\,\epsilon_3 + 32\,\epsilon_1^2\,\epsilon_2\,\epsilon_3 - \frac{35}{2}\,\epsilon_1\,\epsilon_2^2\,\epsilon_3 - \frac{3}{2}\,\epsilon_2^2\,\epsilon_3 - \frac{1}{2}\,\epsilon_2^3\,\epsilon_3 \\ & - 9\,\epsilon_1\,\epsilon_2\,\epsilon_3^2 + 3\,\epsilon_2\,\epsilon_3^2 + \frac{3}{2}\,\epsilon_2^2\,\epsilon_3^2 + \epsilon_2\,\epsilon_3^3 - 9\,\epsilon_1\,\epsilon_2\,\epsilon_3\,\epsilon_4 + 3\,\epsilon_2\,\epsilon_3\,\epsilon_4 + \frac{1}{2}\,\epsilon_2^2\,\epsilon_3\,\epsilon_4 \\ & + 3\,\epsilon_2\,\epsilon_3^2\,\epsilon_4 + \epsilon_2\,\epsilon_3\,\epsilon_4^2 + \epsilon_2\,\epsilon_3\,\epsilon_4\,\epsilon_5 \Big]\, V\,\delta\phi^4 \end{aligned}$$

and it is the last term (in blue) that leads to the dominant contribution.

[29] See, for instance, E. Dimastrogiovanni and N. Bartolo, JCAP **11**, 016 (2008).

Feynman diagrams for the Hamiltonian at the quartic order



The Feynman diagrams that correspond to the Hamiltonian at the quartic order.

Smoothing the transitions

We smooth the transitions by replacing the delta functions we encounter by a Gaussian as follows:

▸ ϵ_2 in the SR-USR-SR scenario

$$\delta^{(1)}(\eta - \tilde{\eta}) = \frac{1}{\sqrt{\pi}\,\Delta\eta}\,\mathrm{e}^{-\frac{(\eta-\tilde{\eta})^2}{\Delta\eta^2}},$$

where $\tilde{\eta}$ is the point of transition and $\Delta\eta$ denote the sharpness of transition.

In such a case, we can express the time-derivative of the second SR parameter ϵ_2 as

$$\epsilon_2' = \begin{cases} \dfrac{\epsilon_2^{\mathrm{II}}}{\sqrt{\pi}\,\Delta\eta}\,\mathrm{e}^{-\frac{(\eta-\eta_1)^2}{\Delta\eta^2}} & \text{around } \eta_1, \\ -\dfrac{\epsilon_2^{\mathrm{II}}}{\sqrt{\pi}\,\Delta\eta}\,\mathrm{e}^{-\frac{(\eta-\eta_2)^2}{\Delta\eta^2}} & \text{around } \eta_2. \end{cases}$$

The higher order SR parameters ϵ_3, ϵ_4 and ϵ_5 can then be obtained to be

$$\epsilon_3 = \mp\frac{\eta}{\sqrt{\pi}\,\Delta\eta}\,\mathrm{e}^{-\frac{(\eta-\eta_{1,2})^2}{\Delta\eta^2}},\quad \epsilon_4 = -1 + \frac{2\,\eta\,(\eta-\eta_{1,2})}{\Delta\eta^2},\quad \epsilon_5 = \frac{2\,\eta\,(2\,\eta-\eta_{1,2})}{\Delta\eta^2 - 2\,\eta\,(\eta-\eta_{1,2})}.$$

The interaction Hamiltonian due to the dominant term

We can express the quartic action in terms of the following gauge invariant variable ζ:

$$\zeta = \frac{H}{\dot{\phi}}\,\delta\phi = -\frac{\delta\phi}{M_{_{\mathrm{Pl}}}\,\sqrt{2\,\epsilon_1}}.$$

In such a case, the dominant term in the action at the quartic order is given by[30]

$$S^{(4)}[\zeta(\eta,\boldsymbol{x})] = \frac{1}{72}\int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x}\, a^4\,\epsilon_1\,\epsilon_2\,\epsilon_3\,\epsilon_4\,\epsilon_5\,V\,\zeta^4.$$

The corresponding interaction Hamiltonian can be determined to be

$$H_{\mathrm{int}}^{(4)} = -\frac{1}{72}\int \mathrm{d}^3\boldsymbol{x}\, a^4\,\epsilon_1\,\epsilon_2\,\epsilon_3\,\epsilon_4\,\epsilon_5\,V\,\zeta^4.$$

[30] E. Dimastrogiovanni and N. Bartolo, JCAP **11**, 016 (2008).

