Primordial black holes and scalar-induced, secondary gravitational waves

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Early Universe from Home

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Plan of the talk

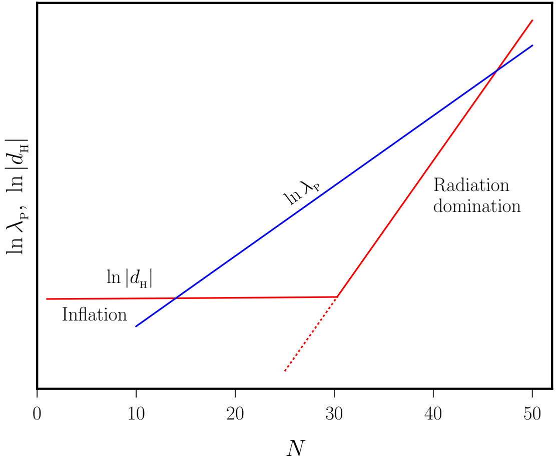
1. Inflation, reheating, and constraints from the CMB
2. Generation of PBHs and secondary GWs by enhanced scalar perturbations
3. NANOGrav 15-year data and the stochastic GW background
4. Summary

This talk is based on. . .

- M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, *Generating PBHs and small-scale GWs in two-field models of inflation*, JCAP **08**, 001 (2020) [arXiv:2005.02895 [astro-ph.CO]].
- H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, *Primordial black holes and secondary gravitational waves from ultra slow roll and punctuated inflation*, Phys. Rev. D **103**, 083510 (2021) [arXiv:2008.12202 [astro-ph.CO]].
- H. V. Ragavendra and L. Sriramkumar, *Observational imprints of enhanced scalar power on small scales in ultra slow roll inflation and associated non-Gaussianities*, Galaxies **11**, 34 (2023) [arXiv:2301.08887 [astro-ph.CO]].
- S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, *Constraining the history of reheating with the NANOGrav 15-year data*, JCAP **01**, 118 (2025) [arXiv:2403.16963 [astro-ph.CO]].

Plan of the talk

Bringing the modes inside the Hubble radius

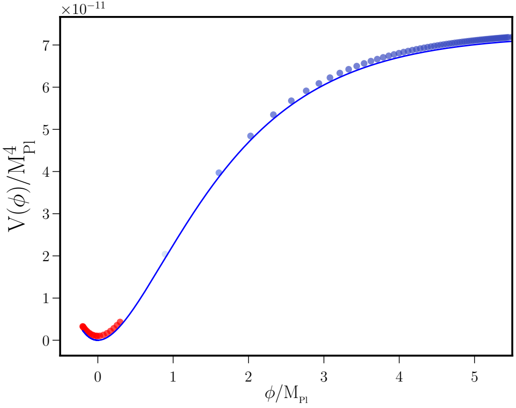


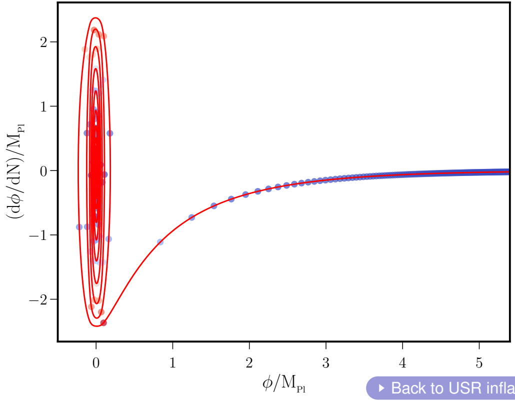
Evolution of comoving lengths

The physical wavelength $\lambda_{\rm P} \propto a$ (in blue) and the Hubble radius $d_{\rm H} = H^{-1}$ (in red) in the inflationary scenario[1]. The scale factor is expressed in terms of e-folds N as $a(N) \propto {\rm e}^{N}$.

[1] See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.

The inflationary attractor





Back to USR inflation

Evolution of the scalar field in the popular Starobinsky model, which leads to slow roll inflation, is indicated (as circles, in blue and red) at regular intervals of time (on the left). Illustration of the behavior of the scalar field in phase space (on the right)[2].

[2]Figure from H. V. Ragavendra, *Observational imprints of non-trivial inflationary dynamics over large and small, scales*, Ph.D. Thesis, Indian Institute of Technology Madras, Chennai, India (2022).

