

Viable near-matter bounces

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Recent Developments in Cosmology

Banaras Hindu University, Varanasi

April 6–8, 2018

Plan of the talk

- 1 Whither inflation?
- 2 Bouncing scenarios
- 3 The tensor power spectrum in a symmetric matter bounce
- 4 A new model for the completely symmetric matter bounce
- 5 The tensor-to-scalar ratio in a matter bounce scenario
- 6 Generating spectral tilt
- 7 The tensor bi-spectrum in a matter bounce
- 8 Summary



This talk is based on...

- ◆ D. Chowdhury, V. Sreenath and L. Sriramkumar, *The tensor bispectrum in a matter bounce*, JCAP **1511**, 002 (2015) [arXiv:1506.06475 [astro-ph.CO]].
- ◆ R. N. Raveendran, D. Chowdhury and L. Sriramkumar, *Viable tensor-to-scalar ratio in a symmetric matter bounce*, JCAP **1801**, 030 (2018) [arXiv:1703.10061 [gr-qc]].
- ◆ R. N. Raveendran and L. Sriramkumar, *Viable scalar spectral tilt and tensor-to-scalar ratio in near-matter bounces*, in preparation.

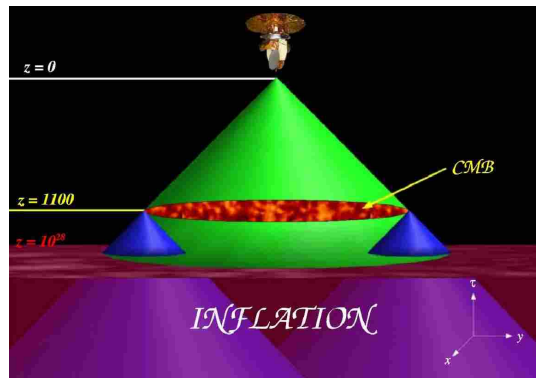
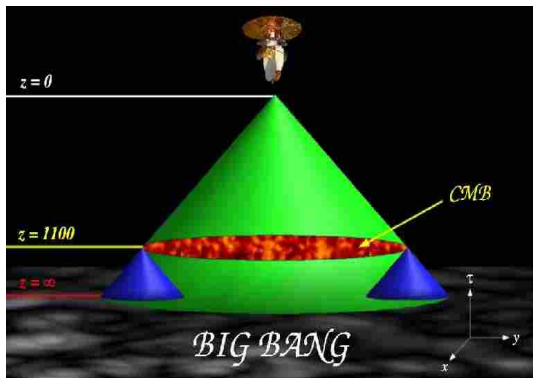


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The resolution of the horizon problem in inflation

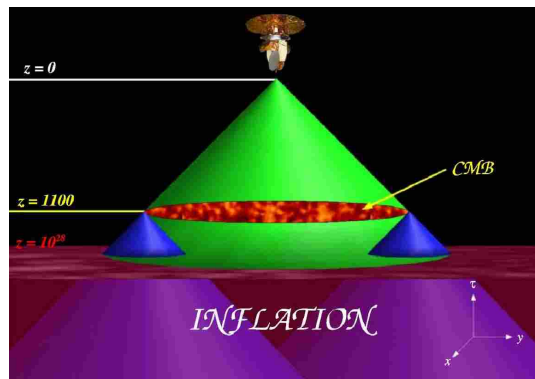
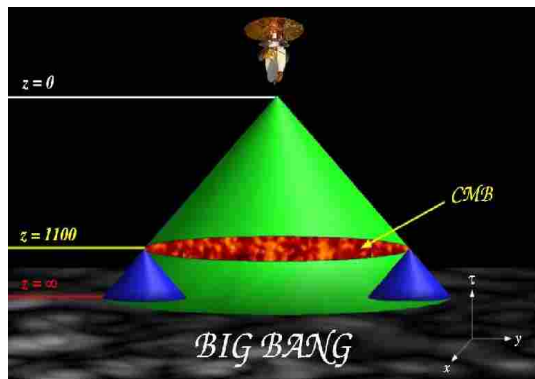


Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

¹Images from [W. Kinney, astro-ph/0301448](#).



The resolution of the horizon problem in inflation



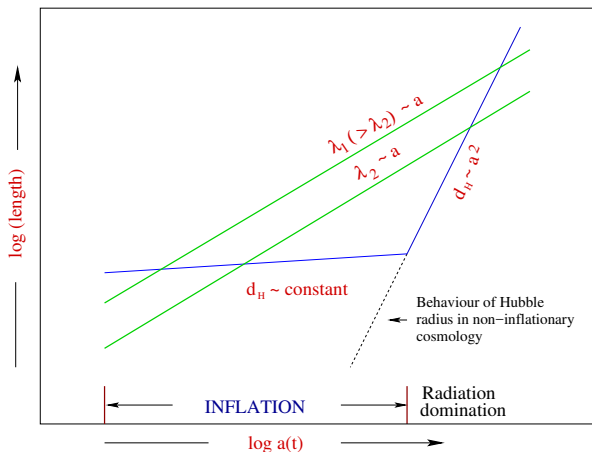
Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

Right: An illustration of how an early and sufficiently long epoch of inflation helps in resolving the horizon problem¹.

¹Images from [W. Kinney, astro-ph/0301448](#).



Bringing the modes inside the Hubble radius



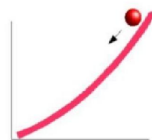
The behavior of the physical wavelength $\lambda_P \propto a$ (the green lines) and the Hubble radius H^{-1} (the blue line) during inflation and the radiation dominated epochs².

[Back to bounce](#)

²See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.



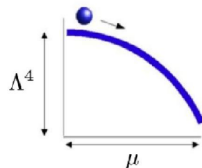
A variety of potentials to choose from



Large field

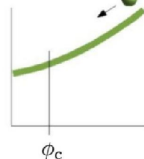
$$V(\phi) = \Lambda^4 (\phi/\mu)^p$$

$$V(\phi) = \Lambda^4 e^{\phi/\mu}$$



Small field

$$V(\phi) = \Lambda^4 [1 - (\phi/\mu)^p]$$



Hybrid

$$V(\phi) = \Lambda^4 [1 + (\phi/\mu)^p]$$

A variety of scalar field potentials have been considered to drive inflation³. Often, these potentials are classified as small field, large field and hybrid models.

³Image from [W. Kinney, astro-ph/0301448](https://arxiv.org/abs/astro-ph/0301448).



Proliferation of inflationary models

5-dimensional assisted inflation	extended open inflation	late-time mild inflation	pre-Big Bang inflation
anisotropic brane inflation	extended warm inflation	low-scale inflation	primary inflation
anomaly-induced inflation	extra dimensional inflation	low-scale supergravity inflation	primordial inflation
assisted inflation	F-term inflation	M-theory inflation	quasi-open inflation
assisted chaotic inflation	F-term hybrid inflation	mass inflation	quintessential inflation
boundary inflation	false vacuum inflation	massive chaotic inflation	R-invariant topological inflation
brane inflation	false vacuum chaotic inflation	moduli inflation	rapid asymmetric inflation
brane-assisted inflation	fast-roll inflation	multi-scalar inflation	running inflation
brane gas inflation	first order inflation	multiple inflation	scalar-tensor gravity inflation
brane-antibrane inflation	gauged inflation	multiple-field slow-roll inflation	scalar-tensor stochastic inflation
braneworld inflation	generalised inflation	multiple-stage inflation	Seiberg-Witten inflation
Brans-Dicke chaotic inflation	generalized assisted inflation	natural inflation	single-bubble open inflation
Brans-Dicke inflation	generalized slow-roll inflation	natural Chaotic inflation	spinodal inflation
bulky brane inflation	gravity driven inflation	natural double inflation	stable starobinsky-type inflation
chaotic hybrid inflation	Hagedorn inflation	natural supergravity inflation	steady-state eternal inflation
chaotic inflation	higher-curvature inflation	new inflation	steep inflation
chaotic new inflation	hybrid inflation	next-to-minimal supersymmetric hybrid inflation	stochastic inflation
D-brane inflation	hyperextended inflation	non-commutative inflation	string-forming open inflation
D-term inflation	induced gravity inflation	non-slow-roll inflation	successful D-term inflation
dilaton-driven inflation	induced gravity open inflation	nonminimal chaotic inflation	supergravity inflation
dilaton-driven brane inflation	intermediate inflation	old inflation	supernatural inflation
double inflation	inverted hybrid inflation	open hybrid inflation	superstring inflation
double D-term inflation	isocurvature inflation	open inflation	supersymmetric hybrid inflation
dual inflation	K inflation	oscillating inflation	supersymmetric inflation
dynamical inflation	kinetic inflation	polynomial chaotic inflation	supersymmetric topological inflator
dynamical SUSY inflation	lambda inflation	polynomial hybrid inflation	supersymmetric new inflation
eternal inflation	large field inflation	power-law inflation	synergistic warm inflation
extended inflation	late D-term inflation		TeV-scale hybrid inflation

A (partial?) list of ever-increasing number of inflationary models⁴. Actually, it may not even be possible to rule out some of these models!