Corrections to power spectrum due to the dominant term

In the in-in formalism, at the leading order, the two point correlation function due to the interaction Hamiltonian $H_{\mathrm{int}}^{(4)}$ is given by the expression

$$\langle\hat{\zeta}_{\boldsymbol{k}}(\eta_{\mathrm{e}})\,\hat{\zeta}_{\boldsymbol{k}'}(\eta_{\mathrm{e}})\rangle = \langle\hat{\zeta}_{\boldsymbol{k}}(\eta_{\mathrm{e}})\,\hat{\zeta}_{\boldsymbol{k}'}(\eta_{\mathrm{e}})\rangle - i\,\langle[\hat{\zeta}_{\boldsymbol{k}}(\eta_{\mathrm{e}})\,\hat{\zeta}_{\boldsymbol{k}'}(\eta_{\mathrm{e}}), \int \mathrm{d}\eta\,\mathcal{T}\{\hat{H}_{\mathrm{int}}^{(4)}(\eta)\}]\rangle.$$

In the vacuum, the dominant loop contribution can be written as as

$$\langle\hat{\zeta}_{\boldsymbol{k}}(\eta_{\mathrm{e}})\,\hat{\zeta}_{\boldsymbol{k}'}(\eta_{\mathrm{e}})\rangle_{\mathrm{C}} \simeq \delta^{(3)}(\boldsymbol{k}+\boldsymbol{k}')\left\{\frac{i}{6}\,f_k(\eta_{\mathrm{e}})\,f_{k'}(\eta_{\mathrm{e}})\,\int \mathrm{d}\eta\,a^4\,\epsilon_1\,\epsilon_2\,\epsilon_3\,\epsilon_4\,\epsilon_5\,V\,f_k^{*}(\eta)\,f_{k'}^{*}(\eta)\right.$$
$$\left.\times\int\frac{\mathrm{d}^3\boldsymbol{q}}{(2\,\pi)^3}|f_q(\eta)|^2 + \text{ complex conjugate}\right\}$$

so that the correction to the scalar power spectrum is given by

$$\mathcal{P}_{\mathrm{C}}^{(4)}(k) = \frac{i}{6}\left(\frac{k^3}{2\,\pi^2}\right)\,f_k^2(\eta_{\mathrm{e}})\,\int \mathrm{d}\eta\,a^4\,\epsilon_1\,\epsilon_2\,\epsilon_3\,\epsilon_4\,\epsilon_5\,V\,\left[f_k^{*}(\eta)\right]^2\int \mathrm{d}\ln q\,\mathcal{P}_{\mathrm{S}}(q,\eta)$$
$$+\text{ complex conjugate.}$$

Loop contributions in the case of slow roll inflation I

In SR inflation, the leading correction to the scalar power spectrum can be written as

$$\mathcal{P}_{\mathrm{C}}^{(4)}(k) = -\frac{i}{8}\,\epsilon_2\,\epsilon_3\,\epsilon_4\,\epsilon_5\,\left(\mathcal{P}_{\mathrm{S}}^{0}\right)^2 \int \frac{\mathrm{d}\eta}{k\,\eta^2}\left(1+\frac{i}{k\eta}\right)^2 \mathrm{e}^{2\,i\,k\,\eta}\int \mathrm{d}\ln q\,(1+q^2\eta^2)$$
$$+ \text{ complex conjugate},$$

where $\mathcal{P}_{\mathrm{S}}^{0} = H^2/(8\pi^2 M_{\mathrm{Pl}}^2\epsilon_1) \simeq 2.1\times 10^{-9}$ is the standard, nearly scale invariant, COBE normalized power spectrum.

If we assume that $\epsilon_1 \simeq \epsilon_2 \simeq \epsilon_3 \simeq \epsilon_4 \simeq \epsilon_5$, after carrying out the integration over the internal momenta q, we obtain

$$\mathcal{P}_{\mathrm{C}}^{(4)}(k) = -\frac{i}{8}\,\epsilon_1^4\,\left(\mathcal{P}_{\mathrm{S}}^{0}\right)^2 \int_{x_{\mathrm{i}}}^{x_{\mathrm{e}}} \frac{\mathrm{d}x}{x^2}\left(1+\frac{i}{x}\right)^2 \mathrm{e}^{2\,i\,x}\left[c_1+\frac{c_2(k)}{2}\,x^2\right] + \text{ complex conjugate},$$

where $x = k\,\eta$, and

$$c_1 = \ln\left(\frac{k_{\mathrm{max}}}{k_{\mathrm{min}}}\right) = \ln\left(\frac{\eta_{\mathrm{i}}}{\eta_{\mathrm{e}}}\right) \simeq 70, \quad c_2(k) = \frac{(k_{\mathrm{max}}^2 - k_{\mathrm{min}}^2)}{k^2} \simeq \left(\frac{k_{\mathrm{max}}}{k}\right)^2 \simeq 10^{42}.$$

