Viable scalar spectral tilt and tensor-to-scalar ratio in near-matter bounces

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Plan of the talk

This talk is based on. . .

✦ D. Chowdhury, V. Sreenath and L. Sriramkumar, *The tensor bispectrum in a matter bounce*, JCAP **1511**, 002 (2015) [arXiv:1506.06475 [astro-ph.CO]].

✦ R. N. Raveendran, D. Chowdhury and L. Sriramkumar, *Viable tensor-to-scalar ratio in a symmetric matter bounce*, arXiv:1703.10061v1 [gr-qc].

✦ R. N. Raveendran and L. Sriramkumar, *Viable scalar spectral tilt and tensor-to-scalar ratio in near-matter bounces*, in preparation.

A few words on our conventions and notations

✦ Units: $c = \hbar = 1$, $M_{\mathrm{Pl}} = (8\,\pi\,G)^{-1/2}$

✦ Background: Spatially flat FLRW metric described by the line-element

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\boldsymbol{x}^2 = a^2(\eta)\,\left(-\mathrm{d}\eta^2 + \mathrm{d}\boldsymbol{x}^2\right)$$

with t – cosmic time and η – conformal time

✦ Scale factor and Hubble parameter: a and $H = \dot{a}/a$

✦ Derivatives: $\dot{} \equiv \mathrm{d}/\mathrm{d}t$, $' \equiv \mathrm{d}/\mathrm{d}\eta$

✦ E-folds: N

✦ E-$\mathcal{N}$-folds: $\mathcal{N}$

✦ Curvature and isocurvature perturbations: $\mathcal{R}$ and $\mathcal{R}_k$, $\mathcal{S}$ and $\mathcal{S}_k$
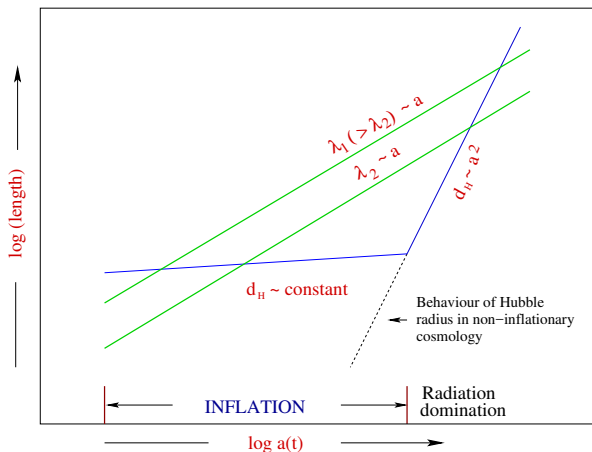
✦ Tensor perturbation: γ_{ij} and h_k

Plan of the talk

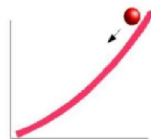# Bringing the modes inside the Hubble radius

The behavior of the physical wavelength $\lambda_{\rm P} \propto a$ (the green lines) and the Hubble radius H^{-1} (the blue line) during inflation and the radiation dominated epochs[1].   ▸ Back to bounce

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[1]See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.

A variety of potentials to choose from

Large field
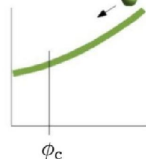$$V(\phi) = \Lambda^4 (\phi/\mu)^p$$
$$V(\phi) = \Lambda^4 e^{\phi/\mu}$$
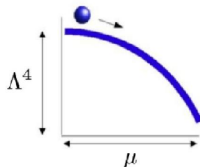
Small field
$$V(\phi) = \Lambda^4 \left[1 - (\phi/\mu)^p\right]$$

Hybrid
$$V(\phi) = \Lambda^4 \left[1 + (\phi/\mu)^p\right]$$

A variety of scalar field potentials have been considered to drive inflation[2]. Often, these potentials are classified as small field, large field and hybrid models.

[2] Image from W. Kinney, astro-ph/0301448.