⁴From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).



The character of the perturbations

In a Friedmann universe, the perturbations in the metric and the matter can be classified according to their behavior with respect to local rotation of the spatial coordinates on hypersurfaces of constant time as follows⁵:

- ◆ Scalar perturbations – Density and pressure perturbations
- ◆ Vector perturbations – Rotational velocity fields
- ◆ Tensor perturbations – Gravitational waves

⁵See, for instance, [L. Sriramkumar, Curr. Sci. 97, 868 \(2009\)](#).



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Inflation does not produce any vector perturbations, while the tensor perturbations can be generated even in the absence of sources.

It is the fluctuations in the inflaton field ϕ that act as the seeds for the scalar perturbations that are primarily responsible for the anisotropies in the CMB and, eventually, the present day inhomogeneities.

⁵See, for instance, [L. Sriramkumar, Curr. Sci. 97, 868 \(2009\)](#).



The quadratic action governing the perturbations

One can show that, at the quadratic order, the action governing the curvature perturbation \mathcal{R} and the tensor perturbation γ_{ij} are given by⁶

$$\mathcal{S}_2[\mathcal{R}] = \frac{1}{2} \int d\eta \int d^3\mathbf{x} z^2 \left[\mathcal{R}'^2 - (\partial\mathcal{R})^2 \right],$$

$$\mathcal{S}_2[\gamma_{ij}] = \frac{M_{\text{Pl}}^2}{8} \int d\eta \int d^3\mathbf{x} a^2 \left[\gamma'_{ij}{}^2 - (\partial\gamma_{ij})^2 \right].$$

▶ [Back to the cubic action](#)

⁶V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).



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These actions lead to the following equations of motion governing the scalar and tensor modes, say, f_k and h_k :

$$f_k'' + 2 \frac{z'}{z} f_k' + k^2 f_k = 0,$$

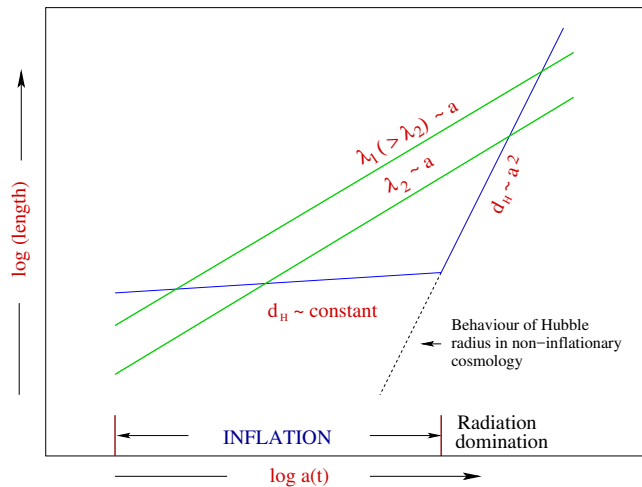
$$h_k'' + 2 \frac{a'}{a} h_k' + k^2 h_k = 0,$$

where $z = a M_{\text{Pl}} \sqrt{2\epsilon_1}$, with $\epsilon_1 = -d \ln H / dN$ being the first slow roll parameter.

⁶V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).



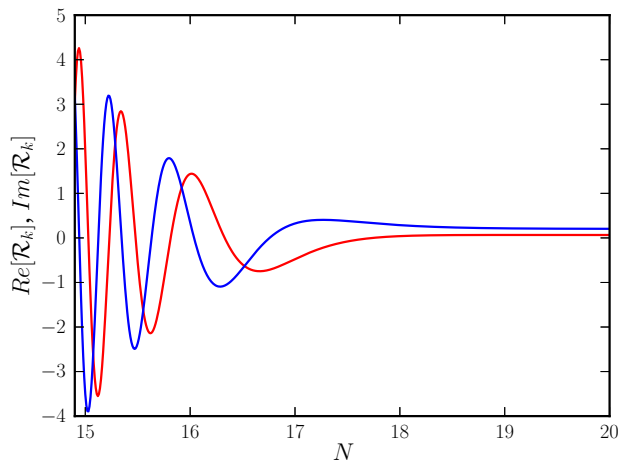
From inside the Hubble radius to super-Hubble scales



The initial conditions are imposed in the sub-Hubble regime when the modes are well inside the Hubble radius (*viz.* when $k/(aH) \gg 1$) and the power spectra are evaluated when they sufficiently outside (*i.e.* as $k/(aH) \ll 1$).



Typical evolution of the scalar modes



Typical evolution of the real and the imaginary parts of the scalar modes during slow roll inflation. The mode considered leaves the Hubble radius at about 18 e-folds⁷.

⁷Figure from V. Sreenath, *Computation and characteristics of inflationary three-point functions*, Ph.D. Thesis, Indian Institute of Technology Madras, Chennai, India (2015).



Spectral indices and the tensor-to-scalar ratio

While comparing with the observations, for convenience, one often uses the following power law, template scalar and the tensor spectra:

$$\mathcal{P}_S(k) = \mathcal{A}_S \left(\frac{k}{k_*} \right)^{n_S - 1}, \quad \mathcal{P}_T(k) = \mathcal{A}_T \left(\frac{k}{k_*} \right)^{n_T},$$

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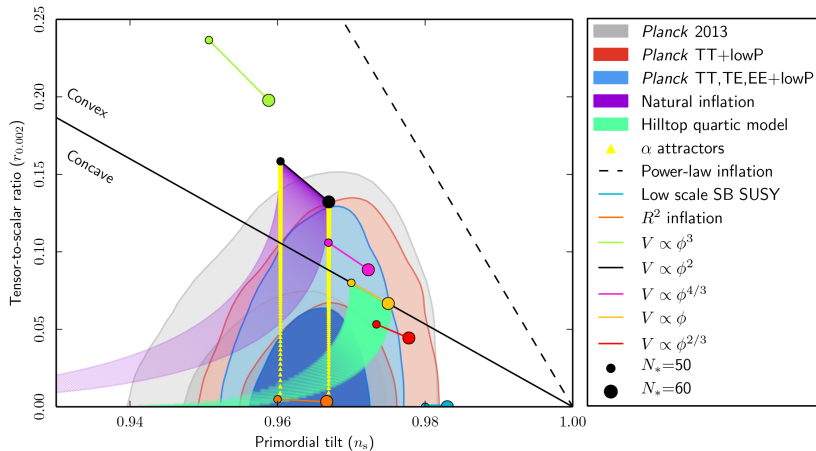
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The tensor-to-scalar ratio r is defined as

$$r(k) = \frac{\mathcal{P}_T(k)}{\mathcal{P}_S(k)}.$$



Performance of models in the n_s - r plane



Marginalized joint **68%** and **95%** CL regions for n_s and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of selected inflationary models⁸.

⁸Planck Collaboration (P. A. R. Ade *et al.*), *Astron. Astrophys.* **594**, A20 (2016).