Loop contributions in the case of slow roll inflation II

After integrating over time, we obtain that[31]

$$\mathcal{P}_{\mathrm{C}}^{(4)}(k) \simeq \frac{1}{8}\,\epsilon_1^4\,\left(\mathcal{P}_{\mathrm{S}}^{0}\right)^2\,\left\{\left[\frac{28}{9}-\frac{4\,\gamma}{3}-\frac{4}{3}\,\ln\left(-2\,x_{\mathrm{e}}\right)\right]\,c_1+3\,c_2(k)\right\},$$

where $\gamma = 0.577$ is Euler-Mascheroni constant.

Upon ignoring the contributions from the sub-Hubble modes (i.e. upon setting $c_2 = 0$), we arrive at (for $\epsilon_1 \simeq 10^{-3}$)

$$\mathcal{P}_{\mathrm{C}}^{(4)}(k) \simeq -\frac{1}{6}\,\epsilon_1^4\,\left(\mathcal{P}_{\mathrm{S}}^{0}\right)^2\,c_1\,\ln\left(\frac{2\,k_*}{k_{\mathrm{max}}}\right) \simeq 10^{-28},$$

where k_* denotes the pivot scale.

[31] S. Maity, H. V. Ragavendra, S. K. Sethi and L. Sriramkumar, arXiv:2307.13636 [astro-ph.CO].

Loop contributions in the SR-USR-SR scenario I

Upon using the form of the SR parameters and, after integrating over time, we obtain the contributions from the two transitions to be

Form of ϵ_2'

$$\mathcal{P}_{_{\rm C}}^{(4)}(k) = i\, \frac{M_{_{\rm Pl}}^2}{H_{_{\rm I}}^2}\, \frac{\epsilon_{1\rm i}\, \epsilon_2^{\rm II}}{\Delta\eta^2}\, \frac{k^3\, f_k^2(\eta_{\rm e})}{2\,\pi^2} \left\{\frac{[f_k^\ast(\eta_1)]^2}{\eta_1} \int_{k_{\rm min}}^{k_1} {\rm d}\ln q\, \mathcal{P}_{_{\rm S}}(q,\eta_1) - \left(\frac{\eta_2}{\eta_1}\right)^6 \frac{[f_k^\ast(\eta_2)]^2}{\eta_2} \int_{k_{\rm min}}^{k_2} {\rm d}\ln q\, \mathcal{P}_{_{\rm S}}(q,\eta_2)\right\} + \text{complex conjugate.}$$

Loop contributions in the SR-USR-SR scenario II

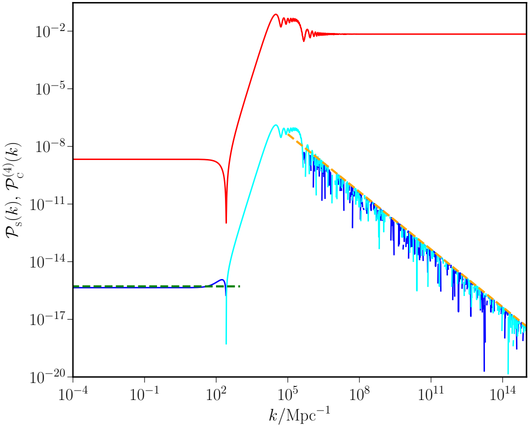
After integrating over momentum, we arrive at

$$\mathcal{P}_{\mathrm{C}}^{(4)}(k) \simeq \frac{i}{4}\left(\frac{H_{\mathrm{I}}^2}{8\,\pi^2\,M_{\mathrm{Pl}}^2\,\epsilon_{1\mathrm{f}}}\right)^2 \frac{\epsilon_2^{\mathrm{II}}}{k^3\,\eta_2\,b\Delta\eta^2}\,\mathcal{F}^2(\alpha_k,\beta_k,\eta_{\mathrm{e}})$$
$$\times\left([\mathcal{F}^*(1,0,\eta_1)]^2\left(\frac{k_1}{k_2}\right)^7\left[\ln\left(\frac{k_1}{k_{\mathrm{min}}}\right)+\frac{1}{2}\left(1-\frac{k_{\mathrm{min}}^2}{k_1^2}\right)\right]\right.$$
$$-[\mathcal{F}^*(\alpha_k,\beta_k,\eta_2)]^2\left\{\left(\frac{k_1}{k_2}\right)^6\left[\ln\left(\frac{k_2}{k_{\mathrm{min}}}\right)-\frac{1}{10}\left(1-\frac{k_{\mathrm{min}}^2}{k_2^2}\right)\right]\right.$$
$$\left.\left.-\left[\frac{2}{5}\left(\frac{k_1}{k_2}\right)-\left(\frac{k_1}{k_2}\right)^4\right]\left[1-\left(\frac{k_{\mathrm{min}}}{k_2}\right)^2\right]\right\}\right)+\text{ complex conjugate,}$$

