Magnetogenesis during inflation: Imprints of non-trivial dynamics

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Plan of the talk

1. Observational evidence for magnetic fields and inflation
2. Generation of magnetic fields in slow roll inflation
3. Challenges in inflationary models leading to features
4. Circumventing the challenges using two field models
5. Amplifying entanglement entropy through violation of parity
6. Summary

This talk is based on. . .

- S. Tripathy, D. Chowdhury, R. K. Jain and L. Sriramkumar, *Challenges in the choice of the nonconformal coupling function in inflationary magnetogenesis*, Phys. Rev. D **105**, 063519 (2022) [arXiv:2111.01478 [astro-ph.CO]].
- S. Tripathy, D. Chowdhury, H. V. Ragavendra, R. K. Jain and L. Sriramkumar, *Circumventing the challenges in the choice of the non-conformal coupling function in inflationary magnetogenesis*, Phys. Rev. D **107**, 043501 (2023) [arXiv:2211.05834 [astro-ph.CO]].
- S. Tripathy, R. N. Raveendran, K. Parattu and L. Sriramkumar, Amplifying quantum discord during inflationary magnetogenesis through violation of parity, arXiv:2306.16168 [gr-qc].

Plan of the talk

Observational evidence for magnetic fields

Magnetic fields are ubiquitous in the universe. They are observed at different strengths over a wide range of scales.

- In galaxies, the strength of the observed magnetic fields is $\mathcal{O}(10^{-6}\,\mathrm{G})$, which is coherent over scales of 1–$10\,\mathrm{Kpc}$[1].
- In clusters of galaxies, the strength of the magnetic fields is $\mathcal{O}(10^{-7}$–$10^{-6}\,\mathrm{G})$ with a coherent length of $10\,\mathrm{Kpc}$–$1\,\mathrm{Mpc}$[2].
- In the intergalactic medium, the strength of the magnetic fields is greater than $10^{-16}\,\mathrm{G}$, which is coherent on scales above $1\,\mathrm{Mpc}$[3].
- The observations of the anisotropies in the cosmic microwave background (CMB) constrain the magnetic fields at the scale of $1\,\mathrm{Mpc}$ to be less than $10^{-9}\,\mathrm{G}$[4].

While astrophysical processes may be sufficient to explain the origin of magnetic fields in galaxies and clusters of galaxies, *one may have to turn to cosmological phenomena to explain the magnetic fields observed in voids*.

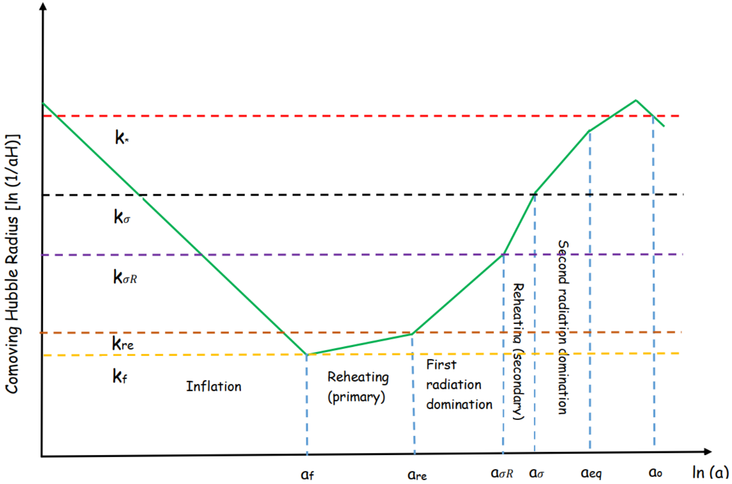
[1] R. Beck, Space Sci. Rev. **99**, 243 (2001).

[2] See, for instance, T. E. Clarke, P. P. Kronberg and H. Böhringer, Astrophys. J. **547**, L111 (2001).

[3] A. Neronov and I. Vovk, Science **328**, 73 (2010).

[4] Planck Collaboration (P. A. R. Ade *et al.*), Astron. Astrophys. **594**, A19 (2016).

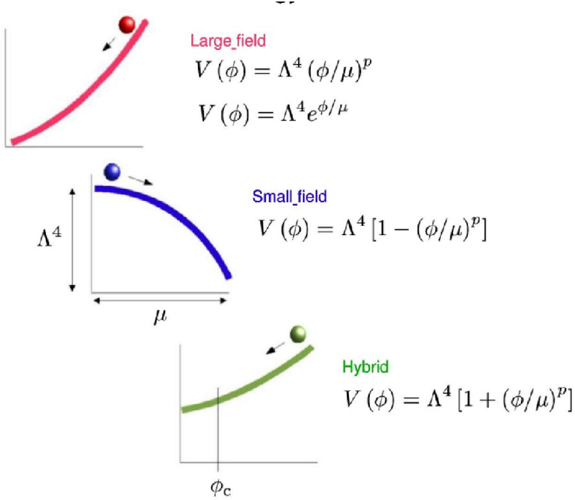
Behavior of the comoving wave numbers and Hubble radius



Behavior of the comoving wave number k (horizontal lines in different colors) and the comoving Hubble radius $d_{\rm H}/a = (a\,H)^{-1}$ (in green) across different epochs[5].

[5] Md. R. Haque, D. Maity, T. Paul and L. Sriramkumar, Phys. Rev. D **104**, 063513 (2021).

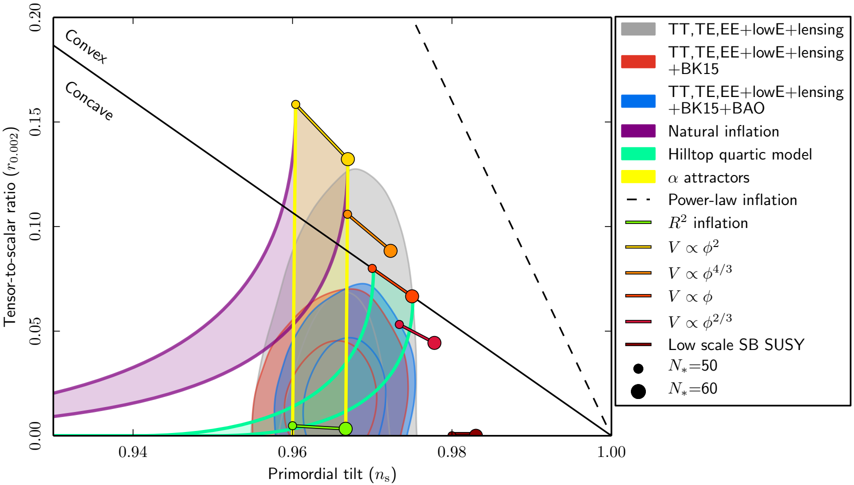
Variety of potentials can drive inflation



A variety of scalar field potentials have been considered to drive inflation[6]. Often, these potentials are classified as small field, large field and hybrid models.