Spectral indices and tensor-to-scalar ratio

While comparing with the observations, for convenience, one often uses the following power law template for the primordial scalar and the tensor spectra:

$$\mathcal{P}_{\rm S}(k) = A_{\rm S}\,\left(\frac{k}{k_*}\right)^{n_{\rm S}-1}, \qquad \mathcal{P}_{\rm T}(k) = A_{\rm T}\,\left(\frac{k}{k_*}\right)^{n_{\rm T}},$$

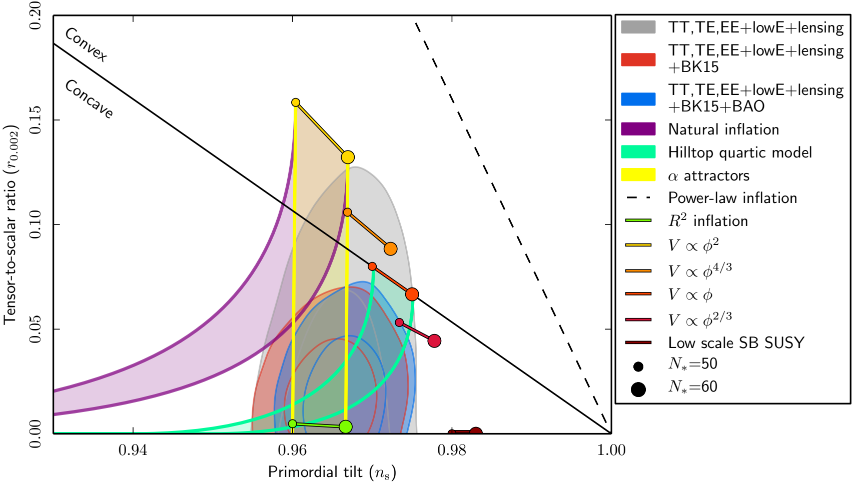
with the spectral indices $n_{\rm S}$ and $n_{\rm T}$ assumed to be constant.

The tensor-to-scalar ratio r is defined as

$$r(k) = \frac{\mathcal{P}_{\rm T}(k)}{\mathcal{P}_{\rm S}(k)}$$

and it is usual to further set $r = -8\,n_{\rm T}$, viz. the so-called consistency relation, which is valid during slow roll inflation.

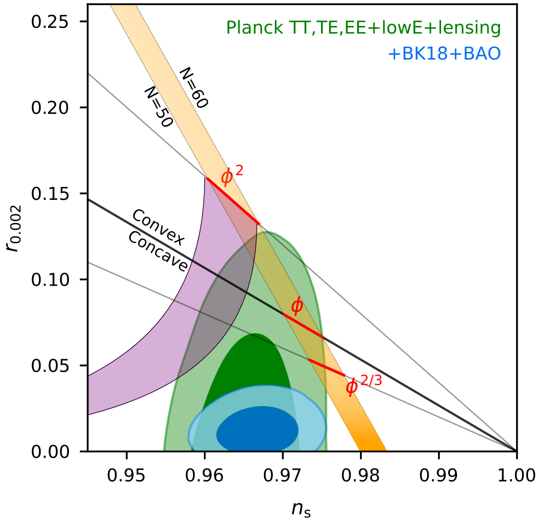
Performance of inflationary models in the $n_{\rm s}$-r plane



Joint constraints on $n_{\rm s}$ and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of some of the popular inflationary models[3].

[3] Planck Collaboration (Y. Akrami *et al.*), Astron. Astrophys. **641**, A10 (2020).

Latest constraints on the tensor-to-scalar ratio r

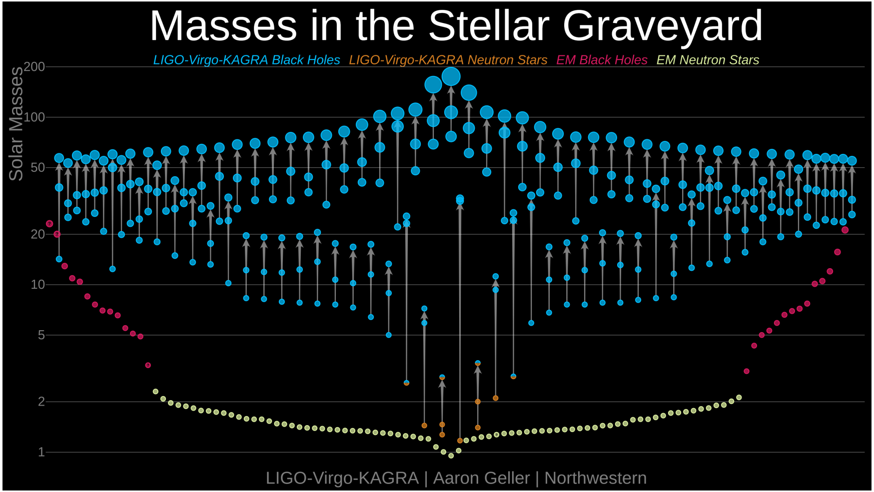


Latest constraints on the tensor-to-scalar ratio r from the BICEP/Keck telescopes[4].

[4] BICEP/Keck Collaboration (P. A. R. Ade *et al.*), arXiv:2203.16556 [astro-ph.CO].

