Magnetogenesis in bouncing universes

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The Plasma Universe and its Structure Formation

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Plan of the talk

This talk is based on. . .

- L. Sriramkumar, K. Atmjeet and R. K. Jain, *Generation of scale invariant magnetic fields in bouncing universes*, JCAP **1509**, 010 (2015) [arXiv:1506.06475 [astro-ph.CO]].
- D. Chowdhury, L. Sriramkumar and R. K. Jain, *Duality and the generation of magnetic fields in bouncing universes*, Phys. Rev. D **94**, 083512 (2016) [arXiv:1604.02143 [gr-qc]].

Bouncing scenarios: An alternative to inflation[1]

- Bouncing models correspond to situations wherein the universe initially goes through a period of contraction until the scale factor reaches a certain minimum value before transiting to the expanding phase.

[1] See, for instance, M. Novello and S. P. Bergliaffa, Phys. Rep. **463**, 127 (2008); D. Battefeld and P. Peter, Phys. Rep. **571**, 1 (2015).

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- Bouncing models correspond to situations wherein the universe initially goes through a period of contraction until the scale factor reaches a certain minimum value before transiting to the expanding phase.
- They offer an alternative to inflation to overcome the horizon problem, as they permit well motivated, Minkowski-like initial conditions to be imposed on the perturbations at early times during the contracting phase.

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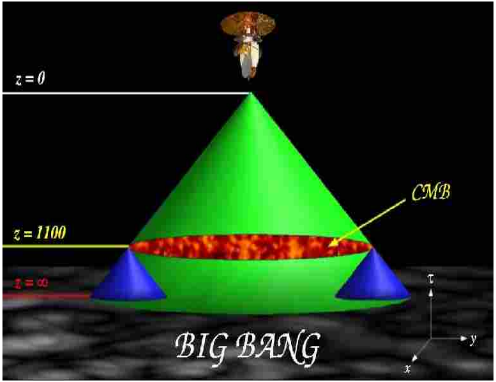
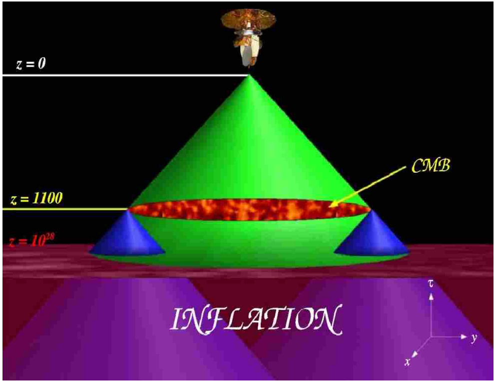
- They offer an alternative to inflation to overcome the horizon problem, as they permit well motivated, Minkowski-like initial conditions to be imposed on the perturbations at early times during the contracting phase.
- However, matter fields may have to violate the null energy condition near the bounce in order to give rise to such a scale factor. Also, there exist (genuine) concerns whether such an assumption about the scale factor is valid in a domain where general relativity is expected to fail and quantum gravitational effects are supposed to take over.

[1] See, for instance, M. Novello and S. P. Bergliaffa, Phys. Rep. **463**, 127 (2008); D. Battefeld and P. Peter, Phys. Rep. **571**, 1 (2015).

The resolution of the horizon problem in inflation





Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

[2] Images from W. Kinney, astro-ph/0301448.

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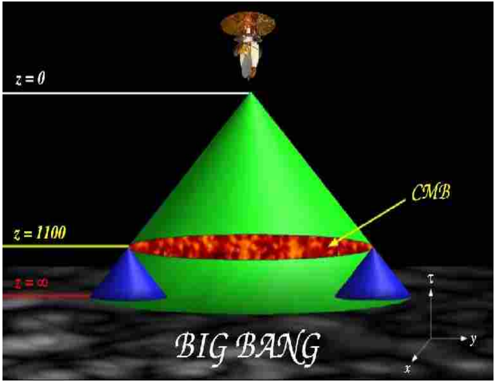
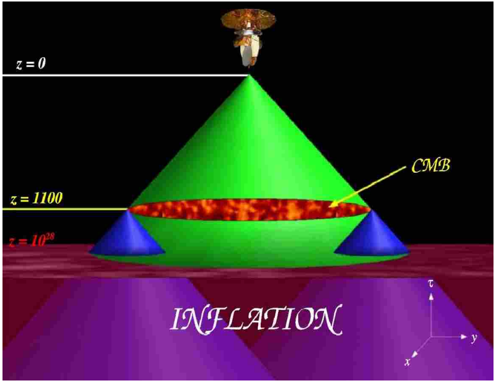


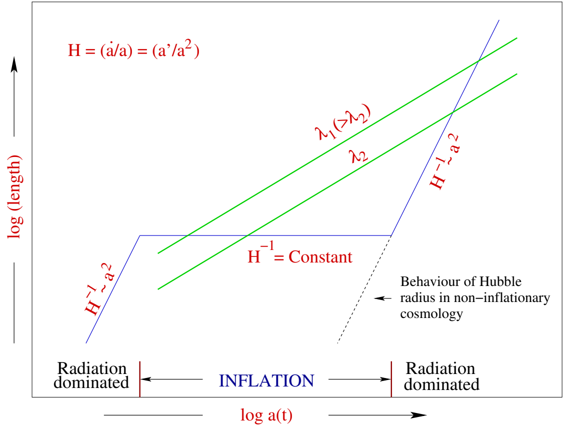


Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

Right: An illustration of how an early and sufficiently long epoch of inflation helps in resolving the horizon problem[2].