[6] Image from W. Kinney, astro-ph/0301448.

Performance of inflationary models in the $n_{\rm s}$-r plane



Joint constraints on $n_{\rm s}$ and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of some of the popular inflationary models[7].

[7] Planck Collaboration (Y. Akrami *et al.*), Astron. Astrophys. **641**, A10 (2020).

Plan of the talk

Non-conformally coupled electromagnetic fields

We shall assume that the electromagnetic field is described by the action[8]

$$S[A^\mu] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\sqrt{-g}\, J^2(\phi)\, F_{\mu\nu}\, F^{\mu\nu},$$

where $J(\phi)$ denotes the coupling function and the field tensor $F_{\mu\nu}$ is expressed in terms of the vector potential A_μ as $F_{\mu\nu} = (\partial_\mu\, A_\nu - \partial_\nu\, A_\mu)$.

On working in the Coulomb gauge wherein $A_\eta = 0$ and $\partial_i\, A^i = 0$, one finds that the Fourier modes, say, $\bar{A}_k$, describing the vector potential satisfy the differential equation

$$\bar{A}_k'' + 2\,\frac{J'}{J}\,\bar{A}_k' + k^2\,\bar{A}_k = 0.$$

If we write $\bar{A}_k = \mathcal{A}_k/J$, then this equation reduces to

$$\mathcal{A}_k'' + \left(k^2 - \frac{J''}{J}\right)\mathcal{A}_k = 0.$$

[8] See, for instance, J. Martin and J. Yokoyama, JCAP **01**, 025 (2008); K. Subramanian, Astron. Nachr. **331**, 110 (2010).

Power spectra of the electromagnetic fields

The power spectra associated with the magnetic and electric fields are defined to be[9]

$$\mathcal{P}_{_{\rm B}}(k) = \frac{{\rm d}\langle 0|\hat{\rho}_{_{\rm B}}|0\rangle}{{\rm d}\ln k} = \frac{J^2(\eta)}{2\,\pi^2}\,\frac{k^5}{a^4(\eta)}\,|\bar{A}_k(\eta)|^2 = \frac{1}{2\,\pi^2}\,\frac{k^5}{a^4(\eta)}\,|\mathcal{A}_k(\eta)|^2,$$

$$\mathcal{P}_{_{\rm E}}(k) = \frac{{\rm d}\langle 0|\hat{\rho}_{_{\rm E}}|0\rangle}{{\rm d}\ln k} = \frac{J^2(\eta)}{2\,\pi^2}\,\frac{k^3}{a^4(\eta)}\,|\bar{A}_k'(\eta)|^2 = \frac{1}{2\,\pi^2}\,\frac{k^3}{a^4(\eta)}\,\left|\mathcal{A}_k'(\eta) - \frac{J'(\eta)}{J(\eta)}\,\mathcal{A}_k(\eta)\right|^2.$$

[9] J. Martin and J. Yokoyama, JCAP **01**, 025 (2008); K. Subramanian, Astron. Nachr. **331**, 110 (2010).

Power spectra in de Sitter inflation

Let us now consider de Sitter inflation wherein the scale factor is given by $a(\eta) = -1/(H_{\rm I}\,\eta)$, with $H_{\rm I}$ denoting the constant Hubble parameter.

Typically, the non-conformal coupling function J is assumed to depend on the scale factor as follows:

$$J(\eta) = [a(\eta)/a(\eta_{\rm e})]^n\,,$$

where $\eta_{\rm e}$ denotes the conformal time at the end of inflation.

The spectrum of the magnetic field is found to scale invariant when $n \simeq -3$ or when $n \simeq 2$. At late times, $n_{\rm E} \simeq -2$ leads to a large backreaction.

In order to avoid such an issue, one often considers the $n = 2$. In this case, the power spectra of the electromagnetic fields reduce to the following simple forms (with $k_{\rm e} = -1/\eta_{\rm e}$):

$$\mathcal{P}_{\rm B}(k) = \frac{9\,H_{\rm I}^4}{4\,\pi^2}, \quad \mathcal{P}_{\rm E}(k) = \frac{H_{\rm I}^4}{4\,\pi^2}\,\left(\frac{k}{k_{\rm e}}\right)^2.$$

Helical electromagnetic fields

The action we had considered earlier can be extended to include a parity violating term as follows[10]:

$$S[A^\mu] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\,\sqrt{-g}\,\left[J^2(\phi)\,F_{\mu\nu}\,F^{\mu\nu} - \frac{\gamma}{2}\,I^2(\phi)\,F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right],$$

where $\widetilde{F}^{\mu\nu} = (\epsilon^{\mu\nu\alpha\beta}/\sqrt{-g})\,F_{\alpha\beta}$, with $\epsilon^{\mu\nu\alpha\beta}$ being the completely anti-symmetric Levi-Civita tensor, and γ is a constant.

Upon decomposing in the helical basis, the Fourier modes, say, $\bar{A}_k^\sigma$ satisfy the differential equation

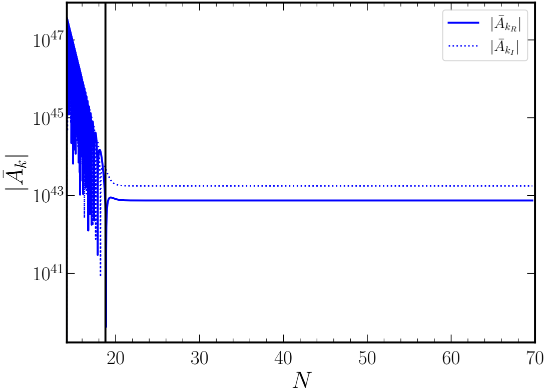
$$\bar{A}_k^{\sigma\,\prime\prime} + 2\,\frac{J'}{J}\,\bar{A}_k^{\sigma\,\prime} + \left(k^2 + \frac{\sigma\,\gamma\,k}{J^2}\,\frac{\mathrm{d}I^{\,2}}{\mathrm{d}\,\eta}\right)\,\bar{A}_k^\sigma = 0.$$

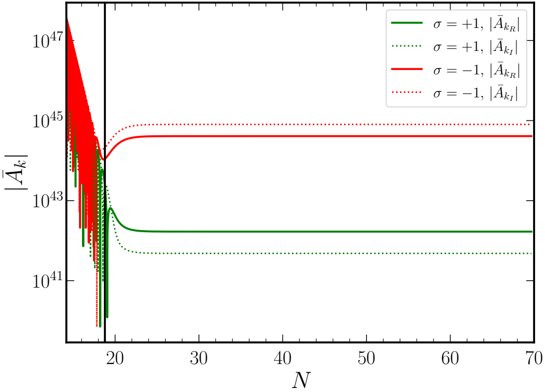
In terms of $\mathcal{A}_k^\sigma = J\,\bar{A}_k^\sigma$, the above equation reduces to

$$\mathcal{A}_k^{\sigma\,\prime\prime} + \left(k^2 + \frac{2\,\sigma\,\gamma\,k\,I\,I'}{J^2} - \frac{J''}{J}\right)\,\mathcal{A}_k^\sigma = 0.$$

[10] M. M. Anber and L. Sorbo, JCAP **10**, 018 (2006); C. Caprini and L. Sorbo, JCAP **10**, 056 (2014); D. Chowdhury, L. Sriramkumar and M. Kamionkowski, JCAP **10**, 031 (2018).