Proliferation of inflationary models

5-dimensional assisted inflation	extended open inflation	late-time mild inflation	pre-Big-Bang inflation
anisotropic brane inflation	extended warm inflation	low-scale inflation	primary inflation
anomaly-induced inflation	extra dimensional inflation	low-scale supergravity inflation	primordial inflation
assisted inflation	F-term inflation	M-theory inflation	quasi-open inflation
assisted chaotic inflation	F-term hybrid inflation	mass inflation	quintessential inflation
boundary inflation	false vacuum inflation	massive chaotic inflation	R-invariant topological inflation
brane inflation	false vacuum chaotic inflation	moduli inflation	rapid asymmetric inflation
brane-assisted inflation	fast-roll inflation	multi-scalar inflation	running inflation
brane gas inflation	first order inflation	multiple inflation	scalar-tensor gravity inflation
brane-antibrane inflation	gauged inflation	multiple-field slow-roll inflation	scalar-tensor stochastic inflation
braneworld inflation	generalised inflation	multiple-stage inflation	Seiberg-Witten inflation
Brans-Dicke chaotic inflation	generalized assisted inflation	natural inflation	single-bubble open inflation
Brans-Dicke inflation	generalized slow-roll inflation	natural Chaotic inflation	spinodal inflation
bulky brane inflation	gravity driven inflation	natural double inflation	stable starobinsky-type inflation
chaotic hybrid inflation	Hagedorn inflation	natural supergravity inflation	steady-state eternal inflation
chaotic inflation	higher-curvature inflation	new inflation	steep inflation
chaotic new inflation	hybrid inflation	next-to-minimal supersymmetric	stochastic inflation
D-brane inflation	hyperextended inflation	hybrid inflation	string-forming open inflation
D-term inflation	induced gravity inflation	non-commutative inflation	successful D-term inflation
dilaton-driven inflation	induced gravity open inflation	non-slow-roll inflation	supergravity inflation
dilaton-driven brane inflation	intermediate inflation	nonminimal chaotic inflation	supernatural inflation
double inflation	inverted hybrid inflation	old inflation	superstring inflation
double D-term inflation	isocurvature inflation	open hybrid inflation	supersymmetric hybrid inflation
dual inflation	K inflation	open inflation	supersymmetric inflation
dynamical inflation	kinetic inflation	oscillating inflation	supersymmetric topological inflation
dynamical SUSY inflation	lambda inflation	polynomial chaotic inflation	supersymmetric new inflation
eternal inflation	large field inflation	polynomial hybrid inflation	synergistic warm inflation
extended inflation	late D-term inflation	power-law inflation	TeV-scale hybrid inflation

A (partial?) list of ever-increasing number of inflationary models[3]. Actually, it may not even be possible to rule out some of these models!

[3]From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).

Performance of models in the n_{s}-r plane
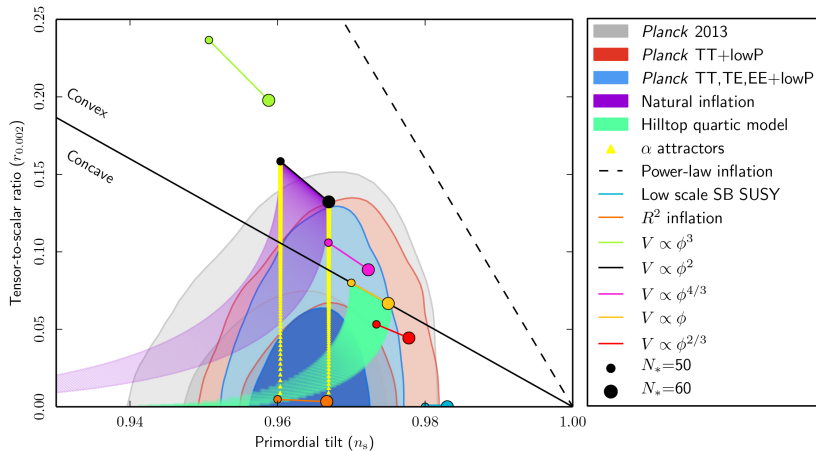
Marginalized joint $68\,\%$ and $95\,\%$ CL regions for n_{s} and $r_{0.002}$ from Planck in combination with other data sets, compared to the theoretical predictions of selected inflationary models[4].