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Bouncing scenarios as an alternative paradigm⁹

- ◆ Bouncing models correspond to situations wherein the universe initially goes through a period of contraction until the scale factor reaches a certain minimum value before transiting to the expanding phase.

⁹See, for instance, [M. Novello and S. P. Bergliaffa, Phys. Rep. **463**, 127 \(2008\);](#)
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Bouncing scenarios as an alternative paradigm⁹

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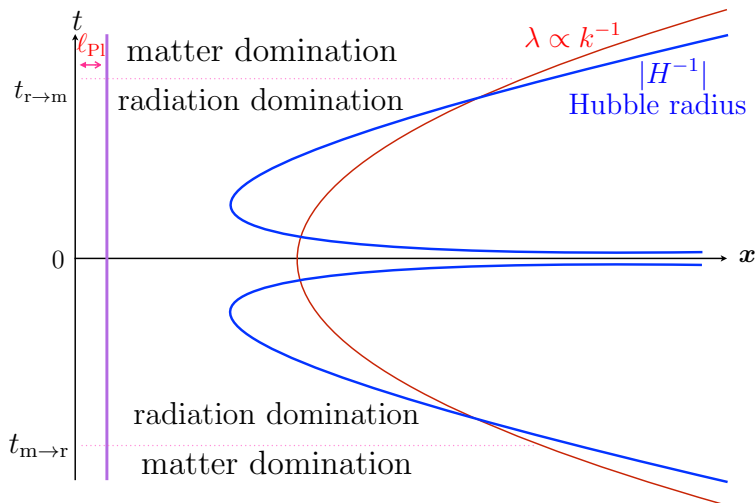
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- ◆ They offer an alternative to inflation to overcome the horizon problem, as they permit well motivated, Minkowski-like initial conditions to be imposed on the perturbations at early times during the contracting phase.
- ◆ However, matter fields *will* have to violate the null energy condition near the bounce in order to give rise to such a scale factor. Also, there exist (genuine) concerns whether such an assumption about the scale factor is valid in a domain where general relativity can be supposed to fail and quantum gravitational effects are expected to take over.

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Overcoming the horizon problem in bouncing models



Evolution of the physical wavelength and the Hubble radius in a bouncing scenario¹⁰

¹⁰Figure from, D. Battefeld and P. Peter, *Phys. Rept.* **571**, 1 (2015).



Violation of the null energy condition near the bounce

Recall that, according to the Friedmann equations

$$\dot{H} = -4\pi G (\rho + p).$$

In any bouncing scenario, the Hubble parameter is negative before the bounce, crosses zero at the bounce and is positive thereafter.



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It can be shown that, if the modes of cosmological interest have to be inside the Hubble radius at early times during the contracting phase, *the universe needs to undergo non-accelerated contraction*.



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In such cases, one finds that \dot{H} will be positive near the bounce, which implies that $(\rho + p)$ has to be negative in this domain. In other words, the null energy condition needs to be violated in order to achieve such bounces.



Classical bounces and sources

Consider for instance, bouncing models of the form

$$a(\eta) = a_0 \left(1 + \frac{\eta^2}{\eta_0^2} \right)^q = a_0 (1 + k_0^2 \eta^2)^q,$$

where a_0 is the value of the scale factor at the bounce (*i.e.* when $\eta = 0$), $\eta_0 = 1/k_0$ denotes the time scale of the duration of the bounce and $q > 0$. We shall assume that the scale k_0 associated with the bounce is of the order of the Planck scale M_{Pl} .



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The above scale factor can be achieved with the help of two fluids with constant equation of state parameters $w_1 = (1-q)/(3q)$ and $w_2 = (2-q)/(3q)$. The energy densities of these fluids behave as $\rho_1 = M_1/a^{(2q+1)/q}$ and $\rho_2 = M_2/a^{2(1+q)/q}$, where $M_1 = 12 k_0^2 M_{\text{Pl}}^2 a_0^{1/q}$ and $M_2 = -M_1 a_0^{1/q}$.



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Note that, when $q = 1$, during very early times wherein $\eta \ll -\eta_0$, the scale factor behaves as in a matter dominated universe (*i.e.* $a \propto \eta^2$). Therefore, the $q = 1$ case is often referred to as the matter bounce scenario. In such a case, $\rho_1 = 12 k_0^2 M_{\text{Pl}}^2 a_0/a^3$ and $\rho_2 = -12 k_0^2 M_{\text{Pl}}^2 a_0^2/a^4$.



E- \mathcal{N} -folds

The conventional e-fold N is defined $N = \log(a/a_i)$ so that $a(N) = a_i \exp N$. However, the function e^N is a monotonically increasing function of N .

¹¹L. Sriramkumar, K. Atmjeet and R. K. Jain, JCAP **1509**, 010 (2015).



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In completely symmetric bouncing scenarios, an obvious choice for the scale factor seems to be¹¹

$$a(\mathcal{N}) = a_0 \exp(\mathcal{N}^2/2),$$

with \mathcal{N} being the new time variable that we shall consider for integrating the differential equation governing the background as well as the perturbations.

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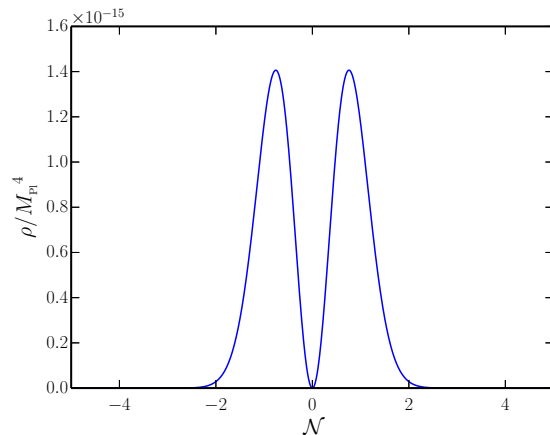
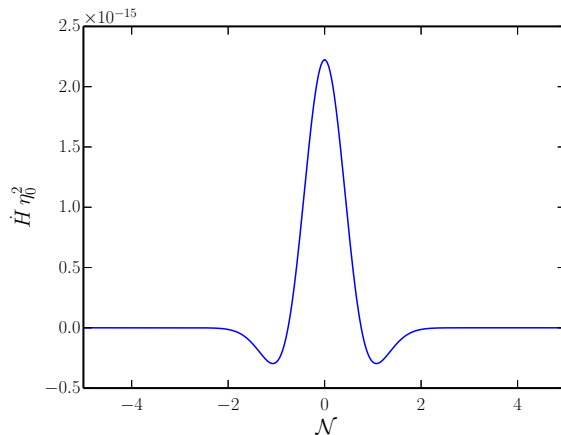
with \mathcal{N} being the new time variable that we shall consider for integrating the differential equation governing the background as well as the perturbations.

We shall refer to the variable \mathcal{N} as e- \mathcal{N} -fold since the scale factor grows roughly by the amount $e^{\mathcal{N}}$ between \mathcal{N} and $(\mathcal{N} + 1)$.

¹¹L. Sriramkumar, K. Atmjeet and R. K. Jain, JCAP **1509**, 010 (2015).



Behavior of \dot{H} and ρ in a matter bounce



The behavior of \dot{H} (on the left) and the total energy density ρ (on the right) in a symmetric matter bounce scenario has been plotted as a function of \mathcal{N} . Note that the maximum value of ρ is much smaller than M_{Pl}^4 , which suggests that the bounce can be treated completely classically.

[▶ Back to scalar perturbations](#)



Duality between de Sitter inflation and matter bounce

It is known that the solutions to the equations of motion governing the scalar and tensor perturbations are invariant under a certain transformation referred to as the duality transformation¹².

¹²D. Wands, *Phys. Rev. D* **60**, 023507 (1999).