Behavior of the non-helical and helical electromagnetic modes





Typical behavior of the real (as solid curves) and imaginary (as dotted curves) parts of the non-helical (in blue, on the left) and helical (in red and green, on the right) electromagnetic modes have been plotted as function of e-folds for a specific wave number. The vertical lines indicates the time when the wave number leaves the Hubble radius[11].

[11] D. Chowdhury, L. Sriramkumar and M. Kamionkowski, JCAP **10**, 031 (2018).

Power spectra of the helical electromagnetic fields

We shall hereafter focus on the case wherein $I = J$.

The power spectra of the magnetic and electric fields can be expressed in terms of the modes $\bar{A}_k^\sigma$ and the coupling function J as follows[12]:

$$\mathcal{P}_{\rm B}(k) = \frac{k^5}{4\,\pi^2}\,\frac{J^2}{a^4}\,\left[\left|\bar{A}_k^+\right|^2 + \left|\bar{A}_k^-\right|^2\right] = \frac{k^5}{4\,\pi^2\,a^4}\,\left[\left|\mathcal{A}_k^+\right|^2 + \left|\mathcal{A}_k^-\right|^2\right],$$

$$\mathcal{P}_{\rm E}(k) = \frac{k^3}{4\,\pi^2}\,\frac{J^2}{a^4}\,\left[\left|\bar{A}_k^{+\prime}\right|^2 + \left|\bar{A}_k^{-\prime}\right|^2\right] = \frac{k^3}{4\,\pi^2\,a^4}\,\left[\left|\mathcal{A}_k^{+\prime} - \frac{J'}{J}\mathcal{A}_k^+\right|^2 + \left|\mathcal{A}_k^{-\prime} - \frac{J'}{J}\mathcal{A}_k^-\right|^2\right].$$

[12] See, for example, C. Caprini and L. Sorbo, JCAP **10**, 056 (2014);
D. Chowdhury, L. Sriramkumar and M. Kamionkowski, JCAP **10**, 031 (2018).

Scale invariant amplitudes and the non-helical limit

When $n=2$, we find that the spectra of the helical magnetic and electric fields, evaluated in the limit $(-k\,\eta_{\rm e}) \ll 1$, can be written as[13]

$$\mathcal{P}_{_{\rm B}}(k) = \frac{9\,H_{_{\rm I}}^4}{4\,\pi^2}\,f(\gamma),$$
$$\mathcal{P}_{_{\rm E}}(k) = \frac{9\,H_{_{\rm I}}^4}{4\,\pi^2}\,f(\gamma)\left[\gamma^2 - \frac{\sinh^2(2\,\pi\,\gamma)}{3\,\pi\,\left(1+\gamma^2\right)\,f(\gamma)}\,(-k\,\eta_{\rm e}) + \frac{1}{9}\,\left(1+23\,\gamma^2+40\,\gamma^4\right)\,(-k\,\eta_{\rm e})^2\right],$$

where the function $f(\gamma)$ is given by

$$f(\gamma) = \frac{\sinh\,(4\,\pi\,\gamma)}{4\,\pi\,\gamma\,\left(1+5\,\gamma^2+4\,\gamma^4\right)}.$$

Note that the non-helical case case corresponds to $\gamma=0$.

Also, note that the power spectra in the helical case are enhanced when compared to the non-helical case.

[13] S. Tripathy, D. Chowdhury, R. K. Jain and L. Sriramkumar, Phys. Rev. D **105**, 063519 (2022).

Strengths of the magnetic fields at the present epoch

In the case of *instantaneous* reheating, the spectrum of the magnetic field today, say, $\mathcal{P}_{_{\rm B}}^0(k)$, is related to the spectrum $\mathcal{P}_{_{\rm B}}(k)$ at the end of inflation as $\mathcal{P}_{_{\rm B}}^0(k) \simeq \mathcal{P}_{_{\rm B}}(k)\,(a_{\rm e}/a_0)^4$, where $a_{\rm e}$ is the scale factor at the end of inflation, while a_0 denotes the scale factor today.

Conservation of entropy post inflation leads to

$$\frac{a_0}{a_{\rm e}} \simeq 2.8 \times 10^{28}\,\left(\frac{H_{_{\rm I}}}{10^{-5}\,M_{_{\rm Pl}}}\right)^{1/2}.$$

Given the scale invariant amplitude for the magnetic field at the end of inflation in the helical case, we can estimate the present day strength of the magnetic field, say, B_0, to be[14]

$$B_0 \simeq 4.5 \times 10^{-12}\,\left(\frac{H_{_{\rm I}}}{10^{-5}\,M_{_{\rm Pl}}}\right)\,f^{1/2}(\gamma)\ {\rm G}.$$

[14] S. Tripathy, D. Chowdhury, R. K. Jain and L. Sriramkumar, Phys. Rev. D **105**, 063519 (2022).

Non-conformal coupling function in the Starobinsky model

The other case that we shall discuss is the Starobinsky model described by the potential

$$V(\phi) = V_0 \left[1 - \exp\left(-\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]^2.$$

The evolution of the field in the slow roll approximation is described by the expression

$$N - N_{\mathrm{e}} \simeq -\frac{3}{4}\left[\exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{\mathrm{Pl}}}\right) - \exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi_{\mathrm{e}}}{M_{\mathrm{Pl}}}\right) - \sqrt{\frac{2}{3}}\left(\frac{\phi}{M_{\mathrm{Pl}}} - \frac{\phi_{\mathrm{e}}}{M_{\mathrm{Pl}}}\right)\right],$$

where ϕ_{e} is the value of the field at the end of inflation which is determined by the relation $\exp\left[\sqrt{(2/3)}\,\phi_{\mathrm{e}}/M_{\mathrm{Pl}}\right] \simeq 1 + 2/\sqrt{3}$.