where the quantity $\mathcal{F}(\alpha_k,\beta_k,\eta)$ is given by

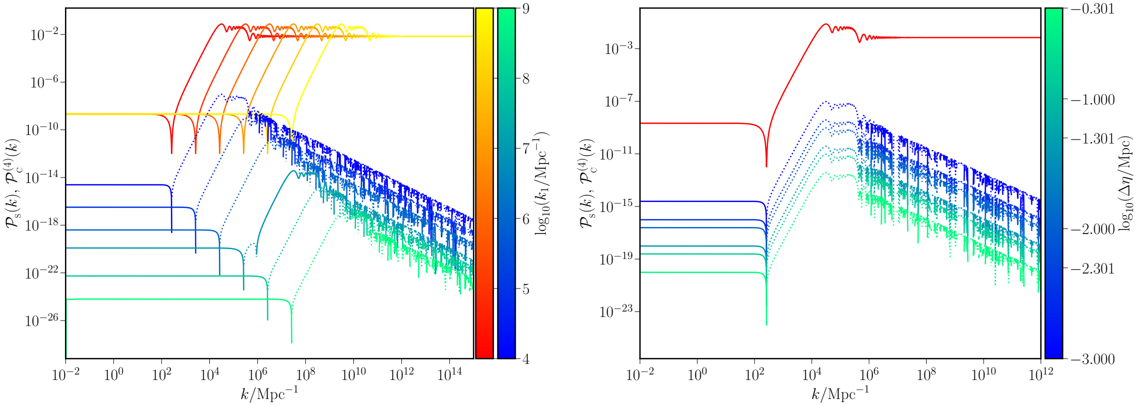
$$\mathcal{F}(\alpha_k,\beta_k,\eta)=\alpha_k\,(k\,\eta-i)\,\mathrm{e}^{-i\,k\,\eta}+\beta_k\,(k\,\eta+i)\,\mathrm{e}^{i\,k\,\eta}.$$

Typical loop contributions to the power spectrum



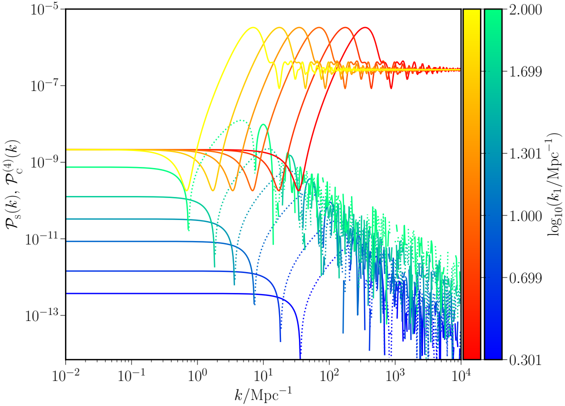
The power spectrum at the first order $\mathcal{P}_{\mathrm{S}}(k)$ (in red) and the loop contributions $\mathcal{P}_{\mathrm{C}}^{(4)}(k)$ (in blue) have been illustrated for the following parameters: $H_{\mathrm{I}} = 1.3 \times 10^{-5}\, M_{\mathrm{Pl}}$, $\epsilon_{1\mathrm{i}} = 10^{-3}$, $\epsilon_2^{\mathrm{II}} = -6$, $\Delta\eta = 10^{-3}\,\mathrm{Mpc}$, $k_1 = 10^4\,\mathrm{Mpc}^{-1}$, $\Delta N = 2.5$, $k_{\mathrm{min}} = 10^{-6}\,\mathrm{Mpc}^{-1}$, and $k_{\mathrm{max}} = 10^{20}\,\mathrm{Mpc}^{-1}$. We have also indicated the domains where $\mathcal{P}_{\mathrm{C}}^{(4)}(k)$ negative (in cyan).