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For instance, recall that the Mukhanov-Sasaki variable corresponding to the tensor perturbations [which is defined as $u_k = (M_{\text{Pl}}/\sqrt{2}) a h_k$] satisfies the differential equation

$$u_k'' + \left(k^2 - \frac{a''}{a} \right) u_k = 0.$$

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Given a scale factor a , the corresponding dual, say, \tilde{a} , which leads to the same equation for the variable u_k is given by

$$a(\eta) \rightarrow \tilde{a}(\eta) = C a(\eta) \int_{\eta_*}^{\eta} \frac{d\bar{\eta}}{a^2(\bar{\eta})},$$

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It is straightforward to show that the dual solution to de Sitter inflation corresponds to the matter bounce. Both these cases lead to scale invariant spectra.

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The matter bounce

We shall assume that the scale factor describing the bouncing scenario is given in terms of the conformal time coordinate η by the relation

$$a(\eta) = a_0 \left(1 + \eta^2/\eta_0^2\right) = a_0 \left(1 + k_0^2 \eta^2\right).$$

As we had discussed earlier, at very early times, *viz.* when $\eta \ll -\eta_0$, the scale factor behaves as in a matter dominated epoch¹³.

¹³See, for example, [R. Brandenberger, arXiv:1206.4196](#).



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As we had discussed earlier, at very early times, *viz.* when $\eta \ll -\eta_0$, the scale factor behaves as in a matter dominated epoch¹³.

The quantity a''/a corresponding to the above scale factor is given by

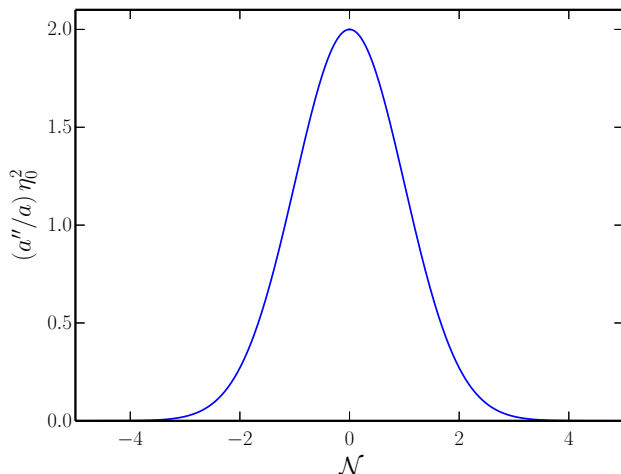
$$\frac{a''}{a} = \frac{2 k_0^2}{1 + k_0^2 \eta^2},$$

which is essentially a Lorentzian profile.

¹³See, for example, [R. Brandenberger, arXiv:1206.4196](#).



The behavior of a''/a



The behavior of the quantity a''/a has been plotted as a function of \mathcal{N} for the matter bounce scenario of interest. Note that the maximum value of a''/a is of the order of k_0^2 .

► Evolution of h_k , sans the details



The tensor modes in the first domain

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In the first domain, we can assume that the scale factor behaves as $a(\eta) \simeq a_0 k_0^2 \eta^2$, so that $a''/a \simeq 2/\eta^2$. Since the condition $k^2 = a''/a$ corresponds to, say, $\eta_k = -\sqrt{2}/k$, the initial conditions can be imposed when $\eta \ll \eta_k$.



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The modes h_k can be easily obtained in such a case and the positive frequency modes that correspond to the vacuum state at early times are given by

$$h_k(\eta) = \frac{\sqrt{2}}{M_{\text{Pl}}} \frac{1}{\sqrt{2k}} \frac{1}{a_0 k_0^2 \eta^2} \left(1 - \frac{i}{k\eta} \right) e^{-ik\eta}.$$



The modes in the second domain

Let us now consider the behavior of the modes in the domain $-\alpha \eta_0 < \eta < \beta \eta_0$, where, say, $\beta \simeq 10^2$. Since we are interested in scales much smaller than k_0 , we shall assume that $\eta_k \ll -\alpha \eta_0$, which corresponds to $k \ll k_0/\alpha$.



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In such a case, upon ignoring the k^2 term, the equation governing h_k can be immediately integrated to yield

$$h_k(\eta) \simeq h_k(\eta_*) + h'_k(\eta_*) a^2(\eta_*) \int_{\eta_*}^{\eta} \frac{d\tilde{\eta}}{a^2(\tilde{\eta})},$$

where η_* is a suitably chosen time and the scale factor $a(\eta)$ is given by the complete expression.



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where η_* is a suitably chosen time and the scale factor $a(\eta)$ is given by the complete expression.

If we choose $\eta_* = -\alpha \eta_0$, we can make use of the solution in the first domain to determine the constants and express the solution in the second domain as follows:

$$h_k = \mathcal{A}_k + \mathcal{B}_k f(k_0 \eta),$$

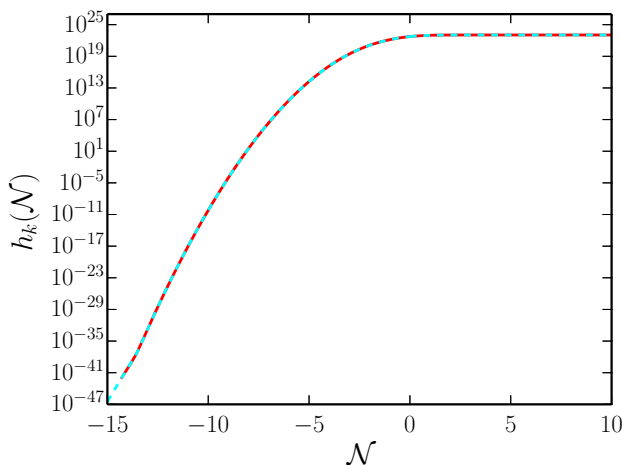
where the function $f(k_0 \eta)$ is given by

$$f(k_0 \eta) = \frac{k_0 \eta}{1 + k_0^2 \eta^2} + \tan^{-1}(k_0 \eta).$$

[▶ Back to scalar perturbations](#)



Evolution of the tensor modes across the bounce



► Tensor power spectrum, sans the details

Comparison of the numerical results (in solid red) with the analytical results (in dashed cyan) for the amplitude of the tensor mode $|h_k|$ corresponding to $k/k_0 = 10^{-20}$. We have set $k_0/(a_0 M_{\text{Pl}}) = 3.3 \times 10^{-8}$ and have chosen $\alpha = 10^5$ for plotting the analytical results¹⁴

¹⁴D. Chowdhury, V. Sreenath and L. Sriramkumar, JCAP **1511**, 002 (2015).



The tensor power spectrum after the bounce

The quantities \mathcal{A}_k and \mathcal{B}_k are given by

$$\mathcal{A}_k = \frac{\sqrt{2}}{M_{\text{Pl}}} \frac{1}{\sqrt{2k}} \frac{1}{a_0 \alpha^2} \left(1 + \frac{i k_0}{\alpha k} \right) e^{i \alpha k/k_0} + \mathcal{B}_k f(\alpha),$$

$$\mathcal{B}_k = \frac{\sqrt{2}}{M_{\text{Pl}}} \frac{1}{\sqrt{2k}} \frac{1}{2 a_0 \alpha^2} (1 + \alpha^2)^2 \left(\frac{3 i k_0}{\alpha^2 k} + \frac{3}{\alpha} - \frac{i k}{k_0} \right) e^{i \alpha k/k_0}.$$



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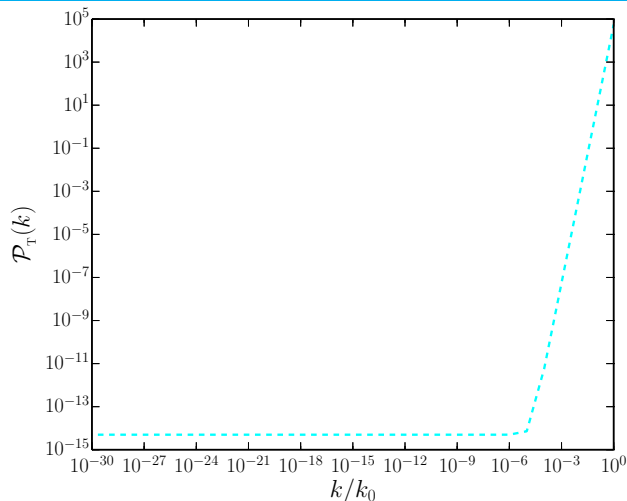
$$\mathcal{B}_k = \frac{\sqrt{2}}{M_{\text{Pl}}} \frac{1}{\sqrt{2k}} \frac{1}{2 a_0 \alpha^2} (1 + \alpha^2)^2 \left(\frac{3 i k_0}{\alpha^2 k} + \frac{3}{\alpha} - \frac{i k}{k_0} \right) e^{i \alpha k/k_0}.$$

If we evaluate the tensor power spectrum after the bounce at $\eta = \beta \eta_0$, we find that it can be expressed as

$$\mathcal{P}_T(k) = 4 \frac{k^3}{2 \pi^2} |\mathcal{A}_k + \mathcal{B}_k f(\beta)|^2.$$



The tensor power spectrum



The behavior of the tensor power spectrum has been plotted as a function of k/k_0 for a wide range of wavenumbers. In plotting this figure, we have set $k_0/(a_0 M_{Pl}) = 3.3 \times 10^7$, $\alpha = 10^5$ and $\beta = 10^2$. Note that the power spectrum is scale invariant for $k \ll k_0/\alpha$.