Therefore, to achieve the desired dependence of the coupling function on the scale factor, we can choose $J(\phi)$ in the model to be

$$J(\phi) = \exp\left\{-\frac{3\,n}{4}\left[\exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{\mathrm{Pl}}}\right) - \exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi_{\mathrm{e}}}{M_{\mathrm{Pl}}}\right) - \sqrt{\frac{2}{3}}\left(\frac{\phi}{M_{\mathrm{Pl}}} - \frac{\phi_{\mathrm{e}}}{M_{\mathrm{Pl}}}\right)\right]\right\}.$$

Strengths of helical electromagnetic fields in slow roll inflation

In the helical case, when $n = 2$, the amplitude of the spectra of the magnetic and electric fields can be expressed as

$$\frac{\mathcal{P}_{_{\rm B}}(k)}{M_{_{\rm Pl}}^4} \simeq \frac{9\,\pi^2}{16}\,(r\,A_{\rm s})^2\,f(\gamma), \quad \frac{\mathcal{P}_{_{\rm E}}(k)}{M_{_{\rm Pl}}^4} \simeq \frac{\mathcal{P}_{_{\rm B}}(k)}{M_{_{\rm Pl}}^4}\,\gamma^2.$$

In these expressions, $A_{\rm s} = 2.1 \times 10^{-9}$ denotes the observed amplitude of the scalar power spectrum at the pivot scale and r represents the tensor-to-scalar ratio.

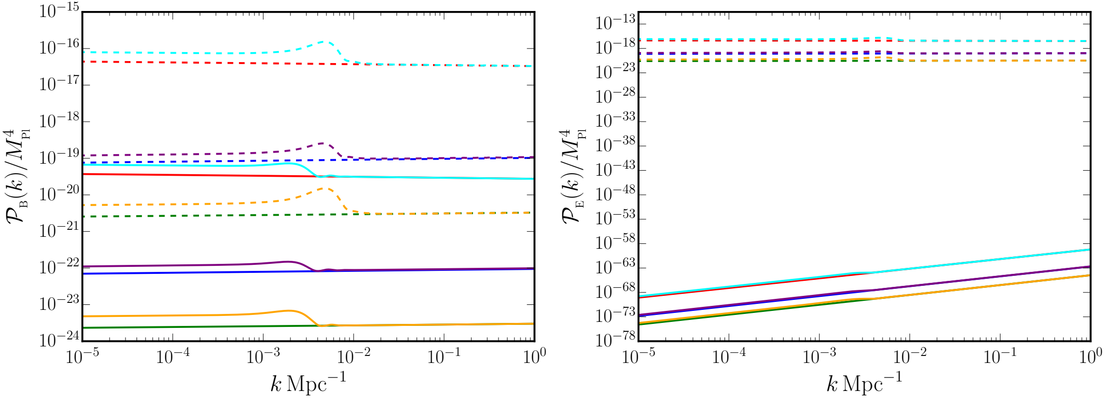
If we need to avoid backreaction due to the helical electromagnetic fields which have been generated, we require that

$$\mathcal{P}_{_{\rm B}}(k) + \mathcal{P}_{_{\rm E}}(k) \ll \rho_{_{\rm I}} = 3\,H_{_{\rm I}}^2\,M_{_{\rm Pl}}^2.$$

Since we are considering inflationary models wherein $H_{_{\rm I}}/M_{_{\rm Pl}} \lesssim 10^{-5}$, we find that the condition for avoiding backreaction leads to $f(\gamma)\,(1+\gamma^2) \lesssim 10^{10}$.

This limits the value of γ to be $\gamma \lesssim 2.5$.

Spectra of electromagnetic fields in slow roll inflation



The spectra of the magnetic (on the left) and electric (on the right) fields in the non-helical (solid curves) and helical (dashed curves) cases, arising in three slow roll inflationary models, have been plotted (in red, blue and green) over the CMB scales. We have also plotted the corresponding spectra when a step has been introduced in these potentials (in cyan, purple and orange). We have set $\gamma = 1$ for which $f(\gamma) \simeq 10^3$.

Back to scalar power spectra with features

Plan of the talk

Potentials with a step

Given an inflationary model described by the potential $V(\phi)$, we can introduce a step in the potential as follows[15]:

$$V_{\rm step}(\phi) = V(\phi)\, \left[1 + \alpha \tanh\left(\frac{\phi - \phi_0}{\Delta\phi}\right)\right],$$

where, evidently, ϕ_0, α and $\Delta\phi$ denote the location, the height and the width of the step.

[15] D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **10**, 008 (2010).

Suppressing power on large scales

In this context, the first example that we shall consider is another model due to Starobinsky, which is governed by the potential[16]

$$V(\phi) = \begin{cases} V_0 + A_+\,(\phi - \phi_0), & \text{for } \phi > \phi_0, \\ V_0 + A_-\,(\phi - \phi_0), & \text{for } \phi < \phi_0. \end{cases}$$

The second model that we shall consider is the so-called punctuated inflationary model described by the potential[17]

$$V(\phi) = \frac{m^2}{2}\,\phi^2 - \frac{2\,m^2}{3\,\phi_0}\,\phi^3 + \frac{m^2}{4\,\phi_0^2}\,\phi^4.$$

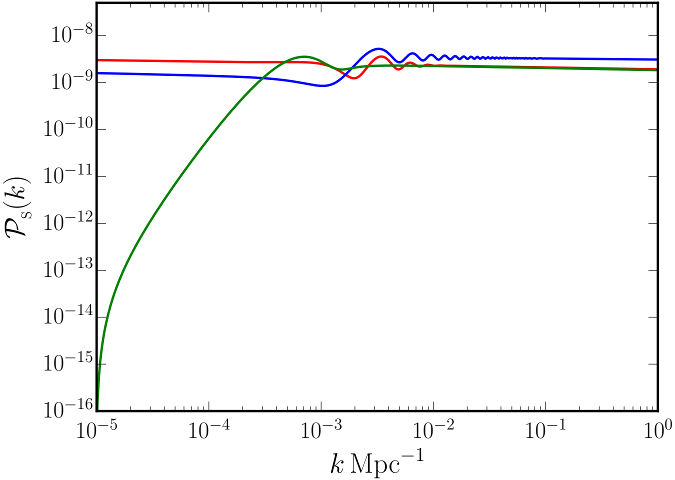
It is easy to show that this potential contains a point of inflection at ϕ_0.

▸ Back to spectra of electromagnetic fields in punctuated inflation

[16] A. A. Starobinsky, JETP Lett. **55**, 489 (1992).

[17] R. K. Jain, P. Chingangbam, J-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **01**, 009 (2009); R. K. Jain, P. Chingangbam, L. Sriramkumar and T. Souradeep, Phys. Rev. D **82**, 023509 (2010); H. V. Ragavendra, D. Chowdhury and L. Sriramkumar, Phys. Rev. D **106**, 043535 (2022).