[4] Planck Collaboration (P. A. R. Ade *et al.*), Astron. Astrophys. **594**, A20 (2016).

Performance of inflationary models against the data
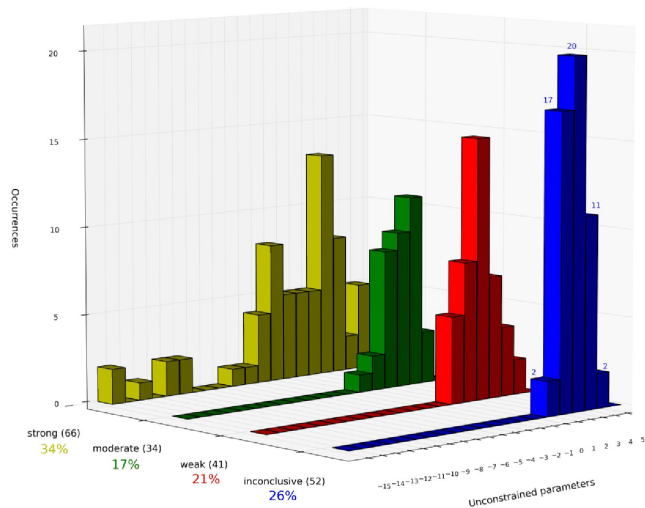
The efficiency of the inflationary paradigm leads to a situation wherein, despite the strong constraints, a variety of models continue to remain consistent with the data[5].

[5] J. Martin, C. Ringeval, R. Trotta and V. Vennin, JCAP **1403**, 039 (2014).

The nature of the cosmological perturbations

If you were to ask what current observations are telling us about the fluctuations that led to large-scale structure formation, then few would argue with the following four key properties:

- Primordial,
- Scale-invariant,
- Adiabatic,
- Gaussian.

Spelt backwards these words form a useful acronym because there would be a collective 'gasp' if any were disproved (though some may need to be qualified with 'very nearly' or 'predominantly')[6].

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[6]From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).

Features and non-Gaussianities
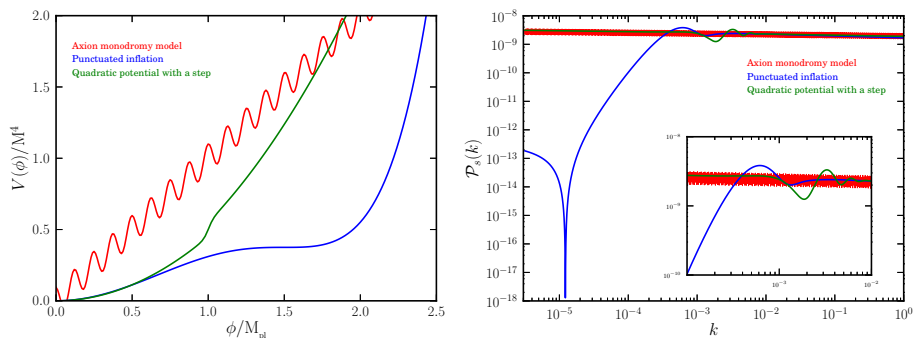
Left: Illustration of potentials that admit departures from slow roll.

Right: The scalar power spectra in different inflationary models that lead to a better fit to the CMB data than the conventional power law spectrum[7]. But, these models lead to large non-Gaussianities, typically beyond the present constraints[8].

[7] R. K. Jain, P. Chingangbam, J.-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **0901**, 009 (2009);

D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **1010**, 008 (2010);

M. Aich, D. K. Hazra, L. Sriramkumar and T. Souradeep, Phys. Rev. D **87**, 083526 (2013).

[8] D. K. Hazra, L. Sriramkumar and J. Martin, JCAP **1305**, 026, (2013).

Can inflation be falsified?

Izz all well with inflation?

Even before the Planck satellite flies we can rest assured that a single field inflation model will match almost any possible observation of the scalar and tensor power spectra!

While we await compelling observational confirmation of more specific inflationary signatures, the search for alternative explanations of these four simple fluctuation properties remains a legitimate enterprise and will continue to prove lively and controversial[a].

[a]From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).

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The difficulty with the inflationary paradigm

A theory that predicts everything predicts nothing[a].