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- 2 Bouncing scenarios
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- 4 A new model for the completely symmetric matter bounce**
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A new model for the completely symmetric matter bounce

As we had discussed, the matter bounce scenario described by the scale factor

$$a(\eta) = a_0 \left(1 + \eta^2/\eta_0^2\right) = a_0 \left(1 + k_0^2 \eta^2\right)$$

can be driven with the aid of two fluids, one which is matter and another fluid which behaves like radiation, but has negative energy density.

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can be driven with the aid of two fluids, one which is matter and another fluid which behaves like radiation, but has negative energy density.

We find that the behavior can also be achieved with the help of two scalar fields, say, ϕ and χ , that are governed by the following action¹⁵:

$$S[\phi, \chi] = - \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + U_0 \left(-\frac{1}{2} \partial_\mu \chi \partial^\mu \chi \right)^2 \right],$$

where U_0 is a constant and the potential $V(\phi)$ is given by

$$V(\phi) = \frac{6 M_{\text{Pl}}^2 (k_0^2/a_0^2)}{\cosh^6[\phi/(\sqrt{12} M_{\text{Pl}})]}.$$

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The scalar perturbations

When the scalar perturbations are taken into account, the FLRW line element can be written as

$$ds^2 = -(1 + 2A) dt^2 + 2a(t) (\partial_i B) dt dx^i + a^2(t) [(1 - 2\psi) \delta_{ij} + 2(\partial_i \partial_j E)] dx^i dx^j,$$

where, evidently, the quantities A , ψ , B and E represent the metric perturbations.

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The gauge invariant curvature and isocurvature perturbations \mathcal{R} and \mathcal{S} can be defined in terms of the above metric perturbations and the perturbations $\delta\phi$ and $\delta\chi$ in the scalar fields as follows¹⁶:

$$\mathcal{R} = \frac{H}{\dot{\phi}^2 - U_0 \dot{\chi}^4} \left(\dot{\phi} \bar{\delta\phi} - U_0 \dot{\chi}^3 \bar{\delta\chi} \right), \quad \mathcal{S} = \frac{H \sqrt{\alpha \dot{\chi}^2}}{\dot{\phi}^2 - U_0 \dot{\chi}^4} \left(\dot{\chi} \bar{\delta\phi} - \dot{\phi} \bar{\delta\chi} \right).$$

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The quantities $\overline{\delta\phi}$ and $\overline{\delta\chi}$ denote the gauge invariant versions of the perturbations in the scalar fields, and are given by

$$\overline{\delta\phi} = \delta\phi + \frac{\dot{\phi} \psi}{H}, \quad \overline{\delta\chi} = \delta\chi + \frac{\dot{\chi} \psi}{H}.$$

¹⁶R. N. Raveendran, D. Chowdhury and L. Sriramkumar, In preparation.



Equations governing the curvature and isocurvature perturbations

We obtain the equations of motion describing the gauge invariant perturbations \mathcal{R}_k and \mathcal{S}_k in our model to be

$$\begin{aligned}
 \mathcal{R}_k'' &+ \frac{2(7 + 9k_0^2\eta^2 - 6k_0^4\eta^4)}{\eta(1 - 3k_0^2\eta^2)^2(1 + k_0^2\eta^2)} \mathcal{R}_k' - \frac{k^2(5 + 9k_0^2\eta^2)}{3(1 - 3k_0^2\eta^2)} \mathcal{R}_k \\
 &= \frac{4(5 + 12k_0^2\eta^2)}{\sqrt{3}\eta(1 - 3k_0^2\eta^2)\sqrt{1 + k_0^2\eta^2}} \mathcal{S}_k' - \frac{4\left[5 - 22k_0^2\eta^2 - 24k_0^4\eta^4 + k^2\eta^2(1 + k_0^2\eta^2)^2\right]}{\sqrt{3}\eta^2(1 + k_0^2\eta^2)^{3/2}(1 - 3k_0^2\eta^2)} \mathcal{S}_k, \\
 \mathcal{S}_k'' &- \frac{2(9 + 7k_0^2\eta^2 + 6k_0^4\eta^4)}{\eta(1 - 3k_0^2\eta^2)(1 + k_0^2\eta^2)} \mathcal{S}_k' \\
 &- \frac{18 - 85k_0^2\eta^2 - 25k_0^4\eta^4 - 6k_0^6\eta^6 + k^2\eta^2(3 - k_0^2\eta^2)(1 + k_0^2\eta^2)^2}{\eta^2(1 - 3k_0^2\eta^2)(1 + k_0^2\eta^2)^2} \mathcal{S}_k \\
 &= \frac{4\sqrt{3}(3 - 2k_0^2\eta^2)}{\eta\sqrt{1 + k_0^2\eta^2}(1 - 3k_0^2\eta^2)} \mathcal{R}_k' + \frac{4k^2\sqrt{1 + k_0^2\eta^2}}{\sqrt{3}(1 - 3k_0^2\eta^2)} \mathcal{R}_k.
 \end{aligned}$$

► Behavior of \dot{H}

However, some of the coefficients diverge when \dot{H} and/or H vanish.

► Evolution of \mathcal{R}_k and \mathcal{S}_k , sans the details



The uniform- χ gauge

The issue of diverging coefficients can be avoided by working in a gauge wherein $\delta\chi = 0$ ¹⁷.

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The issue of diverging coefficients can be avoided by working in a gauge wherein $\delta\chi = 0$ ¹⁷.

In this gauge, the equations of motion for the metric perturbations A_k and ψ_k can be obtained to be

$$A_k'' + 4\mathcal{H}A_k' + \left(\frac{k^2}{3} - \frac{20a_0^2k_0^2}{a^2}\right)A_k = -3\mathcal{H}\psi_k' + \frac{4k^2}{3}\psi_k,$$

$$\psi_k'' - 2\mathcal{H}\psi_k' + k^2\psi_k = 2\mathcal{H}A_k' - \frac{20a_0^2k_0^2}{a^2}A_k,$$

where $\mathcal{H} = a'/a$. These equations prove to be helpful in evolving the scalar perturbations across the bounce.

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where $\mathcal{H} = a'/a$. These equations prove to be helpful in evolving the scalar perturbations across the bounce.

Also, in the uniform- χ gauge, the curvature and isocurvature perturbations simplify to be

$$\mathcal{R}_k = \psi_k + \frac{2 H M_{\text{Pl}}^2}{\dot{\phi}^2 - U_0 \dot{\chi}^4} \left(\dot{\psi}_k + H A_k \right), \quad \mathcal{S}_k = \frac{2 H M_{\text{Pl}}^2 \sqrt{U_0} \dot{\chi}^4}{\left(\dot{\phi}^2 - U_0 \dot{\chi}^4 \right) \dot{\phi}} \left(\dot{\psi}_k + H A_k \right).$$

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Solutions for \mathcal{R}_k and \mathcal{S}_k in the first domain

As in the case of tensors, we shall be interested in evaluating the power spectrum after the bounce at $\eta = \beta \eta_0$. Also, to arrive at the analytical approximations, as earlier, we shall divide period of interest into two domains, viz. $-\infty < \eta < -\alpha \eta_0$ and $-\alpha \eta_0 < \eta < \beta \eta_0$.