Plan of the talk

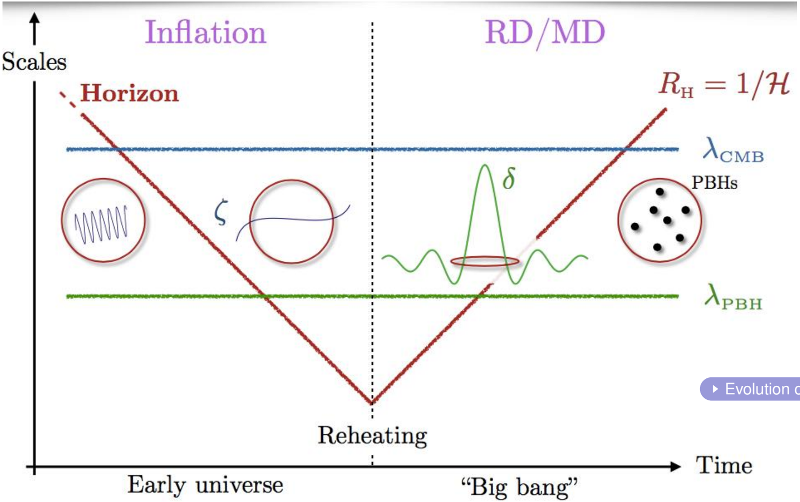
Coalescence of compact binaries observed by LIGO



The third GW Transient Catalog of mergers involving black holes and neutron stars observed by the LIGO-Virgo-KAGRA collaboration[5].

[5] Image from https://www.ligo.caltech.edu/LA/image/ligo20211107a.

Formation of primordial black holes (PBHs)

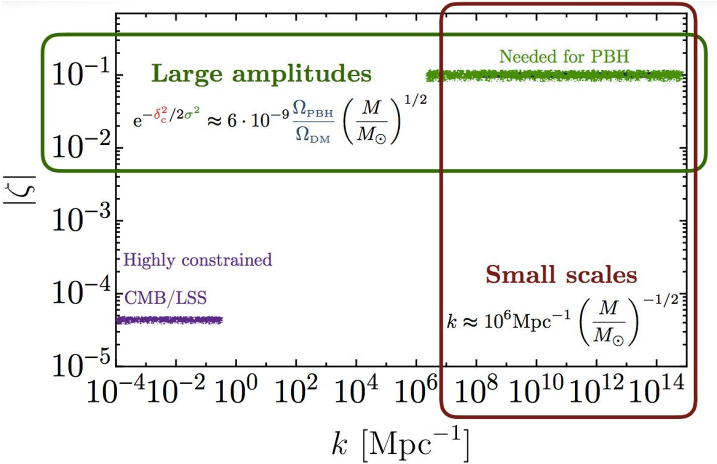


▸ Evolution of physical wavelengths

BHs can form in the primordial universe when perturbations with significant amplitudes on small scales reenter the Hubble radius during the radiation dominated epoch[6].

[6]Figure from G. Franciolini, arXiv:2110.06815 [astro-ph.CO].

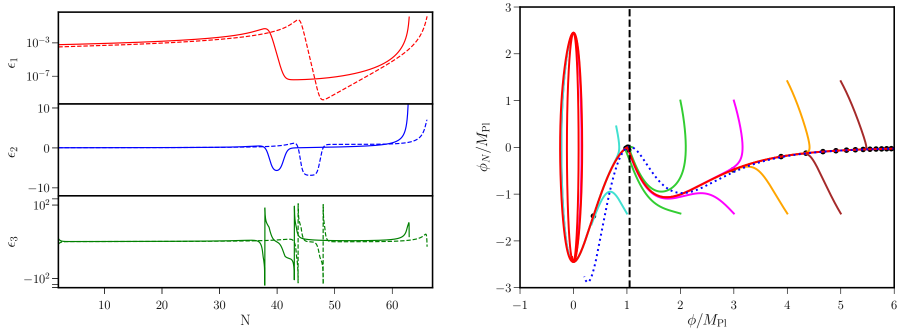
Amplitude required to form significant number of PBHs



In order to form significant number of black holes, the amplitude of the perturbations on small scales has to be large enough such that the dimensionless amplitude of the scalar perturbation is close to unity[7].

[7] Figure credit G. Franciolini.