[a]P. J. Steinhardt, Sci. Am. **304**, 36 (2011).

The possibilities seem endless!

A recent work, which modifies a currently popular model to always remain beyond the upper bounds on the tensor-to-scalar ratio!

Plan of the talk

Bouncing scenarios as an alternative paradigm[9]

✦ Bouncing models correspond to situations wherein the universe initially goes through a period of contraction until the scale factor reaches a certain minimum value before transiting to the expanding phase.

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[9]See, for instance, M. Novello and S. P. Bergliaffa, Phys. Rep. **463**, 127 (2008);
D. Battefeld and P. Peter, Phys. Rep. **571**, 1 (2015).

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- ✦ However, matter fields *will* have to violate the null energy condition near the bounce in order to give rise to such a scale factor. Also, there exist (genuine) concerns whether such an assumption about the scale factor is valid in a domain where general relativity can be supposed to fail and quantum gravitational effects are expected to take over.

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Overcoming the horizon problem in bouncing models
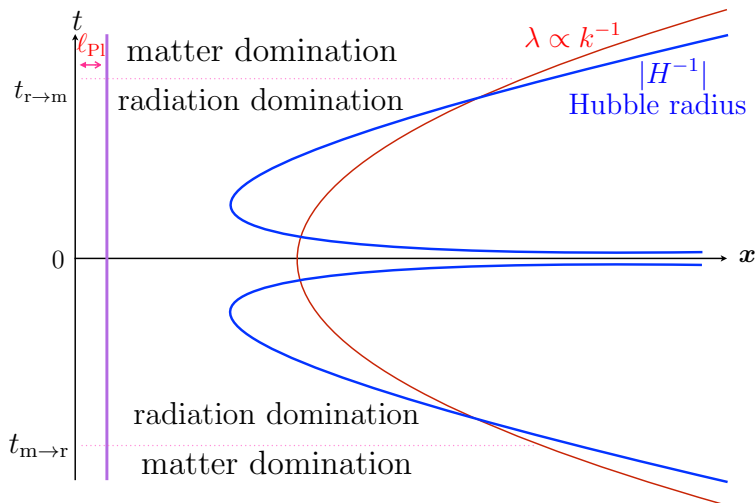
Evolution of the physical wavelength and the Hubble radius in a bouncing scenario[10]

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[10]Figure from, D. Battefeld and P. Peter, Phys. Rept. **571**, 1 (2015).

Violation of the null energy condition near the bounce

Recall that, according to the Friedmann equations

$$\dot{H} = -4\pi G \, (\rho + p).$$

In any bouncing scenario, the Hubble parameter is negative before the bounce, crosses zero at the bounce and is positive thereafter.

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It can be shown that, if the modes of cosmological interest have to be inside the Hubble radius at early times during the contracting phase, *the universe needs to undergo non-accelerated contraction*.

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It can be shown that, if the modes of cosmological interest have to be inside the Hubble radius at early times during the contracting phase, *the universe needs to undergo non-accelerated contraction*.

In such cases, one finds that $\dot{H}$ *will* be positive near the bounce, which implies that $(\rho + p)$ has to be negative in this domain. In other words, the null energy condition needs to be violated in order to achieve such bounces.

Classical bounces and sources

Consider, for instance, bouncing models of the form

$$a(\eta) = a_0 \left(1 + \frac{\eta^2}{\eta_0^2}\right)^q = a_0 \left(1 + k_0^2 \eta^2\right)^q,$$

where a_0 is the value of the scale factor at the bounce (*i.e.* when $\eta = 0$), $\eta_0 = 1/k_0$ denotes the time scale of the duration of the bounce and $q > 0$. We shall assume that the scale k_0 associated with the bounce is of the order of the Planck scale M_{Pl}.

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The above scale factor can be achieved with the help of two fluids with constant equation of state parameters $w_1 = (1-q)/(3\,q)$ and $w_2 = (2-q)/(3\,q)$. The energy densities of these fluids behave as $\rho_1 = M_1/a^{(2\,q+1)/q}$ and $\rho_2 = M_2/a^{2\,(1+q)/q}$, where $M_1 = 12\,k_0^2\,M_{\mathrm{Pl}}^2\,a_0^{1/q}$ and $M_2 = -M_1\,a_0^{1/q}$.