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In the first domain, we find that the solution to the curvature perturbation can be arrived at as in the case of tensors and is given by

$$\mathcal{R}_k(\eta) \simeq \frac{1}{\sqrt{6k} M_{\text{Pl}} a_0 k_0^2 \eta^2} \left(1 - \frac{i}{k\eta} \right) e^{-ik\eta}.$$



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In the first domain, we find that the solution to the curvature perturbation can be arrived at as in the case of tensors and is given by

$$\mathcal{R}_k(\eta) \simeq \frac{1}{\sqrt{6} k M_{\text{Pl}} a_0 k_0^2 \eta^2} \left(1 - \frac{i}{k \eta} \right) e^{-i k \eta}.$$

Using this solution, it is straightforward to obtain the following solution for the isocurvature perturbation at early times:

$$\begin{aligned} \mathcal{S}_k(\eta) \simeq & \frac{1}{9 \sqrt{2} k^3 a_0 k_0^3 M_{\text{Pl}} \eta^4} \left(-12 i (1 + i k \eta) e^{-i k \eta} + \frac{9}{3^{1/4}} k k_0 \eta^2 e^{-i k \eta / \sqrt{3}} \right. \\ & \left. + 4 k^2 \eta^2 e^{-i k \eta / \sqrt{3}} \left\{ \pi + i \text{Ei} \left[e^{-i (3 - \sqrt{3}) k \eta / 3} \right] \right\} \right). \end{aligned}$$



Solutions for ψ_k and A_k in the second domain

In the second domain, upon ignoring the k^2 dependent terms, one finds that the combination $A_k + \psi_k$ satisfies the same equation of motion as the tensor modes.



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This feature helps us obtain the solutions for A_k and ψ_k , and they are given by

$$A_k(\eta) + \psi_k(\eta) \simeq \frac{C_k}{2 a_0^2} f(k_0 \eta) + \mathcal{D}_k,$$

$$A_k(\eta) \simeq \frac{C_k k_0 \eta}{4 a_0^2 (1 + k_0^2 \eta^2)} + \mathcal{E}_k e^{-2\sqrt{5} \tan^{-1}(k_0 \eta)} + \mathcal{F}_k e^{2\sqrt{5} \tan^{-1}(k_0 \eta)},$$

where $f(k_0 \eta)$ is the same function that we had encountered earlier in the case of tensors, and C_k , \mathcal{D}_k , \mathcal{E}_k and \mathcal{F}_k are four constants of integration.

► Function $f(k_0 \eta)$



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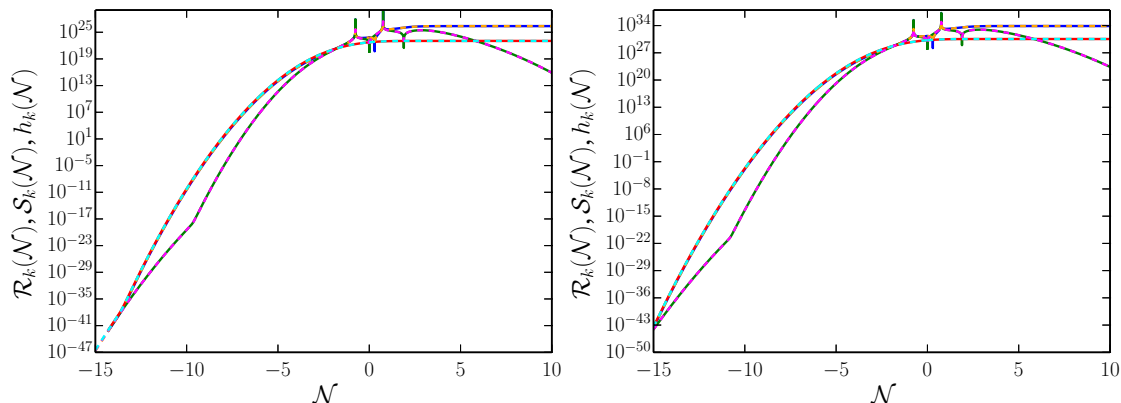
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where $f(k_0 \eta)$ is the same function that we had encountered earlier in the case of tensors, and C_k , \mathcal{D}_k , \mathcal{E}_k and \mathcal{F}_k are four constants of integration. ▶ Function $f(k_0 \eta)$

The four constants, viz. C_k , \mathcal{D}_k , \mathcal{E}_k and \mathcal{F}_k , are determined by matching the above solutions with the solutions for \mathcal{R}_k and \mathcal{S}_k in the first domain at $\eta = -\alpha \eta_0$.

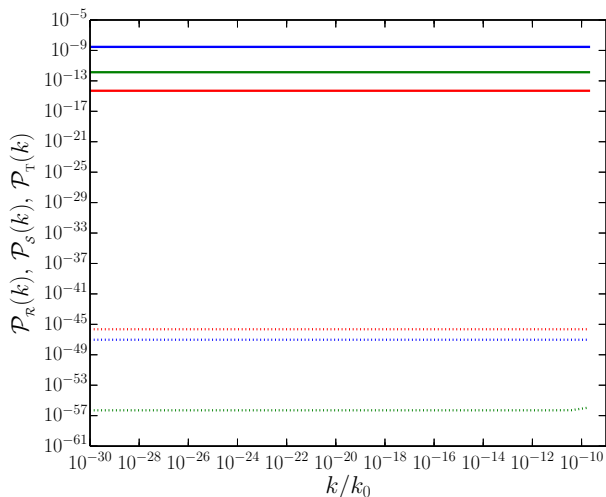


Evolution of \mathcal{R}_k , \mathcal{S}_k and h_k 

The evolution of the curvature, isocurvature and tensor perturbations, viz. \mathcal{R}_k (in blue and orange), \mathcal{S}_k (in green and magenta) and h_k (in red and cyan) across the bounce for the modes $k/k_0 = 10^{-20}$ (on the left) and $k/k_0 = 10^{-25}$ (on the right). We have set $k_0/(a_0 M_{\text{Pl}}) = 3.3 \times 10^{-8}$, $\alpha = 10^5$ and $\beta = 10^2$. The solid lines denote the results obtained numerically, while the dashed lines represent the analytical approximations.



The scalar and tensor power spectra in the matter bounce

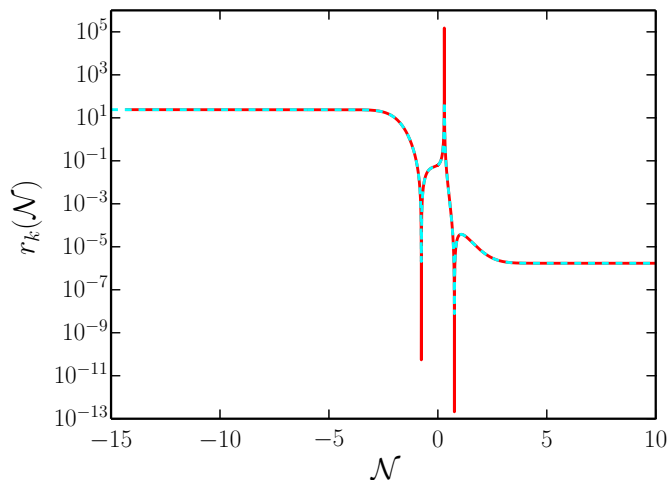


The scalar (curvature as blue and isocurvature as green) and tensor (as red) power spectra have been plotted before (as dotted lines) as well as after (as solid lines) the bounce¹⁸

¹⁸R. N. Raveendran, D. Chowdhury and L. Sriramkumar, arXiv:1703.10061v1 [gr-qc].