Single-field models admitting ultra slow roll inflation



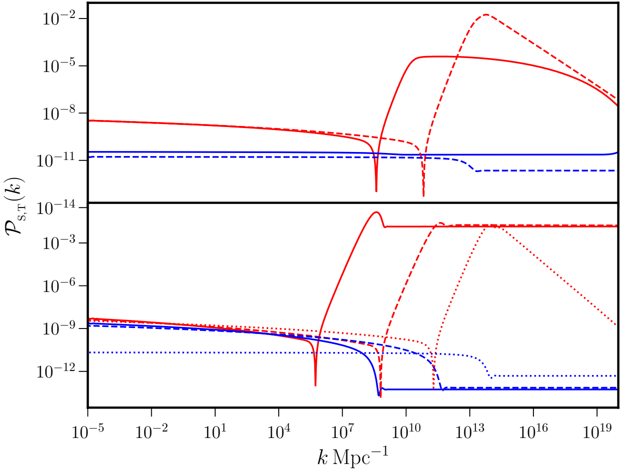
Potentials leading to ultra slow roll inflation (with $x = \phi/v$, v being a constant)[8]: ▸ Inflationary attractor

$$\mathrm{M1}: V(\phi) = V_0\, \frac{6\,x^2 - 4\,\alpha\,x^3 + 3\,x^4}{(1+\beta\,x^2)^2},$$

$$\mathrm{M4}: V(\phi) = V_0\, \left\{\tanh\left(\frac{\phi}{\sqrt{6}\,M_{\rm Pl}}\right) + A\,\sin\left[\frac{\tanh\left[\phi/\left(\sqrt{6}\,M_{\rm Pl}\right)\right]}{f_\phi}\right]\right\}^2.$$

[8] C. Germani and T. Prokopec, Phys. Dark Univ. **18**, 6 (2017);
I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).

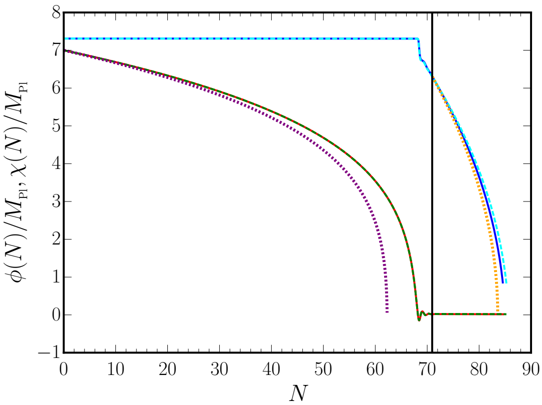
Power spectra in different models leading to ultra slow roll inflation

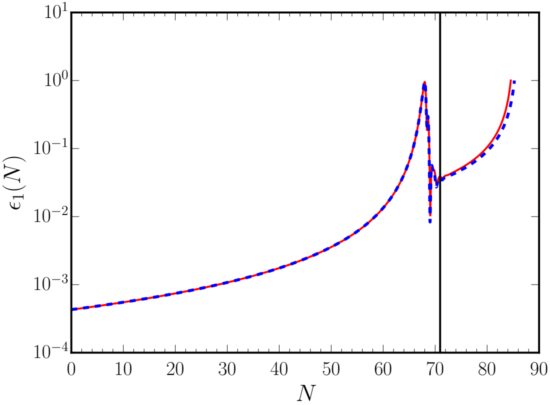


Scalar (in red) and the tensor (in blue) power spectra arising in various single field models that permit a period of ultra slow roll inflation[9].

[9] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021); Also see H. V. Ragavendra and L. Sriramkumar, Galaxies **11**, 34 (2023).

Behavior of the background in a two-field model

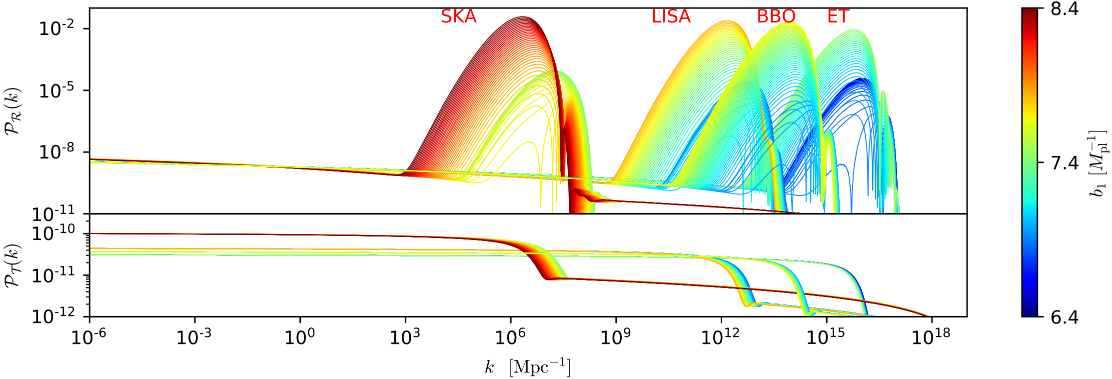




Behavior of the two scalar fields ϕ and χ (in blue and red, on the left) and the first slow roll parameter ϵ_1 (on the right) in the two field model of our interest[10]. Note that there arises a turn in the field space around $N = 70$, when the first slow roll parameter begins to decrease before increasing again, leading to the termination of inflation.