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Note that, when $q = 1$, during very early times wherein $\eta \ll -\eta_0$, the scale factor behaves as in a matter dominated universe (*i.e.* $a \propto \eta^2$). Therefore, the $q = 1$ case is often referred to as the matter bounce scenario. In such a case, $\rho_1 = 12\,k_0^2\,M_{\mathrm{Pl}}^2\,a_0/a^3$ and $\rho_2 = -12\,k_0^2\,M_{\mathrm{Pl}}^2\,a_0^2/a^4$.

E-$\mathcal{N}$-folds

The conventional e-fold N is defined $N = \log{(a/a_\mathrm{i})}$ so that $a(N) = a_\mathrm{i} \exp N$. However, the function e^N is a monotonically increasing function of N.

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In completely symmetric bouncing scenarios, an obvious choice for the scale factor seems to be[11]

$$a(\mathcal{N}) = a_0 \exp\left(\mathcal{N}^2/2\right),$$

with $\mathcal{N}$ being the new time variable that we shall consider for integrating the differential equation governing the background as well as the perturbations.

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We shall refer to the variable $\mathcal{N}$ as e-$\mathcal{N}$-fold since the scale factor grows roughly by the amount $\mathrm{e}^{\mathcal{N}}$ between $\mathcal{N}$ and $(\mathcal{N}+1)$.

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[11] L. Sriramkumar, K. Atmjeet and R. K. Jain, JCAP **1509**, 010 (2015).

Behavior of $\dot{H}$ and ρ in a matter bounce
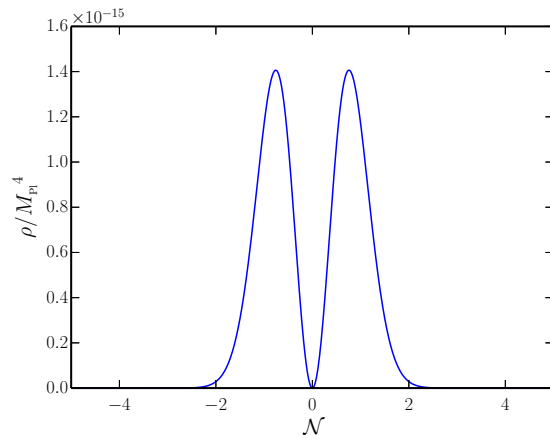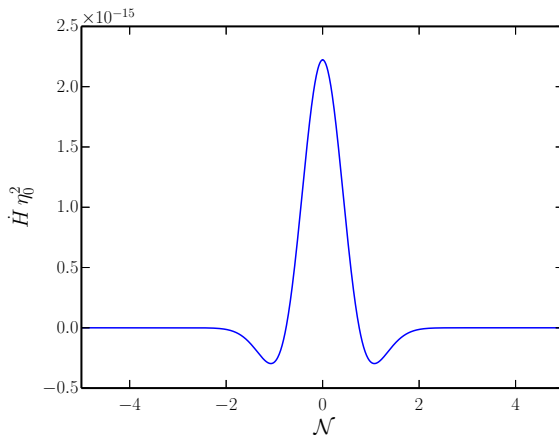
The behavior of $\dot{H}$ (on the left) and the total energy density ρ (on the right) in a symmetric matter bounce scenario has been plotted as a function of $\mathcal{N}$. Note that the maximum value of ρ is much smaller than $M_{\rm Pl}^4$, which suggests that the bounce can be treated completely classically.

▸ Back to scalar perturbations

Duality between de Sitter inflation and matter bounce

It is known that the solutions to the equations of motion governing the scalar and tensor perturbations are invariant under a certain transformation referred to as the duality transformation[12].