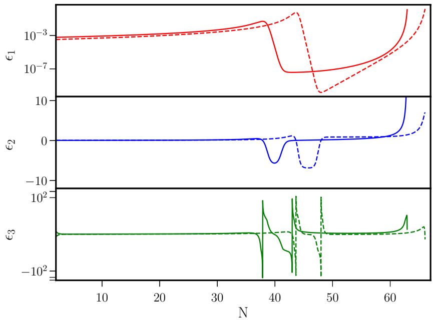
Scalar power spectra with features over the CMB scales

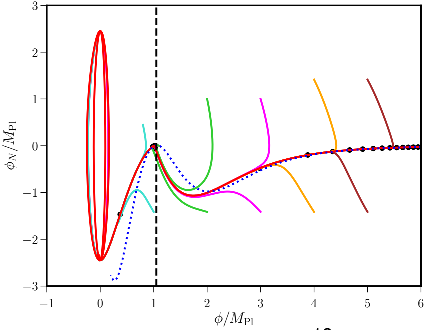


The scalar power spectra with features over the CMB scales in the cases of the quadratic potential with a step (in red), the second Starobinsky model (in blue), and the model of punctuated inflation (in green).

▸ Spectra of electromagnetic fields in slow roll inflation

Potentials admitting ultra slow roll inflation





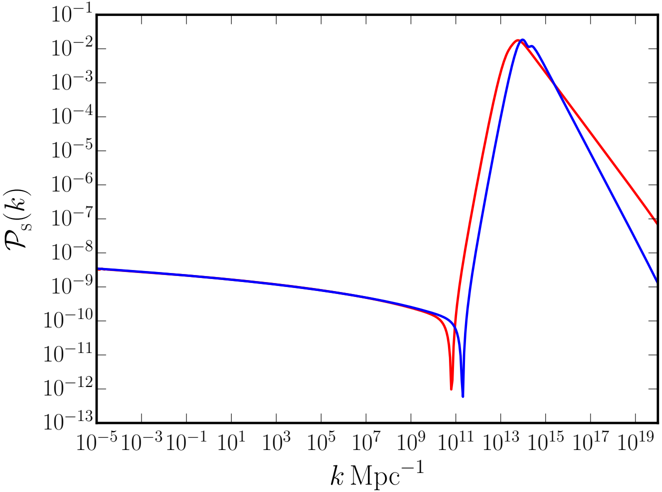
Potentials leading to ultra slow roll inflation (with $x = \phi/v$, v being a constant)[18]:

$$\mathrm{USR1}: V(\phi) = V_0\, \frac{6\, x^2 - 4\, \alpha\, x^3 + 3\, x^4}{(1 + \beta\, x^2)^2},$$

$$\mathrm{USR2}: V(\phi) = V_0\, \left\{\tanh\left(\frac{\phi}{\sqrt{6}\, M_{_{\rm Pl}}}\right) + A\, \sin\left[\frac{\tanh\left[\phi/\left(\sqrt{6}\, M_{_{\rm Pl}}\right)\right]}{f_\phi}\right]\right\}^2.$$

[18] J. Garcia-Bellido and E. R. Morales, Phys. Dark Univ. **18**, 47 (2017); I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).

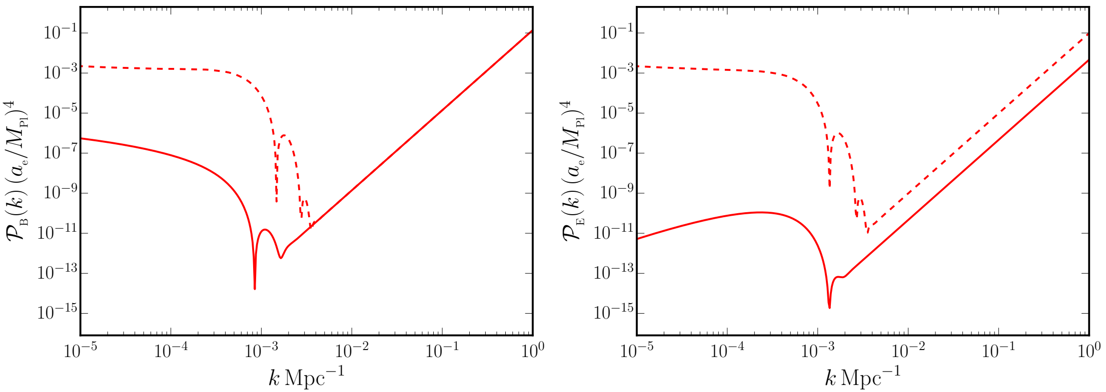
Scalar power spectra with enhanced power on small scales



The scalar power spectra with enhanced power on small scales that arise in the inflationary models USR2 (in red) and PI3 (in blue)[19].

[19] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

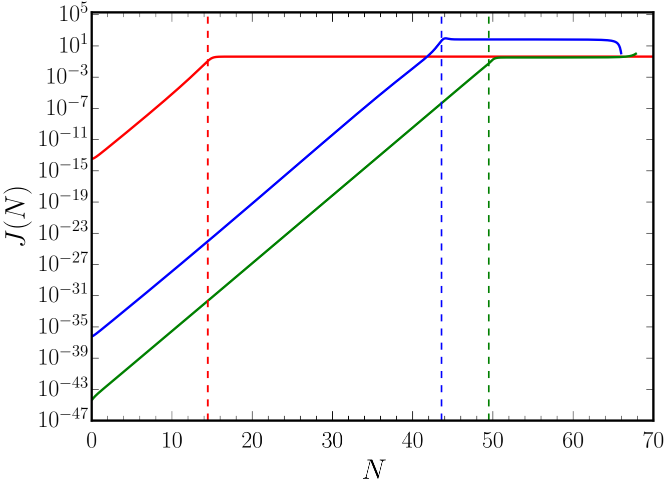
Spectra of electromagnetic fields in punctuation inflation



The spectra of the magnetic (on the left) and electric (on the right) fields arising in the case of the first punctuated inflation model have been plotted for both the non-helical (as solid curves) and helical (as dashed curves) cases.

▸ Models leading to suppression of power on large scales

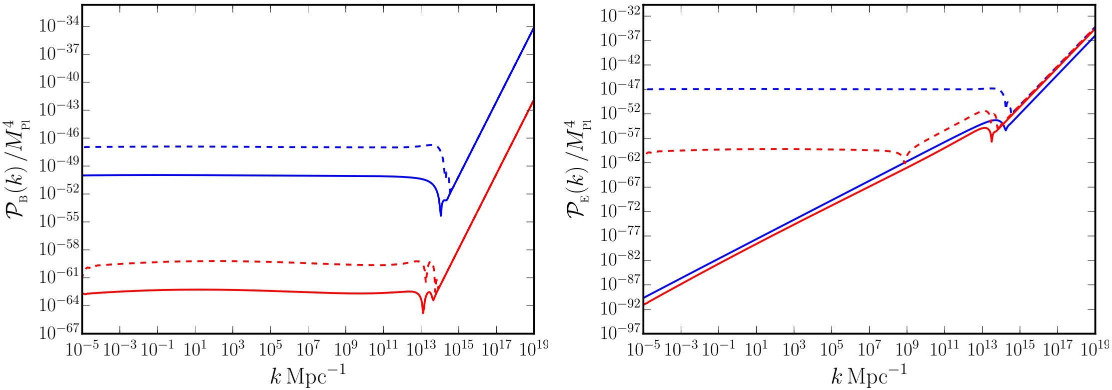
Behavior of the coupling function in models permitting ultra slow roll



The evolution of the non-conformal coupling function J in inflationary models leading to ultra slow roll. Note that the coupling function does not change appreciably once ultra slow roll sets in (indicated by the vertical lines).