The evolution of the tensor-to-scalar ratio



The evolution of the tensor-to-scalar ratio r across the symmetric matter bounce for a typical mode of cosmological interest. The solid (in red) and dashed (in cyan) lines represent the numerical and analytical results, respectively.



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Near matter bounces

Near-matter bounces can be described by the scale factor

$$a(\eta) = a_0 (1 + k_0^2 \eta^2)^{(1+\lambda)}$$

and, one finds that the index λ leads to a tilt in the power spectra.

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and, one finds that the index λ leads to a tilt in the power spectra.

Such a scale factor can also be achieved with the aid of two scalar fields, say, ϕ and χ , governed by the action¹⁹

$$S[\phi, \chi] = - \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + U_0 \left(-\frac{1}{2} \partial_\mu \chi \partial^\mu \chi \right)^\alpha \right],$$

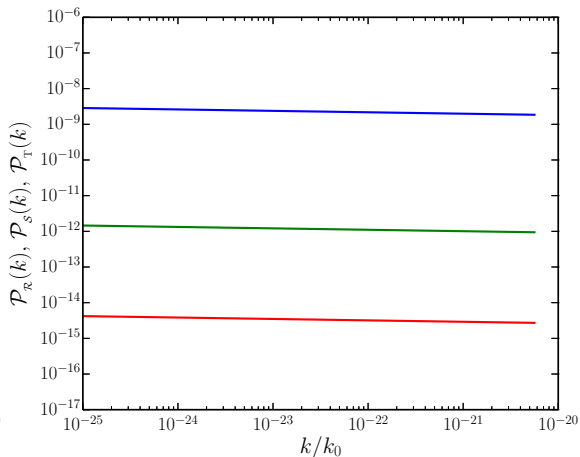
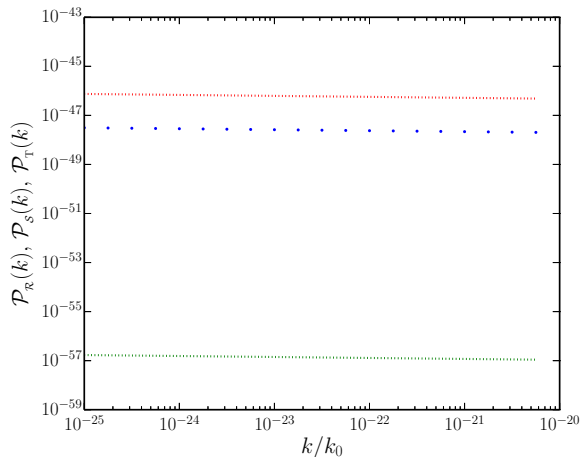
where U_0 is a constant, $\alpha = (2 + \lambda)/(1 + \lambda)$ and the potential $V(\phi)$ is given by

$$V(\phi) = \frac{2(3 + 4\lambda)}{1 + \lambda} \left(\frac{M_{\text{Pl}} k_0 (1 + \lambda)}{a_0} \right)^2 \cosh^{-2(3+2\lambda)} \left(\frac{\phi}{2 \sqrt{(1 + \lambda)(3 + 2\lambda)} M_{\text{Pl}}} \right).$$

¹⁹R. N. Raveendran and L. Sriramkumar, in preparation.



Red-tilted scalar and tensor power spectra in a near-matter bounce



The scalar (curvature as blue and isocurvature as green) and tensor (as red) power spectra for $\lambda \simeq 10^{-2}$ (corresponding to a scalar spectral tilt of $n_s = 0.96$) have been plotted before (as dotted lines, on the left) and after (as solid lines, on the right) the bounce²⁰.

²⁰R. N. Raveendran, and L. Sriramkumar, in preparation.



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- 8 Summary



Tensor bi-spectrum and non-Gaussianity parameter

The tensor bi-spectrum, evaluated at the conformal time, say, η_e , is defined as

$$\langle \hat{\gamma}_{m_1 n_1}^{\mathbf{k}_1}(\eta_e) \hat{\gamma}_{m_2 n_2}^{\mathbf{k}_2}(\eta_e) \hat{\gamma}_{m_3 n_3}^{\mathbf{k}_3}(\eta_e) \rangle = (2\pi)^3 \mathcal{B}_{\gamma\gamma\gamma}^{m_1 n_1 m_2 n_2 m_3 n_3}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \\ \times \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$

and, for convenience, we shall set

$$\mathcal{B}_{\gamma\gamma\gamma}^{m_1 n_1 m_2 n_2 m_3 n_3}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = (2\pi)^{-9/2} G_{\gamma\gamma\gamma}^{m_1 n_1 m_2 n_2 m_3 n_3}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3).$$

²¹V. Sreenath, R. Tibrewala and L. Sriramkumar, JCAP **1312**, 037 (2013).



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As in the scalar case, one can define a dimensionless non-Gaussianity parameter to characterize the amplitude of the tensor bi-spectrum as follows²¹:

$$h_{\text{NL}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = - \left(\frac{4}{2\pi^2} \right)^2 [k_1^3 k_2^3 k_3^3 G_{\gamma\gamma\gamma}^{m_1 n_1 m_2 n_2 m_3 n_3}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)] \times \left[\Pi_{m_1 n_1, m_2 n_2}^{\mathbf{k}_1} \Pi_{m_3 n_3, \bar{m} \bar{n}}^{\mathbf{k}_2} k_3^3 \mathcal{P}_{\text{T}}(k_1) \mathcal{P}_{\text{T}}(k_2) + \text{five permutations} \right]^{-1}.$$

²¹V. Sreenath, R. Tibrewala and L. Sriramkumar, JCAP **1312**, 037 (2013).



The third order action and the tensor bi-spectrum

The third order action that leads to the tensor bi-spectrum is given by²²

► The quadratic action

$$S_{\gamma\gamma\gamma}^3[\gamma_{ij}] = \frac{M_{\text{Pl}}^2}{2} \int d\eta \int d^3\mathbf{x} \left[\frac{a^2}{2} \gamma_{lj} \gamma_{im} \partial_l \partial_m \gamma_{ij} - \frac{a^2}{4} \gamma_{ij} \gamma_{lm} \partial_l \partial_m \gamma_{ij} \right].$$

²²J. Maldacena, JHEP **0305**, 013 (2003).



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The tensor bi-spectrum calculated in the perturbative vacuum using the Maldacena formalism, can be written in terms of the modes h_k as follows:

$$\begin{aligned} & G_{\gamma\gamma\gamma}^{m_1 n_1 m_2 n_2 m_3 n_3}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \\ &= M_{\text{Pl}}^2 \left[\left(\Pi_{m_1 n_1, ij}^{\mathbf{k}_1} \Pi_{m_2 n_2, im}^{\mathbf{k}_2} \Pi_{m_3 n_3, lj}^{\mathbf{k}_3} - \frac{1}{2} \Pi_{m_1 n_1, ij}^{\mathbf{k}_1} \Pi_{m_2 n_2, ml}^{\mathbf{k}_2} \Pi_{m_3 n_3, ij}^{\mathbf{k}_3} \right) k_{1m} k_{1l} \right. \\ & \quad \left. + \text{five permutations} \right] \\ & \quad \times [h_{k_1}(\eta_e) h_{k_2}(\eta_e) h_{k_3}(\eta_e) \mathcal{G}_{\gamma\gamma\gamma}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + \text{complex conjugate}], \end{aligned}$$

where $\mathcal{G}_{\gamma\gamma\gamma}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is described by the integral

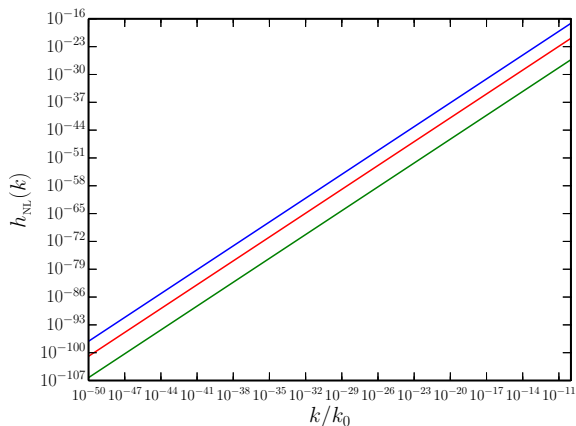
$$\mathcal{G}_{\gamma\gamma\gamma}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = -\frac{i}{4} \int_{\eta_i}^{\eta_e} d\eta a^2 h_{k_1}^* h_{k_2}^* h_{k_3}^*,$$

with η_i denoting the time when the initial conditions are imposed on the perturbations.