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For instance, recall that the Mukhanov-Sasaki variable corresponding to the tensor perturbations [which is defined as $u_k = (M_{\mathrm{Pl}}/\sqrt{2})\, a\, h_k$] satisfies the differential equation

$$u_k'' + \left(k^2 - \frac{a''}{a}\right) u_k = 0.$$

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Given a scale factor a, the corresponding dual, say, $\tilde{a}$, which leads to the same equation for the variable u_k is given by

$$a(\eta) \to \tilde{a}(\eta) = C\, a(\eta) \int\limits_{\eta_*}^{\eta} \frac{\mathrm{d}\bar{\eta}}{a^2(\bar{\eta})},$$

where C and η_* are constants.

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It is straightforward to show that the dual solution to de Sitter inflation corresponds to the matter bounce. Both these cases lead to scale invariant spectra.

[12] D. Wands, Phys. Rev. D **60**, 023507 (1999).

Plan of the talk

The matter bounce

We shall assume that the scale factor describing the bouncing scenario is given in terms of the conformal time coordinate η by the relation

$$a(\eta) = a_0 \left(1 + \eta^2/\eta_0^2\right) = a_0 \left(1 + k_0^2\,\eta^2\right).$$

As we had discussed earlier, at very early times, *viz.* when $\eta \ll -\eta_0$, the scale factor behaves as in a matter dominated epoch[13].

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The quantity a''/a corresponding to the above scale factor is given by

$$\frac{a''}{a} = \frac{2 k_0^2}{1 + k_0^2 \eta^2},$$

which is essentially a Lorentzian profile.

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The behavior of a''/a
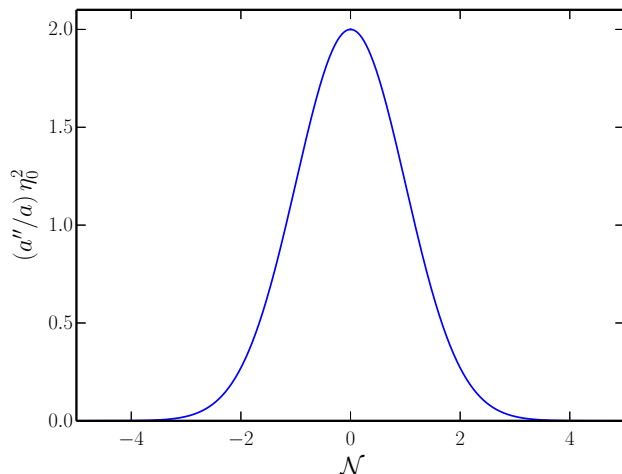
The behavior of the quantity a''/a has been plotted as a function of $\mathcal{N}$ for the matter bounce scenario of interest. Note that the maximum value of a''/a is of the order of k_0^2.

The tensor modes in the first domain

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In the first domain, we can assume that the scale factor behaves as $a(\eta) \simeq a_0\,k_0^2\,\eta^2$, so that $a''/a \simeq 2/\eta^2$. Since the condition $k^2 = a''/a$ corresponds to, say, $\eta_k = -\sqrt{2}/k$, the initial conditions can be imposed when $\eta \ll \eta_k$.

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The modes h_k can be easily obtained in such a case and the positive frequency modes that correspond to the vacuum state at early times are given by

$$h_k(\eta) = \frac{\sqrt{2}}{M_{\mathrm{Pl}}} \frac{1}{\sqrt{2\,k}} \frac{1}{a_0\,k_0^2\,\eta^2} \left(1 - \frac{i}{k\,\eta}\right) \mathrm{e}^{-i\,k\,\eta}.$$

The modes in the second domain

Let us now consider the behavior of the modes in the domain $-\alpha\,\eta_0 < \eta < \beta\,\eta_0$, where, say, $\beta \simeq 10^2$. Since we are interested in scales much smaller than k_0, we shall assume that $\eta_k \ll -\alpha\,\eta_0$, which corresponds to $k \ll k_0/\alpha$.

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In such a case, upon ignoring the k^2 term, the equation governing h_k can be immediately integrated to yield

$$h_k(\eta) \simeq h_k(\eta_*) + h_k'(\eta_*)\, a^2(\eta_*) \int_{\eta_*}^{\eta} \frac{\mathrm{d}\tilde\eta}{a^2(\tilde\eta)},$$

where η_* is a suitably chosen time and the scale factor $a(\eta)$ is given by the complete expression.