Back to spectra of electromagnetic fields in two field models

Resulting spectra of the electromagnetic fields



The spectra of the magnetic (on the left) and electric (on the right) fields arising in two inflationary models permitting a period of ultra slow roll (at late times) have been plotted in the non-helical (as solid lines) and helical (as dashed lines) cases[20].

[20] S. Tripathy, D. Chowdhury, R. K. Jain and L. Sriramkumar, Phys. Rev. D **105**, 063519 (2022).

Plan of the talk

The two field models of interest

It has been shown that two scalar fields ϕ and χ governed by the action

$$S[\phi,\chi] = \int \mathrm{d}^4x\,\sqrt{-g}\,\left[-\frac{1}{2}\,\partial^\mu\phi\,\partial_\mu\phi - \frac{f(\phi)}{2}\,\partial^\mu\chi\,\partial_\mu\chi - V(\phi,\chi)\right]$$

described by *separable* potentials and the non-canonical functions $f(\phi) = \exp(2\,b\,\phi)$ or $f(\phi) = \exp(2\,b\,\phi^2)$ can lead to features in the scalar power spectrum.

While the potential[21]

$$V(\phi,\chi) = \frac{1}{2}\,m_\phi^2\,\phi^2 + V_0\,\frac{\chi^2}{\chi_0^2+\chi^2}$$

leads to a suppression in power on large scales, the potential[22]

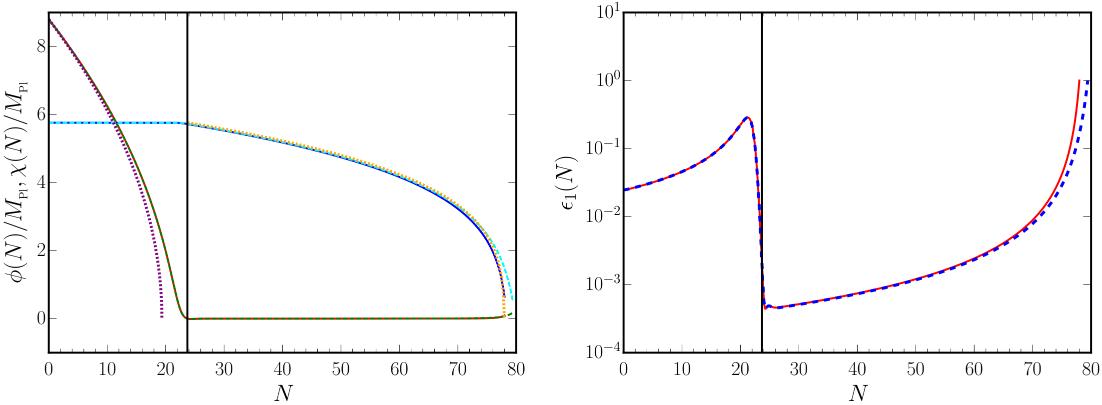
$$V(\phi,\chi) = V_0\,\frac{\phi^2}{\phi_0^2+\phi^2} + \frac{1}{2}\,m_\chi^2\,\chi^2$$

generates enhanced power on small scales.

[21] M. Braglia, D. K. Hazra, L. Sriramkumar and F. Finelli, JCAP **08**, 025 (2020).

[22] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

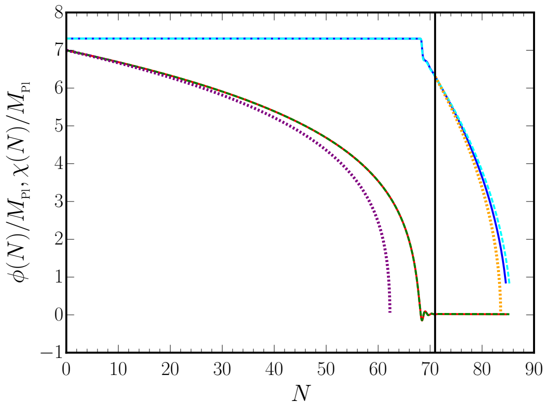
Behavior of the scalar fields and the first slow roll parameter I

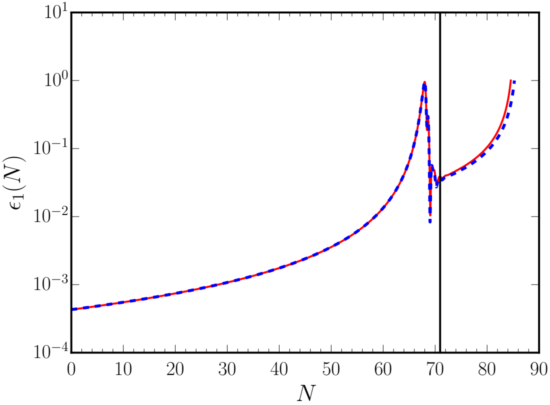


Behavior of the two scalar fields ϕ and χ (in blue and red, on the left) and the first slow roll parameter ϵ_1 (on the right) in the first two field model[23]. Note that there arises a turn in the field space around the e-fold $N = 24$, when the first slow roll parameter begins to decrease before increasing again, leading to the termination of inflation.

[23] M. Braglia, D. K. Hazra, L. Sriramkumar and F. Finelli, JCAP **08**, 025 (2020).

Behavior of the scalar fields and the first slow roll parameter II

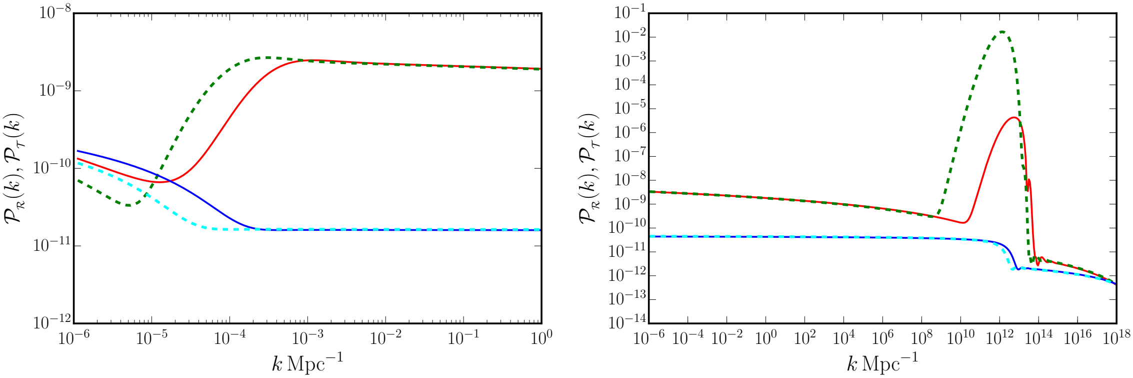




Behavior of the two scalar fields and the first slow roll parameter in the second two field model has been plotted as in the case of the first model[24]. Evidently, barring the time of the turn in field space, the fields broadly behave in the same manner as in the previous example.