[2] Images from W. Kinney, astro-ph/0301448.

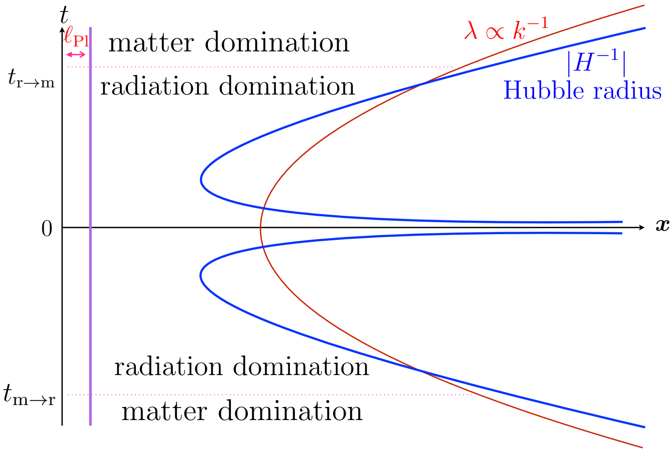
Bringing the modes inside the Hubble radius



A schematic diagram illustrating the behavior of the physical wavelength $\lambda_{\rm P} \propto a$ (the green lines) and the Hubble radius H^{-1} (the blue line) during inflation and the radiation dominated epochs[3].

[3] See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.

Overcoming the horizon problem in bouncing models



The evolution of the physical wavelength and the Hubble radius in a typical bouncing scenario[4].

[4]Figure from, D. Battefeld and P. Peter, Phys. Rept. **571**, 1 (2015).

Violation of the null energy condition

Recall that, according to the Friedmann equations

$$\dot{H} = -4\,\pi\,G\,(\rho + p)\,.$$

In any bouncing scenario, the Hubble parameter is negative before the bounce, crosses zero at the bounce and is positive thereafter.

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In any bouncing scenario, the Hubble parameter is negative before the bounce, crosses zero at the bounce and is positive thereafter.

Evidently, $\dot{H}$ will be positive near the bounce, which implies that $(\rho + p)$ has to be negative in this domain. In other words, the null energy condition needs to be violated in order to achieve a bounce.

Classical bounces and sources

Consider for instance, bouncing models of the form

$$a(\eta) = a_0 \left(1 + \frac{\eta^2}{\eta_0^2}\right)^q = a_0 \left(1 + k_0^2\, \eta^2\right)^q,$$

where a_0 is the value of the scale factor at the bounce (*i.e.* when $\eta = 0$), $\eta_0 = 1/k_0$ denotes the time scale of the duration of the bounce and $q > 0$. We shall assume that the scale k_0 associated with the bounce is of the order of the Planck scale $M_{_{\rm Pl}}$.

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The above scale factor can be achieved with the help of two fluids with constant equation of state parameters $w_1 = (1-q)/(3\,q)$ and $w_2 = (2-q)/(3\,q)$. The energy densities of these fluids behave as $\rho_1 = M_1/a^{(2\,q+1)/q}$ and $\rho_2 = M_2/a^{2\,(1+q)/q}$, where $M_1 = 12\, k_0^2\, M_{\rm Pl}^2\, a_0^{1/q}$ and $M_2 = -M_1\, a_0^{1/q}$.

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Note that, when $q = 1$, during very early times wherein $\eta \ll -\eta_0$, the scale factor behaves as in a matter dominated universe (*i.e.* $a \propto \eta^2$). Therefore, the $q = 1$ case is often referred to as the matter bounce scenario. In such a case, $\rho_1 = 12\,k_0^2\,M_{_{\rm Pl}}^2\,a_0/a^3$ and $\rho_2 = -12\,k_0^2\,M_{_{\rm Pl}}^2\,a_0^2/a^4$.

The non-minimal action and the equation of motion

We shall consider a case wherein the electromagnetic field is coupled non-minimally to a scalar field ϕ and is described by the action

$$S[\phi, A^\mu] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\,\sqrt{-g}\,J^2(\phi)\,F_{\mu\nu}F^{\mu\nu},$$

where $F_{\mu\nu}$ denotes the electromagnetic field tensor which is given in terms of the vector potential A^μ as follows:

$$F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu} = A_{\nu,\mu} - A_{\mu,\nu}.$$

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The scalar field ϕ could be, for instance, the primary matter field that is driving the background evolution and J is an arbitrary function of the field.

Quantization of the electromagnetic field

In a spatially flat, FLRW universe, we can choose to work in the Coulomb gauge wherein $A_0 = 0$ and $\partial_i A^i = 0$. In such a gauge, upon quantization, the vector potential $\hat{A}_i$ can be Fourier decomposed as follows[5]:

$$\hat{A}_i(\eta, \boldsymbol{x}) = \sqrt{4\,\pi} \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}} \sum_{\lambda=1}^{2} \tilde{\epsilon}_{\lambda\,i}(\boldsymbol{k}) \left[\hat{a}_{\boldsymbol{k}}^{\lambda}\,\bar{A}_k(\eta)\,\mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{a}_{\boldsymbol{k}}^{\lambda\dagger}\,\bar{A}_k^{*}(\eta)\,\mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right],$$

where the modes $\bar{A}_k$ satisfy the differential equation

$$\bar{A}_k'' + 2\,\frac{J'}{J}\,\bar{A}_k' + k^2\,\bar{A}_k = 0.$$

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If we define a new variable $\mathcal{A}_k = J\,\bar{A}_k$, then the above equation simplifies to

$$\mathcal{A}_k'' + \left(k^2 - \frac{J''}{J}\right)\mathcal{A}_k = 0,$$

and one can impose the standard Bunch-Davies initial conditions on the modes $\mathcal{A}_k$ at suitably early times.