²²J. Maldacena, JHEP **0305**, 013 (2003).



The contributions due to the three domains



The contributions to the non-Gaussianity parameter h_{NL} in the equilateral limit from the first (in green), the second (in red) and the third (in blue) domains have been plotted as a function of k/k_0 for $k \ll k_0/\alpha$. Clearly, the third domain gives rise to the maximum contribution to h_{NL} ²³.

► Skip the discussion on the squeezed limit



²³D. Chowdhury, V. Sreenath and L. Sriramkumar, JCAP **1511**, 002 (2015)

The effect of the long wavelength tensor modes

Since the amplitude of a long wavelength mode freezes on super-Hubble scales during inflation, such modes can be treated as a background as far as the smaller wavelength modes are concerned. Let us denote the constant amplitude of the long wavelength tensor mode as γ_{ij}^B .

²⁴S. Kundu, JCAP **1404**, 016 (2014).



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In the presence of such a long wavelength mode, the background FLRW metric can be written as

$$ds^2 = -dt^2 + a^2(t) [e^{\gamma^B}]_{ij} d\mathbf{x}^i d\mathbf{x}^j,$$

i.e. the spatial coordinates are modified according to a spatial transformation of the form $\mathbf{x}' = \Lambda \mathbf{x}$, where $\Lambda_{ij} = [e^{\gamma^B/2}]_{ij}$.

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Under such a spatial transformation, the small wavelength tensor perturbation transforms as²⁴

$$\gamma_{ij}^{\mathbf{k}} \rightarrow \det(\Lambda^{-1}) \gamma_{ij}^{\Lambda^{-1} \mathbf{k}},$$

where $\det(\Lambda^{-1}) = 1$.

²⁴S. Kundu, JCAP **1404**, 016 (2014).



The behavior of the two and three-point functions

On using the above results, one finds that the tensor two-point function in the presence of a long wavelength mode denoted by, say, the wavenumber k , can be written as

$$\langle \hat{\gamma}_{m_1 n_1}^{\mathbf{k}_1} \hat{\gamma}_{m_2 n_2}^{\mathbf{k}_2} \rangle_k = \frac{(2\pi)^2}{2k_1^3} \frac{\Pi_{m_1 n_1, m_2 n_2}^{\mathbf{k}_1}}{4} \mathcal{P}_T(k_1) \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \\ \times \left[1 - \left(\frac{n_T - 3}{2} \right) \gamma_{ij}^B \hat{n}_{1i} \hat{n}_{1j} \right],$$

where $\hat{n}_{1i} = k_{1i}/k_1$.

²⁵V. Sreenath and L. Sriramkumar, JCAP **1410**, 021 (2014).



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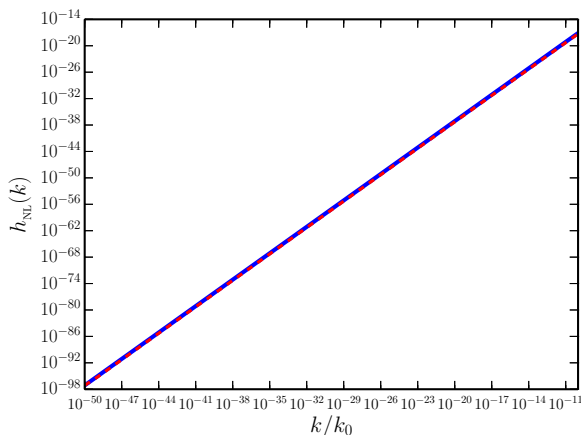
One can also show that, in the presence of a long wavelength mode, the tensor bi-spectrum can be written as²⁵

$$\langle \hat{\gamma}_{m_1 n_1}^{\mathbf{k}_1} \hat{\gamma}_{m_2 n_2}^{\mathbf{k}_2} \hat{\gamma}_{m_3 n_3}^{\mathbf{k}_3} \rangle_{k_3} = - \frac{(2\pi)^{5/2}}{4k_1^3 k_3^3} \left(\frac{n_T - 3}{32} \right) \mathcal{P}_T(k_1) \mathcal{P}_T(k_3) \\ \times \Pi_{m_1 n_1, m_2 n_2}^{\mathbf{k}_1} \Pi_{m_3 n_3, ij}^{\mathbf{k}_3} \hat{n}_{1i} \hat{n}_{1j} \delta^3(\mathbf{k}_1 + \mathbf{k}_2).$$

²⁵V. Sreenath and L. Sriramkumar, JCAP **1410**, 021 (2014).



The complete contribution to h_{NL}



The behavior of h_{NL} in the equilateral (in blue) and the squeezed (in red) limits plotted as a function of k/k_0 for $k \ll k_0/\alpha$. The resulting h_{NL} is considerably small when compared to the values that arise in de Sitter inflation wherein $3/8 \lesssim h_{\text{NL}} \lesssim 1/2$. Moreover, we find that h_{NL} behaves as k^2 in the equilateral and the squeezed limits, with similar amplitudes²⁶.

²⁶D. Chowdhury, V. Sreenath and L. Sriramkumar, JCAP **1511**, 002 (2015).



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- In this work, we have been able to construct a *completely* symmetric matter bounce scenario that leads to nearly scale invariant spectra and a tensor-to-scalar ratio that is consistent with the observations.



Summary

- ◆ Earlier efforts had seemed to suggest that the tensor-to-scalar ratio may naturally be large in symmetric bounces.
- In this work, we have been able to construct a *completely* symmetric matter bounce scenario that leads to nearly scale invariant spectra and a tensor-to-scalar ratio that is consistent with the observations.
- It is also important to examine if the non-Gaussianities generated in such models are in agreement with the recent constraints from Planck.



Issues confronting bouncing models

- ◆ In inflation, any classical perturbations present at the start will decay. In contrast, they grow strongly in bouncing models. So, these need to be assumed to be rather small if smooth bounces have to begin.

²⁷ L. E. Allen and D. Wands, *Phys. Rev.* **70**, 063515 (2004).

²⁸ Y-F. Cai, R. Brandenberger and X. Zhang, *Phys. Letts. B* **703**, 25 (2011).

²⁹ J. Quintin, Z. Sherkatghanad, Y-F. Cai and R. Brandenberger, *Phys. Rev. D* **92**, 062532 (2015).



Issues confronting bouncing models

- ◆ In inflation, any classical perturbations present at the start will decay. In contrast, they grow strongly in bouncing models. So, these need to be assumed to be rather small if smooth bounces have to begin.
- ◆ The growth of the perturbations as one approaches the bounce during the contracting phase causes concerns about the validity of linear perturbation theory near the bounce. Is it, for instance, sufficient if the perturbations remain small in specific gauges? Is a divergent curvature perturbation acceptable?

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- ◆ Is it possible to construct wider classes of completely symmetric bounces with nearly scale invariant spectra and viable tensor-to-scalar ratios²⁷?

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- ◆ After the bounce, the universe needs to transit to a radiation dominated epoch. How can this be achieved? Does this process affect the evolution of the large scale perturbations²⁸?
- Does the growth in the amplitude of the perturbations as one approaches the bounce naturally lead to large levels of non-Gaussianities in such models²⁹?

²⁷ L. E. Allen and D. Wands, *Phys. Rev.* **70**, 063515 (2004).

²⁸ Y-F. Cai, R. Brandenberger and X. Zhang, *Phys. Letts. B* **703**, 25 (2011).

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Collaborators: current and former students



Rathul Nath Raveendran



Debika Chowdhury



V. Sreenath



Thank you for your attention