[24] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

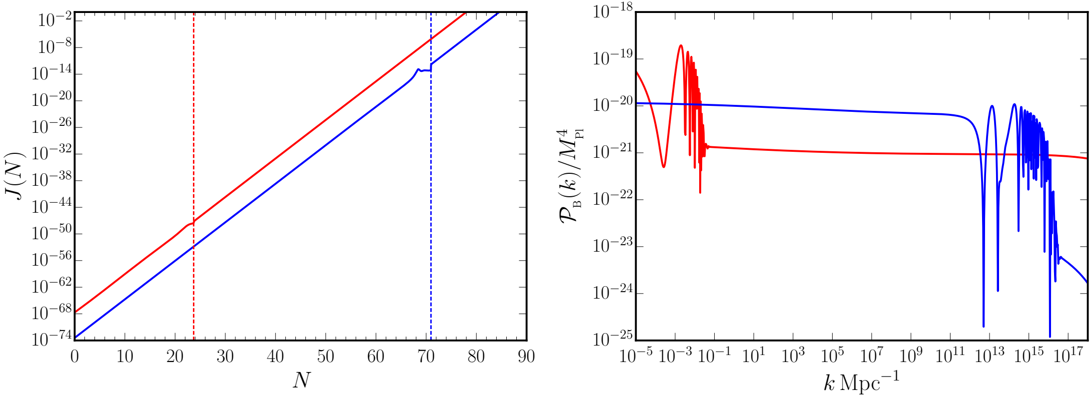
Scalar and tensor power spectra in two field models



The spectra of curvature (in solid red and dashed green) and tensor (in solid blue and dashed cyan) perturbations, viz. $\mathcal{P}_{\mathcal{R}}(k)$ and $\mathcal{P}_{\mathcal{T}}(k)$, have been plotted for the two field inflationary models that we have considered.[25].

[25] S. Tripathy, D. Chowdhury, H. V. Ragavendra, R. K. Jain and L. Sriramkumar, Phys. Rev. D **107**, 043501 (2023).

Circumventing the challenge in two field models



The evolution of the non-conformal coupling function J (on the left) and the corresponding spectrum of magnetic field (on the right) arising in the two field inflationary models leading to features on large (in red) and on small (in blue) scales[26]. The vertical lines (on the left) indicate the time when the turn in the field space takes place.

▸ Evolution of J in ultra slow roll inflation

[26] S. Tripathy, D. Chowdhury, H. V. Ragavendra, R. K. Jain and L. Sriramkumar, Phys. Rev. D **107**, 043501 (2023).

Plan of the talk

Action describing the Fourier modes of the electromagnetic field

The helical electromagnetic modes $\mathcal{A}_{\boldsymbol{k}}^{\sigma}$ are described by the action

$$S[\mathcal{A}_{\boldsymbol{k}}^{\sigma}] = \int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{k} \sum_{\sigma=\pm} \left[\frac{1}{2}\,|\mathcal{A}_{\boldsymbol{k}}^{\sigma\prime}|^2 - \frac{\kappa}{2}\,\left(\mathcal{A}_{\boldsymbol{k}}^{\sigma\prime}\,\mathcal{A}_{\boldsymbol{k}}^{\sigma *} + \mathcal{A}_{\boldsymbol{k}}^{\sigma\prime *}\,\mathcal{A}_{\boldsymbol{k}}^{\sigma}\right) - \frac{\mu^2}{2}\,|\mathcal{A}_{\boldsymbol{k}}^{\sigma}|^2\right],$$

where the quantities μ^2 and κ are given by

$$\mu^2 = k^2 - \left(\frac{J'}{J}\right)^2 + \frac{2\,\sigma\,\gamma\,k\,I^2\,J'}{J^3}, \quad \kappa = \frac{J'}{J} - \frac{\sigma\,\gamma\,k\,I^2}{J^2}.$$

Upon adding the total time derivative $-[(\sigma\,\gamma\,k\,I^2/J^2)\,|\mathcal{A}_{\boldsymbol{k}}^{\sigma}|^2]'$, the action reduces to

$$S[\mathcal{A}_{\boldsymbol{k}}^{\sigma}] = \int \mathrm{d}\eta \int_{\mathbb{R}^3/2} \mathrm{d}^3\boldsymbol{k} \sum_{\sigma=\pm} \left[|\mathcal{A}_{\sigma}'|^2 - \frac{J'}{J}\,\left(\mathcal{A}_{\boldsymbol{k}}^{\sigma\prime}\,\mathcal{A}_{\boldsymbol{k}}^{\sigma *} + \mathcal{A}_{\boldsymbol{k}}^{\sigma\prime *}\,\mathcal{A}_{\boldsymbol{k}}^{\sigma}\right) - \bar{\mu}^2\,|\mathcal{A}_{\boldsymbol{k}}^{\sigma}|^2\right],$$

where $\bar{\mu}^2$ is given by

$$\bar{\mu}^2 = k^2 + \frac{2\,\sigma\,\gamma\,k\,I\,I'}{J^2} - \left(\frac{J'}{J}\right)^2.$$

Hamiltonian governing a pair of modes

Since $\mathcal{A}^\sigma_{-\boldsymbol{k}} = \mathcal{A}^{\sigma\,*}_{\boldsymbol{k}}$, the original action above can be expressed as

$$S[\mathcal{A}^\sigma_{\boldsymbol{k}}, \mathcal{A}^\sigma_{-\boldsymbol{k}}] = \int \mathrm{d}\eta \int_{\mathbb{R}^3/2} \mathrm{d}^3\boldsymbol{k} \sum_{\sigma=\pm} \left[\mathcal{A}^{\sigma\prime}_{\boldsymbol{k}}\, \mathcal{A}^{\sigma\prime}_{-\boldsymbol{k}} - \kappa\, \left(\mathcal{A}^{\sigma\prime}_{\boldsymbol{k}}\, \mathcal{A}^{\sigma}_{-\boldsymbol{k}} + \mathcal{A}^{\sigma\prime}_{-\boldsymbol{k}}\, \mathcal{A}^{\sigma}_{\boldsymbol{k}}\right) - \mu^2\, \mathcal{A}^\sigma_{\boldsymbol{k}}\, \mathcal{A}^\sigma_{-\boldsymbol{k}}\right].$$