[5]See, for instance, J. Martin and J. Yokoyama, JCAP **0801**, 025 (2008); K. Subramanian, Astron. Nachr. **331**, 110 (2010).

Power spectra of the electric and magnetic fields

The energy densities associated with the electric and magnetic fields can be written in terms of the vector potential A_i and its time and spatial derivatives as follows:

$$\rho_{\rm E} = \frac{J^2}{8\,\pi\,a^2}\, g^{ij}\, A_i'\, A_j',$$

$$\rho_{\rm B} = \frac{J^2}{16\,\pi}\, g^{ij}\, g^{lm}\, \left(\partial_j A_m - \partial_m A_j\right)\, \left(\partial_i A_l - \partial_l A_i\right),$$

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The expectation values of the corresponding operators, *i.e.* $\hat{\rho}_{\rm E}$ and $\hat{\rho}_{\rm B}$, can be evaluated in the vacuum state annihilated by the operator $\hat{a}_{\boldsymbol{k}}^{\lambda}$.

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It can be shown that the spectral energy densities of the magnetic and electric fields are given by

$$\mathcal{P}_{\rm B}(k) = \frac{{\rm d}\langle 0|\hat{\rho}_{\rm B}|0\rangle}{{\rm d}\ln k} = \frac{J^2(\eta)}{2\,\pi^2}\, \frac{k^5}{a^4(\eta)}\, |\bar{A}_k(\eta)|^2,$$

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Power spectra in power law inflation

For power law inflation described by the scale factor $a(\eta) = a_1\,(-\eta/\eta_1)^{\beta+1}$ and for coupling function of the form $J(\eta) = J_0\,a^n(\eta)$, one can show that the power spectrum of the magnetic field is given by[6]

$$\mathcal{P}_{_{\rm B}}(k) = \mathcal{F}(m)\,H^4\,(-k\,\eta)^{4+2\,m},$$

where $m = (\beta+1)\,n = \alpha$ for $\alpha \le 1/2$ and $m = 1-\alpha$ for $\alpha \ge 1/2$, while

$$\mathcal{F}(m) = \left[(2\,\pi)\,2^{2\,m+1}\,\Gamma^2(m+1/2)\,\cos^2(\pi\,m)\right]^{-1}.$$

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The corresponding spectrum for the electric field can be obtained to be

$$\mathcal{P}_{_\mathrm{E}}(k) = \frac{\mathcal{G}(m)}{2\,\pi^2}\, H^4 (-k\,\eta)^{4+2\,m},$$

where $m = 1+\alpha$ if $\alpha \leq -1/2$ and $m = -\alpha$ for $\alpha \geq -1/2$, while

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It is evident that $m = -2$ leads to a scale invariant spectrum for the magnetic field which corresponds to either $\alpha = 3$ or $\alpha = -2$.

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Modeling the bounce and the non-minimal coupling

We shall model the bounce by assuming that the scale factor $a(\eta)$ behaves as follows:

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We shall assume that the coupling function can be expressed in terms of the scale factor as

$$J(\eta) = J_0\, a^n(\eta).$$

E-$\mathcal{N}$-folds

The conventional e-fold N is defined $N = \log(a/a_0)$ so that $a(N) = a_0 \exp N$. However, the function e^N is a monotonically increasing function of N.

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In a bouncing scenario, an obvious choice for the scale factor seems to be[7]

$$a(\mathcal{N}) = a_0 \exp\left(\mathcal{N}^2/2\right),$$

with $\mathcal{N}$ being the new time variable that we shall consider for integrating the differential equation governing $\bar{A}_k$.

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with $\mathcal{N}$ being the new time variable that we shall consider for integrating the differential equation governing $\bar{A}_k$.

For want of a better name, we shall refer to the variable $\mathcal{N}$ as e-$\mathcal{N}$-fold since the scale factor grows roughly by the amount $\mathrm{e}^{\mathcal{N}}$ between $\mathcal{N}$ and $(\mathcal{N}+1)$.

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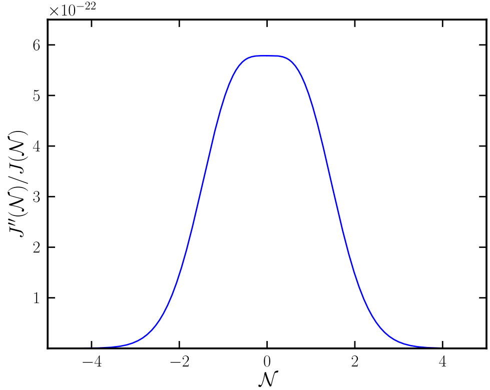
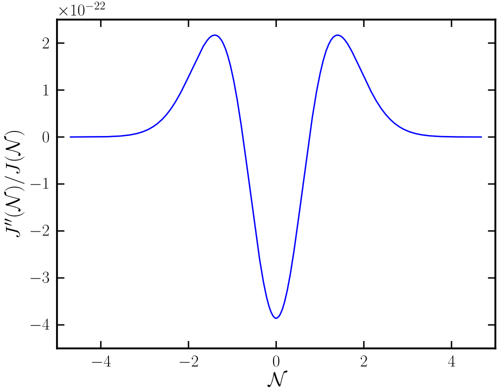
The behavior of J''/J





The behavior of the quantity J''/J has been plotted as a function of $\mathcal{N}$ for $q = 1$ and $n = 3/2$ (on the left) and $n = -1$ (on the right). Note that the maximum value of J''/J is roughly of the order of k_0^2.