[10] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

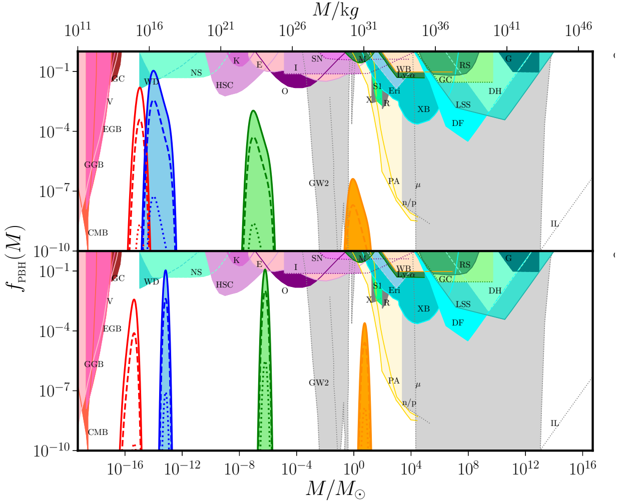
Enhanced power on small scales in the two-field model



The scalar (on top) and the tensor (at the bottom) power spectra evaluated at the end of inflation have been plotted for a few different sets of initial conditions for the fields and a range of values of the parameter b_1[11].

[11] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

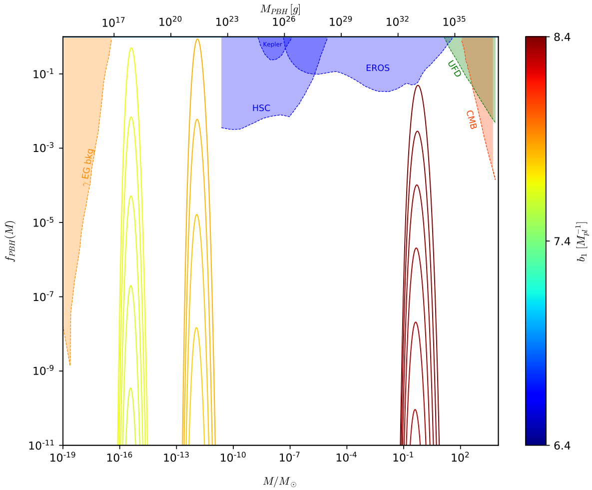
$f_{\rm PBH}(M)$ in ultra slow roll and punctuated inflation



The fraction of PBHs contributing to the cold dark matter density today $f_{\rm PBH}(M)$ has been plotted for different models, viz. USR2 (on top, in red) and PI3 (at the bottom, in red)[12].

[12] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

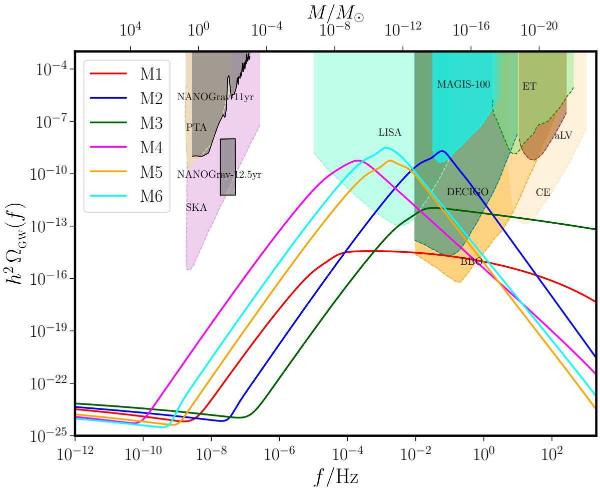
$f_{\rm PBH}(M)$ in the two-field model



The fraction of PBHs contributing to the dark matter density today $f_{\rm PBH}(M)$ in the two-field model of our interest[13].

[13] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

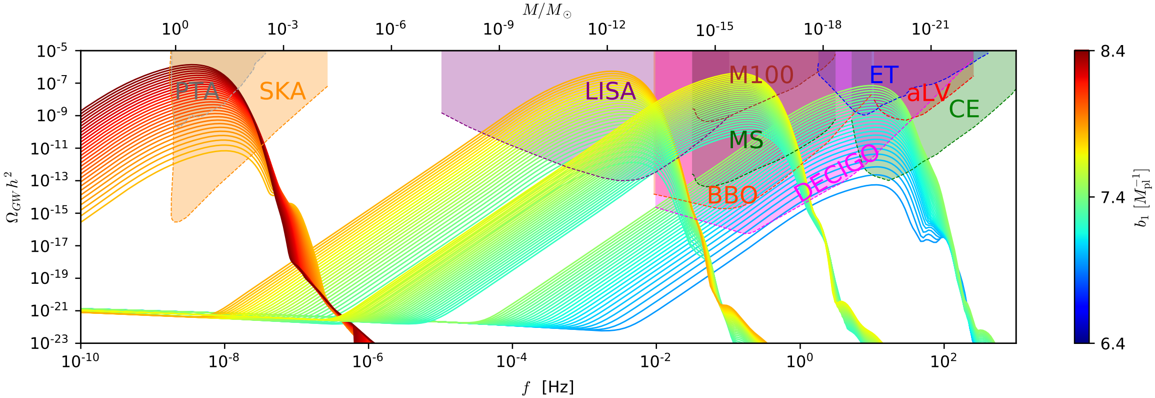
$\Omega_{\rm GW}(f)$ in ultra slow roll inflation



The dimensionless spectral density of scalar-induced, secondary GWs $\Omega_{\rm GW}(f)$ in different single-field models of inflation that permit a period of ultra slow roll[14].