Motivated by the approach adopted in the case of the scalar perturbations, we can define the new variables $(x^\sigma_{\boldsymbol{k}}, p^\sigma_{\boldsymbol{k}})$ in terms of $(\mathcal{A}^\sigma_{\boldsymbol{k}}, \mathcal{P}^\sigma_{\boldsymbol{k}})$ and $(\mathcal{A}^\sigma_{-\boldsymbol{k}}, \mathcal{P}^\sigma_{-\boldsymbol{k}})$ as follows[27]:

$$x^\sigma_{\boldsymbol{k}} = \frac{1}{2}\, \left(\mathcal{A}^\sigma_{\boldsymbol{k}} + \mathcal{A}^\sigma_{-\boldsymbol{k}}\right) + \frac{i}{2\,\widetilde{\omega}}\, \left(\mathcal{P}^\sigma_{\boldsymbol{k}} - \mathcal{P}^\sigma_{-\boldsymbol{k}}\right), \quad p^\sigma_{\boldsymbol{k}} = \frac{1}{2}\, \left(\mathcal{P}^\sigma_{\boldsymbol{k}} + \mathcal{P}^\sigma_{-\boldsymbol{k}}\right) - \frac{i\,\widetilde{\omega}}{2}\, \left(\mathcal{A}^\sigma_{\boldsymbol{k}} - \mathcal{A}^\sigma_{-\boldsymbol{k}}\right),$$

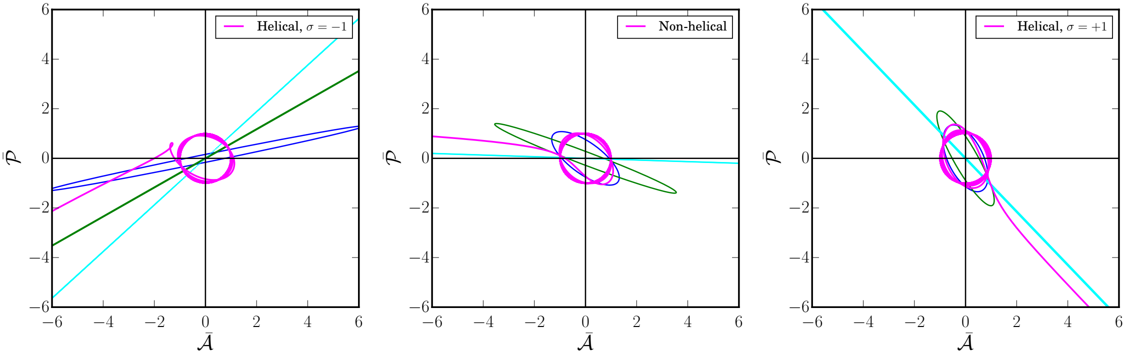
where $\widetilde{\omega}^2 = k^2\,(1 + \gamma^2\, I^4/J^4)$.

The Hamiltonian density in Fourier space is then given by

$$\mathcal{H} = \frac{1}{2}\, \left(p^\sigma_{\boldsymbol{k}}\, p^\sigma_{\boldsymbol{k}} + p^\sigma_{-\boldsymbol{k}}\, p^\sigma_{-\boldsymbol{k}}\right) + \kappa\, \left(x^\sigma_{\boldsymbol{k}}\, p^\sigma_{-\boldsymbol{k}} + x^\sigma_{-\boldsymbol{k}}\, p^\sigma_{\boldsymbol{k}}\right) + \frac{\widetilde{\omega}^2}{2}\, \left(x^\sigma_{\boldsymbol{k}}\, x^\sigma_{\boldsymbol{k}} + x^\sigma_{-\boldsymbol{k}}\, x^\sigma_{-\boldsymbol{k}}\right).$$

[27] J. Martin and V. Vennin, Phys. Rev. D **93**, (2016) 2, 023505 (2016); J. Martin, Universe **5**, 92 (2019).

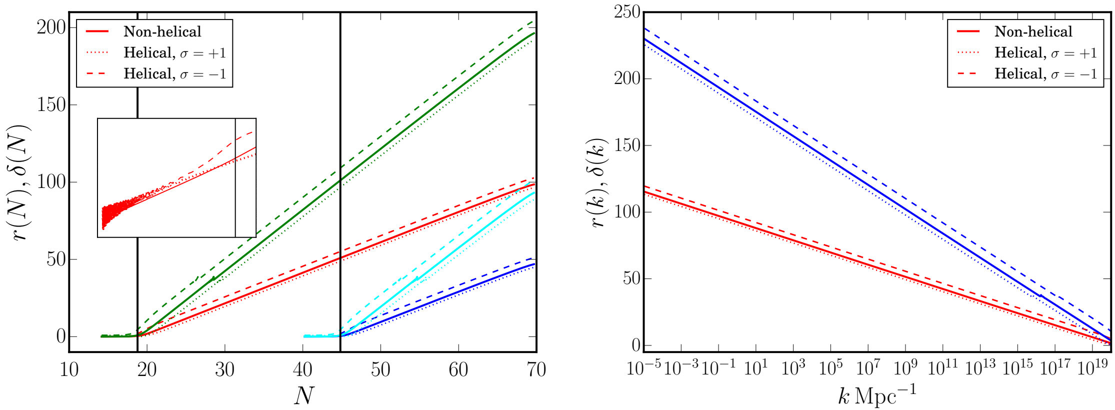
Behavior of the Wigner ellipse



Evolution of the Wigner ellipse (in red, blue, green and cyan) and the classical trajectory (in magenta) in the phase space $\bar{A}$-$\bar{\mathcal{P}}$ has been plotted for the electromagnetic mode in slow roll inflation with the wave number corresponding to the CMB pivot scale[28].

[28] S. Tripathy, R. N. Raveendran, K. Parattu and L. Sriramkumar, arXiv:2306.16168 [gr-qc].

'Spectra' of the squeezing amplitude and entanglement entropy



Evolution of the squeezing amplitude $r(N)$ (in red and blue) and the entanglement entropy $\delta(N)$ (in green and cyan) have been plotted (on the left) for electromagnetic modes with two different wave numbers in slow roll inflation. We have also plotted (on the right) the 'spectra' of the squeezing amplitude $r(k)$ and the entanglement entropy $\delta(k)$[29].

[29] S. Tripathy, R. N. Raveendran, K. Parattu and L. Sriramkumar, arXiv:2306.16168 [gr-qc].

Plan of the talk

Summary

- In single field models of inflation, substantial departures from slow roll inflation can lead to strong features in the spectra of magnetic fields.
- Strong departures from slow roll inflation can also suppress the strength of the magnetic field on large scales.
- Some of these challenges can be overcome in two field models of inflation. However, there always seems to be an element of fine tuning involved.
- It seems necessary to examine the behavior of additional quantities such as the three-point functions and the corresponding observables to arrive at constraints on the nature and form of the non-conformal coupling function.
- On a different note, we find that violation of parity leads to an enhancement of the squeezing amplitude and the entanglement entropy associated with one of the two states of polarization of the electromagnetic field.

Collaborators

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Thank you for your attention