Analytical solutions for the modes at early times

At very early times (*i.e.* for $-\eta \gg \eta_0$), the scale factor simplifies to the power law form $a(\eta) \propto \eta^{2\,q}$. During such times, the non-minimal coupling function J also behaves as $J(\eta) \propto \eta^{\gamma}$, where we have set $\gamma = 2\,n\,q$.

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In such a case, we have $J''/J \simeq \gamma\,(\gamma-1)/\eta^2$ and it is easy to show that the solutions to the modes of the electromagnetic vector potential $\mathcal{A}_k$ can be expressed as

$$\mathcal{A}_k(\eta) = \sqrt{-k\,\eta}\,\left[C_1(k)\,J_{\gamma-1/2}(-k\,\eta) + C_2(k)\,J_{-\gamma+1/2}(-k\,\eta)\right].$$

One finds that, for the Bunch-Davies initial conditions, $C_1(k)$ and $C_2(k)$ are given by

$$C_1(k) = \sqrt{\frac{\pi}{4\,k}}\,\frac{\mathrm{e}^{-i\,\pi\,\gamma/2}}{\cos\,(\pi\,\gamma)} \quad \text{and} \quad C_2(k) = \sqrt{\frac{\pi}{4\,k}}\,\frac{\mathrm{e}^{i\,\pi\,(\gamma+1)/2}}{\cos\,(\pi\,\gamma)}.$$

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It can also be shown that

$$\mathcal{A}_k'(\eta) - \frac{J'}{J}\,\mathcal{A}_k(\eta) = k\,\sqrt{-k\,\eta}\,\left[C_1(k)\,J_{\gamma+1/2}(-k\,\eta) - C_2(k)\,J_{-\gamma-1/2}(-k\,\eta)\right].$$

Analytical solutions near the bounce

Note that, when $n > 0$, J''/J has a maximum at the bounce. In such a case, for $k \ll k_0$, $k^2 \ll J''/J$ around the bounce. Hence, upon ignoring the k^2 in the equation governing $\bar{A}_k$, we can integrate the equation to yield

$$\bar{A}_k'(\eta) \simeq \bar{A}_k'(\eta_*) \, \frac{J^2(\eta_*)}{J^2(\eta)},$$

where η_* is a time when $k^2 \ll J''/J$ before the bounce. The above equation can be integrated to arrive at

$$\bar{A}_k(\eta) \simeq \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*) \int_{\eta_*}^{\eta} \mathrm{d}\eta \, \frac{J^2(\eta_*)}{J^2(\eta)} = \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*) \, a^{2\,n}(\eta_*) \int_{\eta_*}^{\eta} \frac{\mathrm{d}\eta}{a^{2\,n}(\eta)},$$

where we have set the constant of integration to be $\bar{A}_k(\eta_*)$.

Analytical solutions near the bounce

Note that, when $n > 0$, J''/J has a maximum at the bounce. In such a case, for $k \ll k_0$, $k^2 \ll J''/J$ around the bounce. Hence, upon ignoring the k^2 in the equation governing $\bar{A}_k$, we can integrate the equation to yield

$$\bar{A}_k'(\eta) \simeq \bar{A}_k'(\eta_*)\, \frac{J^2(\eta_*)}{J^2(\eta)},$$

where η_* is a time when $k^2 \ll J''/J$ before the bounce. The above equation can be integrated to arrive at

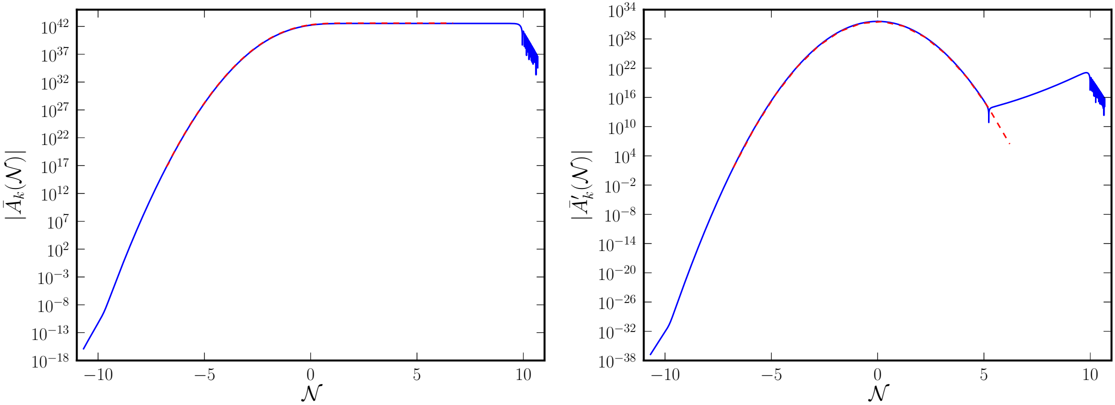
$$\bar{A}_k(\eta) \simeq \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*) \int\limits_{\eta_*}^{\eta} \mathrm{d}\eta\, \frac{J^2(\eta_*)}{J^2(\eta)} = \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*)\, a^{2\,n}(\eta_*) \int\limits_{\eta_*}^{\eta} \frac{\mathrm{d}\eta}{a^{2\,n}(\eta)},$$

where we have set the constant of integration to be $\bar{A}_k(\eta_*)$.