[14] H. V. Ragavendra and L. Sriramkumar, Galaxies **11**, 34 (2023).

$\Omega_{\rm GW}(f)$ in the two-field model

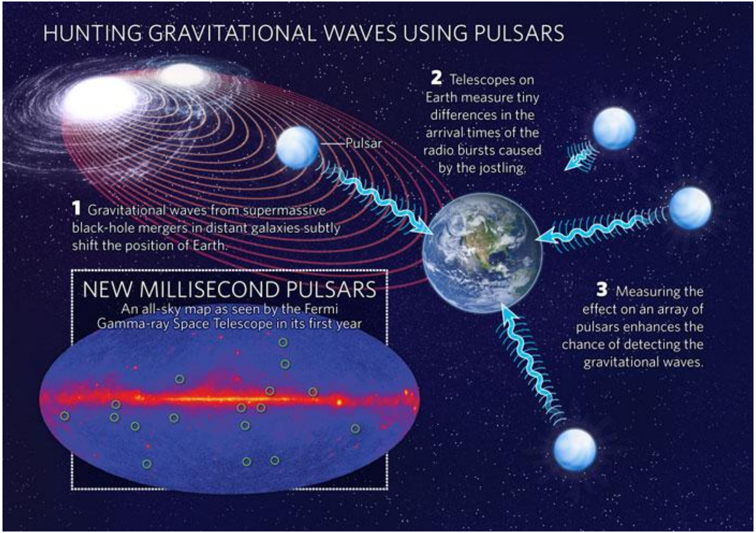


The dimensionless spectral density of GWs $\Omega_{\rm GW}(f)$ arising in the two-field model has been plotted for a set of initial conditions for the background fields as well as a range of values of the parameter b_1[15].

[15] M. Braglia, n D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

Plan of the talk

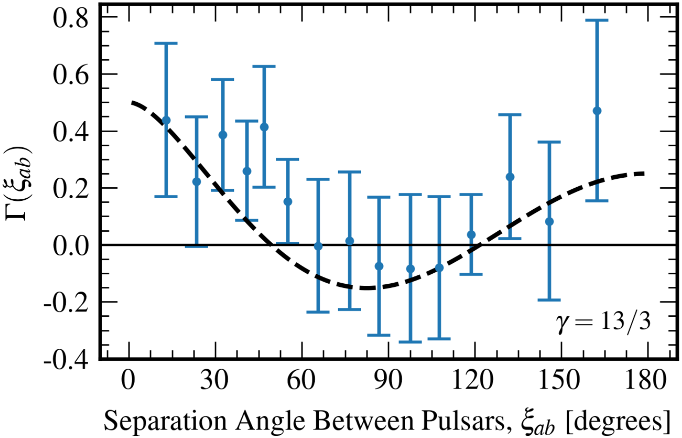
Pulsar timing arrays (PTAs)



The PTAs monitor an array of millisecond pulsars[16].

[16] See https://ipta.github.io/mock_data_challenge/.