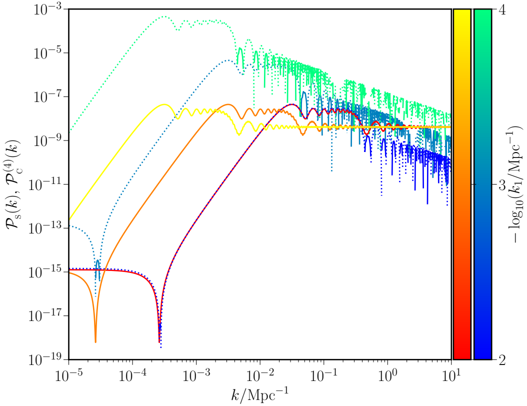
Loop contributions for late onset of USR



The power spectrum at the first order $\mathcal{P}_{\rm S}(k)$ and the loop contributions $\mathcal{P}_{\rm C}^{(4)}(k)$ have been illustrated for different values of k_1 (on the left) and different values of $\Delta\eta$ (on the right).

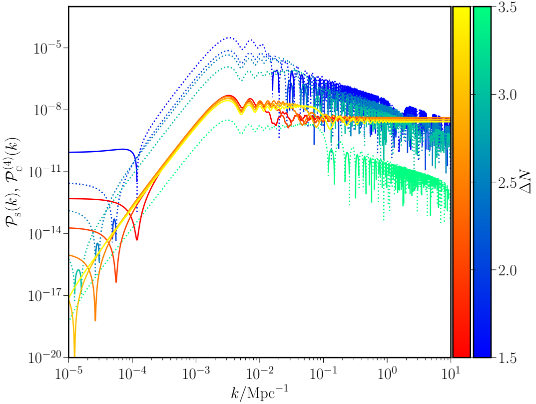
Loop contributions for intermediate and early onsets of USR

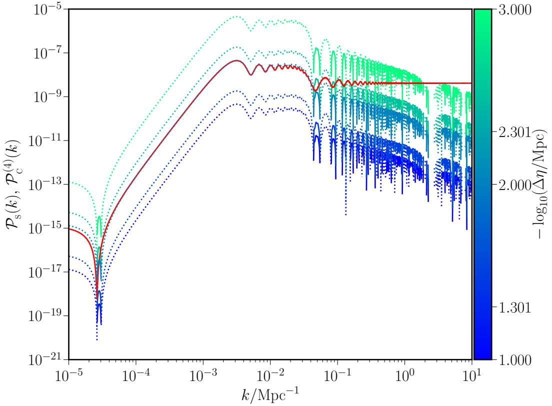




The power spectrum at the first order $\mathcal{P}_{\rm S}(k)$ and the loop contributions $\mathcal{P}_{\rm C}^{(4)}(k)$ have been illustrated for intermediate (on the left) and early (on the right) onsets of USR. Note that, as the USR sets in earlier and earlier, the contributions from the loops prove to be larger and larger.

Contributions for early USR: Effects of duration and sharpness





We find that the earlier is the phase of USR and the sharper is the transition, the larger are the loop contributions to the scalar power spectrum[32]. These raise the specter of the breakdown of perturbation theory!

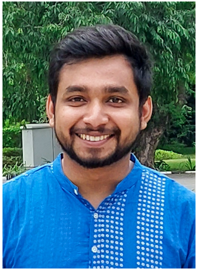
[32] S. Maity, H. V. Ragavendra, S. K. Sethi and L. Sriramkumar, arXiv:2307.13636 [astro-ph.CO].

Plan of the talk

Summary

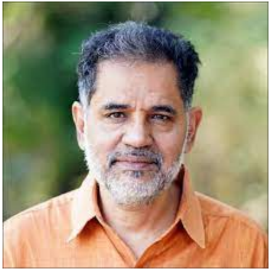
- Inflationary models permitting an epoch of ultra slow roll lead to enhanced power on small scales, resulting in significant production of PBHs and increased strength of secondary GWs. The non-Gaussianity parameter $f_{\rm NL}$ in such single field models is of order unity near the peaks in the power spectrum.
- When the contributions due to the loops are taken into account, there seem to exist a range in the parameter space wherein the contributions due to the loops become significant suggesting the breakdown of perturbation theory.
- Counter-intuitively, we find that the leading quartic term in the interaction Hamiltonian leads to significant loop contributions on large scales.

Collaborators



Suvashis Maity

H. V. Ragavendra



Shiv Sethi

Pankaj Saha

Joseph Silk

Thank you for your attention