When $\gamma = 3$, we can evaluate the above integral to obtain that

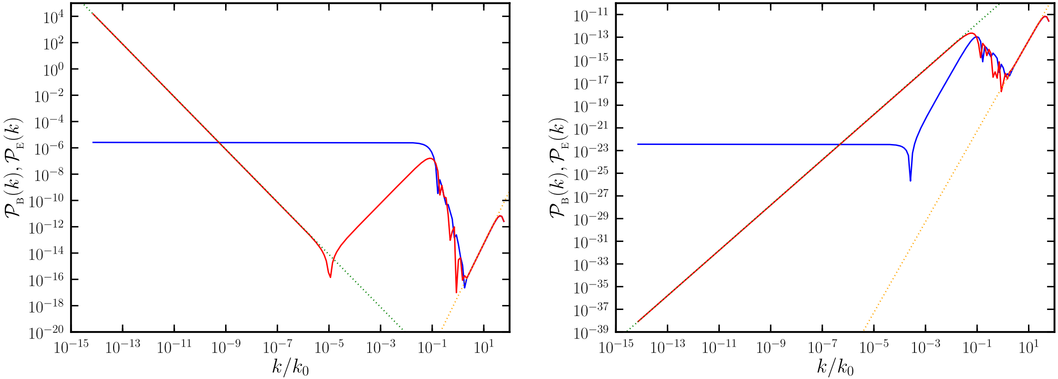
$$\begin{aligned} \bar{A}_k(\eta) \quad &\simeq \quad \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*)\, \frac{a^{2\,n}(\eta_*)}{a_0^{2\,n}}\, \frac{\eta_0}{8} \left\{\frac{\eta}{\eta_0}\, \frac{5 + 3\,(\eta/\eta_0)^2}{\left[1 + (\eta/\eta_0)^2\right]^2} + 3\tan^{-1}\left(\frac{\eta}{\eta_0}\right) \right. \\ &\left. - \frac{\eta_*}{\eta_0}\, \frac{5 + 3\,(\eta_*/\eta_0)^2}{\left[1 + (\eta_*/\eta_0)^2\right]^2} - 3\tan^{-1}\left(\frac{\eta_*}{\eta_0}\right)\right\}. \end{aligned}$$

Comparison of the numerical and analytical results



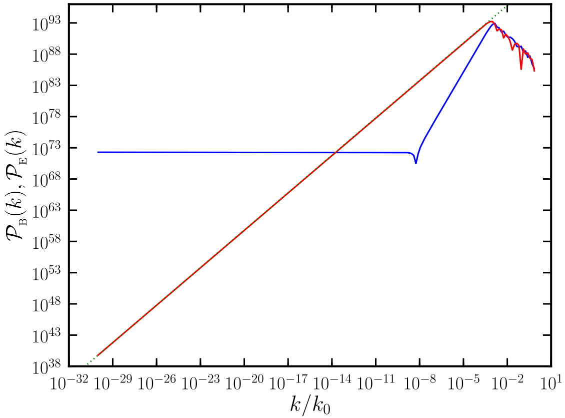
The behavior of the absolute values of $\bar{A}_k$ (on the left) and its derivative $\bar{A}'_k$ (on the right) has been plotted for the mode $k = 10^{-10}\, k_0$ with $k_0/M_{\rm Pl} = {\rm e}^{-25} = 1.389 \times 10^{-11}$ for the case wherein $n = 3/2$, $q = 1$, $a_0 = 10^{-10}$ and $J_0 = 10^4$. The dashed red curves represent the analytical approximation around the bounce that can be arrived at for modes such that $k \ll k_0$.

Power spectra of magnetic and electric fields



The power spectra of the magnetic (in blue) and the electric (in red) fields for the cases wherein $(q, n) = (1, 3/2)$ (corresponding to $\gamma = 3$, on the left) and $(q, n) = (1, -1)$ (corresponding to $\gamma = -2$, on the right). We have worked with the same values of η_0, a_0 and J_0 as in the previous figure. The power spectra of the electric field are along expected lines, behaving as $k^{4-2\gamma} = k^{-2}$ when $\gamma = 3$ and $k^{6+2\gamma} = k^{2}$ when $\gamma = -2$ (indicated by the dotted green lines).

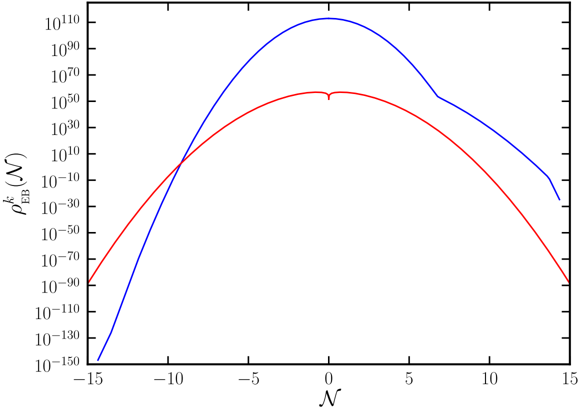
Spectrum of observable strengths



The power spectra with $q = 1$ and $n = -1$, corresponding to $\gamma = -2$ has been plotted for a wide range of wavenumbers. We have set $k_0/M_{\mathrm{Pl}} = 1$, $a_0 = 4 \times 10^{-29}$ and $J_0 = 10^4$, which lead to magnetic fields in the early universe that correspond to observable strengths today[8].