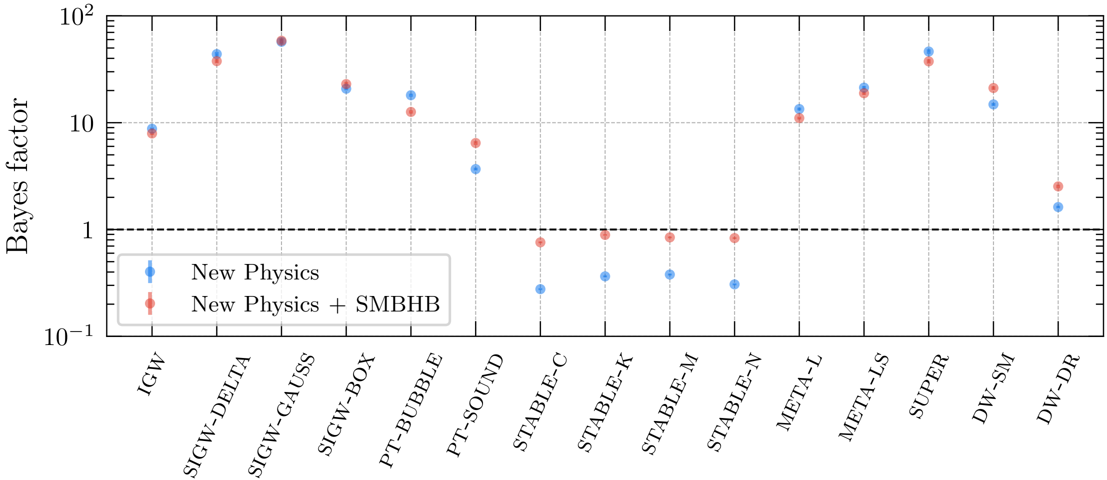
Hellings-Downs curve



The inter-pulsar correlations measured from 2,211 distinct pairings in the 67-pulsar array of the NANOGrav 15-year data. The dashed black line shows the Hellings-Downs correlation pattern[17].

[17] NANOGrav Collaboration (G. Agazie *et al.*), Astrophys. J. Lett. **951**, 1 (2023).

Stochastic GW background observed by the PTAs



The Bayesian evidence for a variety of astrophysical and cosmological sources for the stochastic GW background suggested by the observations of the PTAs[18].

[18] NANOGrav Collaboration (G. Agazie *et al.*), Astrophys. J. Lett. **951**, 1 (2023).

Form of the inflationary scalar power spectrum

We assume that the inflationary scalar power spectrum is given by[19]

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\mathrm{S}} \left(\frac{k}{k_*}\right)^{n_{\mathrm{S}}-1} + A_0 \begin{cases} \left(\dfrac{k}{k_{\mathrm{peak}}}\right)^4 & k \leq k_{\mathrm{peak}} \\ \left(\dfrac{k}{k_{\mathrm{peak}}}\right)^{n_0} & k \geq k_{\mathrm{peak}} \end{cases},$$

where A_{S} and n_{S} are the amplitude and spectral index of the power spectrum at the CMB pivot scale of $k_* = 0.05\,\mathrm{Mpc}^{-1}$.

We set the reheating temperature to the rather low value of $T_{\mathrm{re}} = 50\,\mathrm{MeV}$.

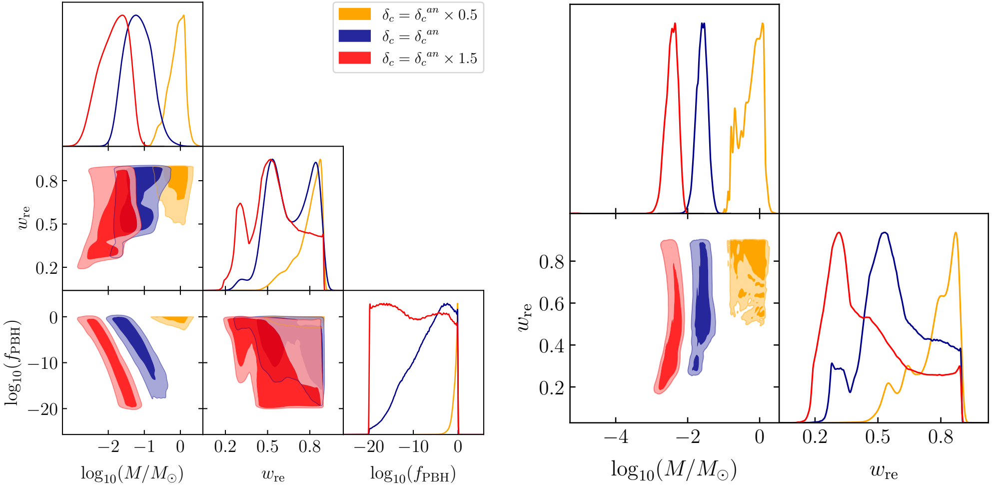
We shall assume that the threshold value of the density contrast for the formation of PBHs is given by[20]

$$\delta_{\mathrm{c}}^{\mathrm{an}} = \frac{3\,(1+w_{\mathrm{re}})}{5+3\,w_{\mathrm{re}}}\,\sin^2\left(\frac{\pi\,\sqrt{w_{\mathrm{re}}}}{1+3\,w_{\mathrm{re}}}\right).$$

[19] For other forms of spectra, see G. Domènech, S. Pi, A. Wang and J. Wang, arXiv:2402.18965 [astro-ph.CO].

[20] In this context, see T. Harada, C.-M. Yoo, and K. Kohri, Phys. Rev. D **88**, 084051 (2013).