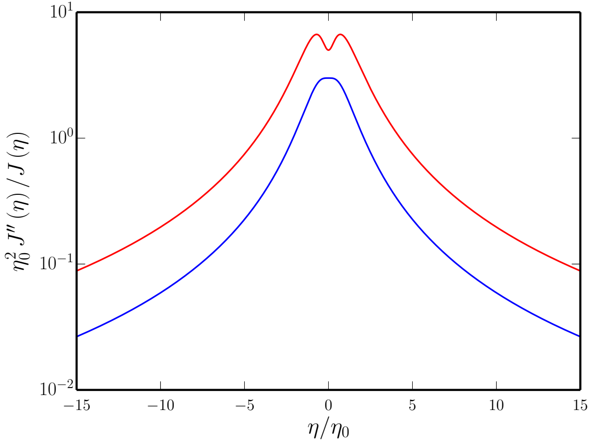
[8] L. Sriramkumar, K. Atmjeet and R. K. Jain, JCAP **1509**, 010 (2015).

The issue of backreaction



The behavior of the energy density in the electric and magnetic fields for the mode $k = 10^{-20}\,k_0$ has been plotted (in blue) along with the energy density of the background (in red). We have worked with the same values of the various parameters as in the last figure.

The behavior of J''/J



The behavior of $\eta_0^2\, J''/J$, which depends only on η/η_0, has been plotted for $\gamma = 3$ (in blue) and $\gamma = 5$ (in red). The figure has been plotted over a very narrow range of η/η_0 in order to illustrate the presence of a single maximum for $\gamma = 3$ and two maxima and one minimum for $\gamma = 5$.

Analytical solutions near the bounce for arbitrary γ

Recall that, near the bounce, when $n > 0$, for scales of cosmological interest such that $k \ll k_0$, we had obtained that

$$\bar{A}_k(\eta) \simeq \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*) \int_{\eta_*}^{\eta} \mathrm{d}\tilde{\eta}\, \frac{J^2(\eta_*)}{J^2(\tilde{\eta})} = \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*)\, a^{2\,n}(\eta_*) \int_{\eta_*}^{\eta} \frac{\mathrm{d}\tilde{\eta}}{a^{2\,n}(\tilde{\eta})},$$

where η_* is a time when $k^2 \ll J''/J$ before the bounce and we have set the constant of integration to be $\bar{A}_k(\eta_*)$.

[9] D. Chowdhury, L. Sriramkumar and R. K. Jain, Phys. Rev. D **94**, 083512 (2016) [arXiv:1604.02143 [gr-qc]].

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where η_* is a time when $k^2 \ll J''/J$ before the bounce and we have set the constant of integration to be $\bar{A}_k(\eta_*)$.

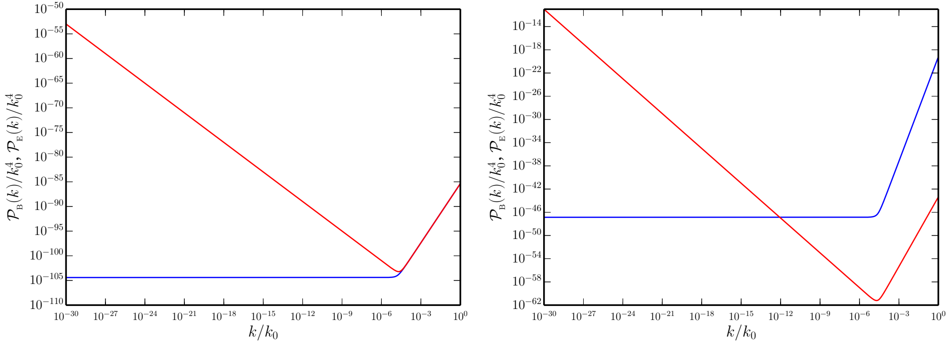
The above integral can, in fact, be carried out for an arbitrary γ to arrive at

$$\begin{aligned} \bar{A}_k(\eta) \quad &\simeq \quad \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*)\, \frac{a^{2\,n}(\eta_*)}{a_0^{2\,n}} \\ &\times \left[\eta\; {}_2F_1\left(\frac{1}{2}, \gamma; \frac{3}{2}; -\frac{\eta^2}{\eta_0^2}\right) - \eta_*\; {}_2F_1\left(\frac{1}{2}, \gamma; \frac{3}{2}; -\frac{\eta_*^2}{\eta_0^2}\right)\right], \end{aligned}$$

where ${}_2F_1(a, b, c, z)$ denotes the hypergeometric function[9].

[9] D. Chowdhury, L. Sriramkumar and R. K. Jain, Phys. Rev. D **94**, 083512 (2016) [arXiv:1604.02143 [gr-qc]].

Power spectra before and after the bounce



Left: The dimensionless power spectra of the magnetic (in blue) and electric (in red) fields, evaluated before the bounce at $\eta = -\alpha\,\eta_0$ have been plotted as a function of k/k_0 for $\gamma = 3$, $q = 1$, $a_0 = 8.73 \times 10^{10}$ and $\alpha = 10^5$.

[10] D. Chowdhury, L. Sriramkumar and R. K. Jain, Phys. Rev. D **94**, 083512 (2016) [arXiv:1604.02143 [gr-qc]].

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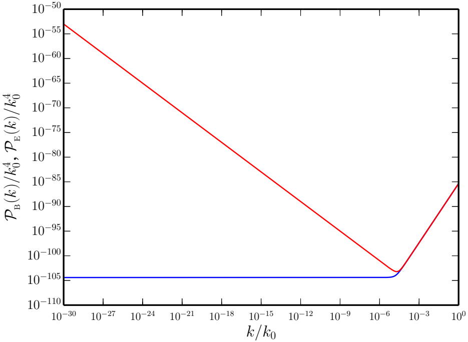

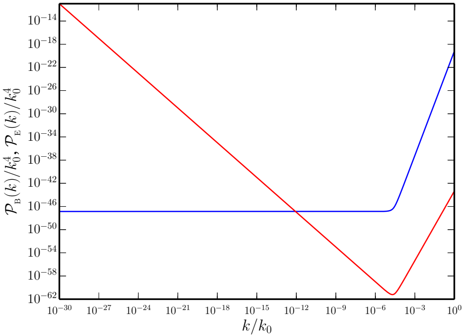

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Right: The corresponding power spectra evaluated after the bounce at $\eta = \beta\,\eta_0$, with $\beta = 10^2$. We should mention that the values of the parameters we have worked with lead to magnetic fields of observed strengths today corresponding to a few femto gauss[10].

[10] D. Chowdhury, L. Sriramkumar and R. K. Jain, Phys. Rev. D **94**, 083512 (2016) [arXiv:1604.02143 [gr-qc]].

Duality transformations

It is known that the solutions to the equations of motion governing the scalar and tensor perturbations are invariant under a certain transformation referred to as the duality transformation[11]. For instance, it can be shown that the dual solution to the de Sitter case corresponds to the matter bounce. Both these cases lead to scale invariant spectra.

[11] D. Wands, Phys. Rev. D **60**, 023507 (1999).

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In the case of electromagnetic fields of our interest here, given a coupling function J, its dual function, say, $\tilde{J}$, which leads to the same $\tilde{J}''/\tilde{J}$ is found to be

$$J(\eta) \to \tilde{J}(\eta) = C\, J(\eta) \int_{\eta_*}^{\eta} \frac{\mathrm{d}\bar{\eta}}{J^2(\bar{\eta})},$$

where C and η_* are constants. These constants can be suitably chosen to arrive at a physically reasonable form for $\tilde{J}$

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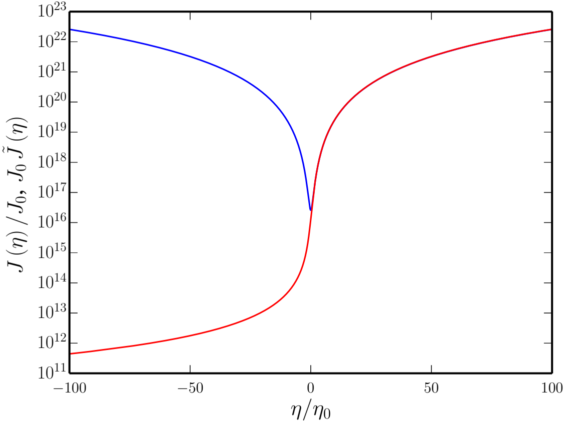
It can be shown that the cases corresponding to $\gamma = 3$ and $\gamma = -2$ in the bouncing models which had led to scale invariant spectra are dual to each other.

[11] D. Wands, Phys. Rev. D **60**, 023507 (1999).

A symmetric coupling function and its asymmetric dual



The coupling function J (in blue) and its dual $\tilde{J}$ (in red) have been plotted as a function of η/η_0 for $\gamma = 3$ and $\eta_* \to -\infty$. Also, we have chosen the constant C to be $C/k_0 = 5.7 \times 10^{32}$ so that the dual function $\tilde{J}$ matches the original coupling function J after the bounce[12].

[12] D. Chowdhury, L. Sriramkumar and R. K. Jain, arXiv:1604.02143 [gr-qc].

The non-minimal action

We shall consider an action of the form[13]

$$S_{\rm em}[A^\mu, \phi] = -\frac{1}{16\,\pi} \int {\rm d}^4x\,\sqrt{-g}\,\left[J^2(\phi)\,F_{\mu\nu}\,F^{\mu\nu} - \frac{\delta}{2}\,I^2(\phi)\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}\right],$$

where the dual of the electromagnetic tensor $\tilde{F}^{\mu\nu}$ is defined as

$$\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}\,F_{\alpha\beta},$$

with $\epsilon^{\mu\nu\alpha\beta} = (1/\sqrt{-g})\,\mathcal{A}^{\mu\nu\alpha\beta}$, $\mathcal{A}^{\mu\nu\alpha\beta}$ is the totally antisymmetric Levi-Civita tensor and $\mathcal{A}^{0123} = 1$.

[13] See, for instance, C. Caprini and L. Sorbo, JCAP **1410**, 056 (2014).