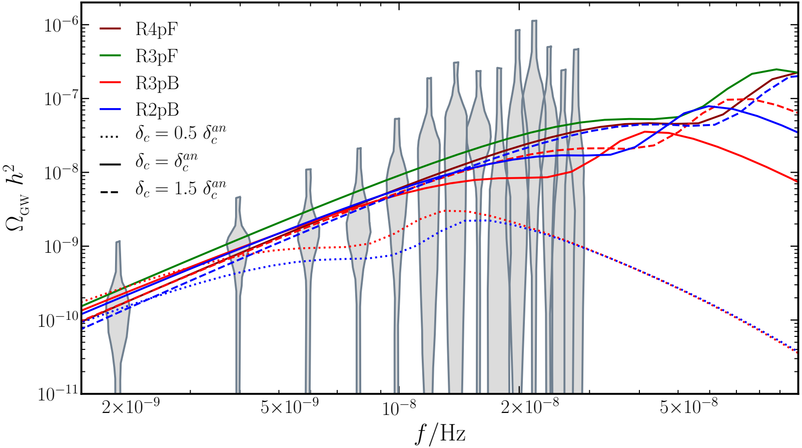
Constraints on the epoch of reheating



Constraints on the parameters in the models `R3pB` (on the left) and `R2pB` (on the right), arrived at upon comparison with the NANOGrav 15-year data[21].

[21] S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, JCAP **01**, 118 (2025).

Generation of secondary GWs during the epoch of reheating

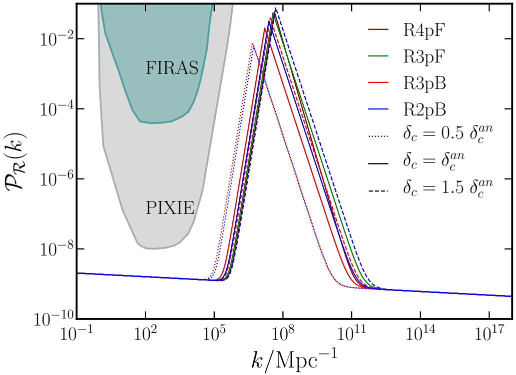


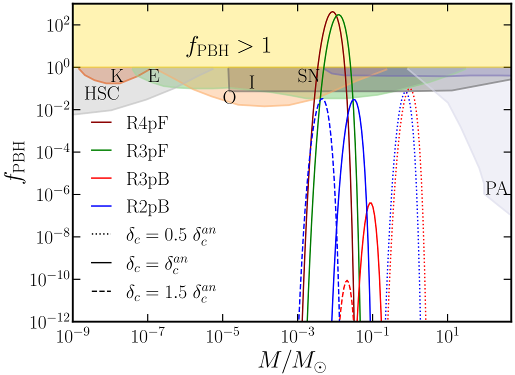
Epoch of reheating

The dimensionless spectral energy density of the secondary GWs today $\Omega_{\rm GW}(f)$ is plotted for a given reheating temperature and the best-fit values of the parameters in the different models[22].

[22] S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, JCAP **01**, 118 (2025).

Power spectra and the extent of PBHs formed





Scalar power spectra (on the left) and the extent of PBHs formed (on the right). We have assumed a specific reheating temperature and have plotted the fraction of PBHs that constitute the dark matter density today, viz. $f_{\mathrm{PBH}}(M)$, for the best-fit values of the parameters in the different models[23].

[23] S. Maity, N. Bhaumik, Md. R. Haque, D. Maity and L. Sriramkumar, JCAP **01**, 118 (2025).

Bayesian evidence

Model X	Model Y	$\mathrm{BF}_{Y,X}$		
		$\delta_\mathrm{c} = 0.5\,\delta_\mathrm{c}^\mathrm{an}$	$\delta_\mathrm{c} = \delta_\mathrm{c}^\mathrm{an}$	$\delta_\mathrm{c} = 1.5\,\delta_\mathrm{c}^\mathrm{an}$
SMBHB	R2pB	$1.7 \pm .06$	260.04 ± 19.21	350.61 ± 27.36

The Bayesian factors $\mathrm{BF}_{Y,X}$ for the model R2pB that invokes primordial physics as the source of the stochastic GW background observed by the NANOGrav 15-year data, when compared to the astrophysical SMBHB model.

Bayesian factors $\mathrm{BF}_{Y,X}$ that far exceed unity indicate strong evidence for the model Y with respect to the model X.

Clearly, when $\delta_\mathrm{c} = \delta_\mathrm{c}^\mathrm{an}$ and $\delta_\mathrm{c} = 1.5\,\delta_\mathrm{c}^\mathrm{an}$, the NANOGrav 15-year data strongly favors the model R2pB when compared to the astrophysical SMBHB model.

Plan of the talk

Summary

- The amplitude of the scalar power spectrum can be enhanced significantly on small scales if there arise strong departures from slow roll inflation. Specifically, such an enhancement can be achieved with the aid of a brief phase of ultra slow roll inflation during which the first slow roll parameter decreases rapidly.
- In addition to producing significant levels of PBHs, the enhanced scalar power on small scales also induce secondary GWs of strengths that are, in principle, detectable by ongoing and forthcoming GW observatories.
- Under certain conditions, the scalar-induced secondary GWs generated in models of USR inflation can explain the stochastic GW background observed by the PTAs.

Thank you for your attention