Quantization and the equation of motion

For each comoving wave vector $\boldsymbol{k}$, we can define the right-handed orthonormal basis $(\varepsilon_1^{\boldsymbol{k}}, \varepsilon_2^{\boldsymbol{k}}, \hat{\boldsymbol{k}})$, where

$$|\varepsilon_i^{\boldsymbol{k}}|^2 = 1, \quad \varepsilon_1^{\boldsymbol{k}} \times \varepsilon_2^{\boldsymbol{k}} = \hat{\boldsymbol{k}}, \quad \varepsilon_1^{\boldsymbol{k}} \cdot \varepsilon_2^{\boldsymbol{k}} = \hat{\boldsymbol{k}} \cdot \varepsilon_1^{\boldsymbol{k}} = \hat{\boldsymbol{k}} \cdot \varepsilon_2^{\boldsymbol{k}} = 0.$$

We can combine the two transverse directions to form the helicity basis as follows:

$$\varepsilon_\pm^{\boldsymbol{k}} = \frac{1}{\sqrt{2}} \left(\varepsilon_1^{\boldsymbol{k}} \pm i\, \varepsilon_2^{\boldsymbol{k}}\right).$$

On quantization, the vector potential A_i can be Fourier decomposed as

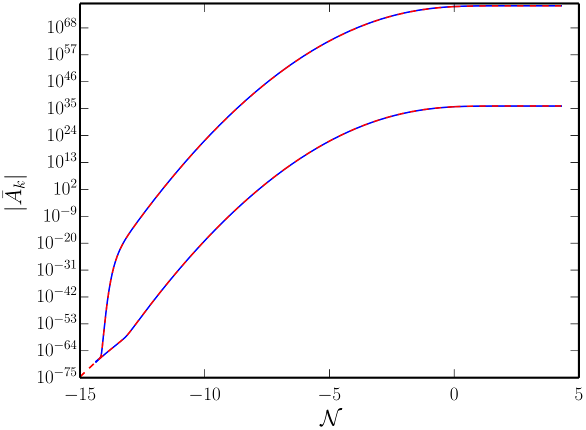
$$A_i(\eta, \boldsymbol{x}) = \sqrt{4\,\pi} \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\,\pi)^{3/2}} \sum_{\sigma=\pm} \left[\varepsilon_{\sigma i}^{\boldsymbol{k}}\, \hat{b}_{\boldsymbol{k}}^{\sigma}\, \bar{A}_k^{\sigma}(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \varepsilon_{\sigma i}^{\boldsymbol{k}\,*}\, \hat{b}_{\boldsymbol{k}}^{\sigma\dagger}\, \bar{A}_k^{\sigma}(\eta)\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right],$$

where the Fourier modes $\bar{A}_k^{\sigma}$ satisfy the differential equation

$$\bar{A}_k^{\sigma\,\prime\prime} + 2\,\frac{J'}{J}\,\bar{A}_k^{\sigma\,\prime} + \left(k^2 + \frac{\sigma\,\gamma\,k}{J^2}\,\frac{\mathrm{d}I^{\,2}}{\mathrm{d}\,\eta}\right)\bar{A}_k^{\sigma} = 0,$$

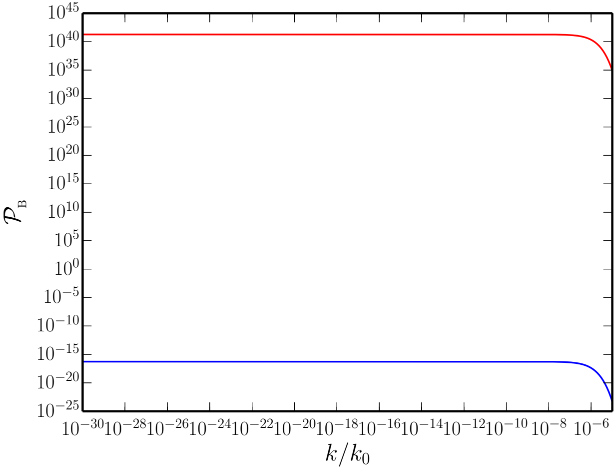
with $\sigma = \pm 1$ denoting the positive and negative helicity modes respectively.

Evolution of the modes



Comparison of the numerical results (in blue) with the analytical results (in red) for the amplitude of the positive and negative helicity modes corresponding to the wavenumber $k/k_0 = 10^{-20}$ for a case wherein $J = I$. We have set $\delta = 10$. Note that, as in inflationary scenario, the positive helicity mode is amplified relative to the negative helicity mode.

Power spectrum before and after the bounce



The power spectra of the magnetic field, evaluated before the bounce (in blue), and after the bounce (in red), have been plotted as a function of k/k_0 for $q = 1$ and $n = 3/2$. The power spectra are evidently scale invariant for the modes of our interest.

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Summary

- Scale invariant magnetic fields of observable strengths can be generated in a class of bouncing models.
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- Preliminary investigating suggest that scale invariant helical magnetic fields can also be generated in bouncing scenarios.

Summary

- Scale invariant magnetic fields of observable strengths can be generated in a class of bouncing models.
- As in the case of the scalar and tensor perturbations, the power spectrum of the magnetic field remains invariant under a two parameter family of transformations (called the duality transformations) of the non-minimal coupling function.
- Preliminary investigating suggest that scale invariant helical magnetic fields can also be generated in bouncing scenarios.
- However, as in the inflationary context, the generation of magnetic fields are also plagued by the problem of backreaction.

Collaborators

Debika Chowdhury

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Thank you for your attention