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Let us now consider the behavior of the modes in the domain $-\alpha\,\eta_0 < \eta < \beta\,\eta_0$, where, say, $\beta \simeq 10^2$. Since we are interested in scales much smaller than k_0, we shall assume that $\eta_k \ll -\alpha\,\eta_0$, which corresponds to $k \ll k_0/\alpha$.

In such a case, upon ignoring the k^2 term, the equation governing h_k can be immediately integrated to yield

$$h_k(\eta) \simeq h_k(\eta_*) + h_k'(\eta_*)\,a^2(\eta_*) \int_{\eta_*}^{\eta} \frac{\mathrm{d}\tilde{\eta}}{a^2(\tilde{\eta})},$$

where η_* is a suitably chosen time and the scale factor $a(\eta)$ is given by the complete expression.

If we choose $\eta_* = -\alpha\,\eta_0$, we can make use of the solution in the first domain to determine the constants and express the solution in the second domain as follows:

$$h_k = \mathcal{A}_k + \mathcal{B}_k\,f(k_0\,\eta),$$

where the function $f(k_0\,\eta)$ is given by

$$f(k_0\,\eta) = \frac{k_0\,\eta}{1 + k_0^2\,\eta^2} + \tan^{-1}(k_0\,\eta).$$

Evolution of the tensor modes across the bounce
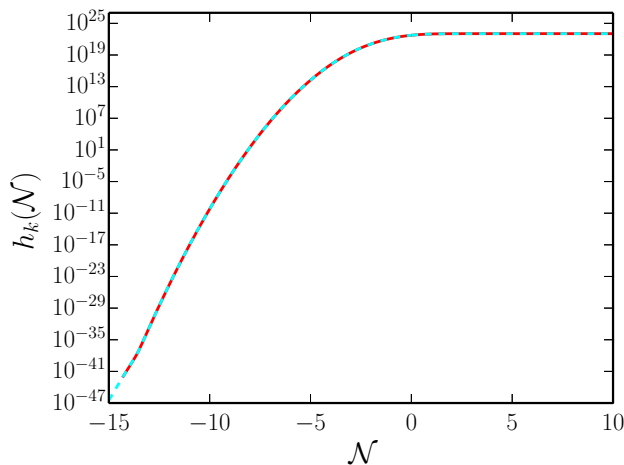
A comparison of the numerical results (in solid red) with the analytical results (in dashed cyan) for the amplitude of the tensor mode $|h_k|$ corresponding to $k/k_0 = 10^{-20}$. We have set $k_0 = M_{\rm Pl}$, $a_0 = 3 \times 10^7$, and we have chosen $\alpha = 10^5$ for plotting the analytical results.[14]

[14] D. Chowdhury, V. Sreenath and L. Sriramkumar, JCAP **1511**, 002 (2015).

The tensor power spectrum after the bounce

The quantities $\mathcal{A}_k$ and $\mathcal{B}_k$ are given by

$$
\begin{aligned}
\mathcal{A}_k &= \frac{\sqrt{2}}{M_{\mathrm{Pl}}} \frac{1}{\sqrt{2\,k}} \frac{1}{a_0\,\alpha^2} \left(1 + \frac{i\,k_0}{\alpha\,k}\right) e^{i\,\alpha\,k/k_0} + \mathcal{B}_k\, f(\alpha), \\
\mathcal{B}_k &= \frac{\sqrt{2}}{M_{\mathrm{Pl}}} \frac{1}{\sqrt{2\,k}} \frac{1}{2\,a_0\,\alpha^2} \left(1 + \alpha^2\right)^2 \left(\frac{3\,i\,k_0}{\alpha^2\,k} + \frac{3}{\alpha} - \frac{i\,k}{k_0}\right) e^{i\,\alpha\,k/k_0}.
\end{aligned}
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\end{aligned}
$$

If we evaluate the tensor power spectrum after the bounce at $\eta = \beta\,\eta_0$, we find that it can be expressed as

$$
\mathcal{P}_{\text{T}}(k) = 4\,\frac{k^3}{2\,\pi^2}\,|\mathcal{A}_k + \mathcal{B}_k\,f(\beta)|^2.
$$

The tensor power spectrum
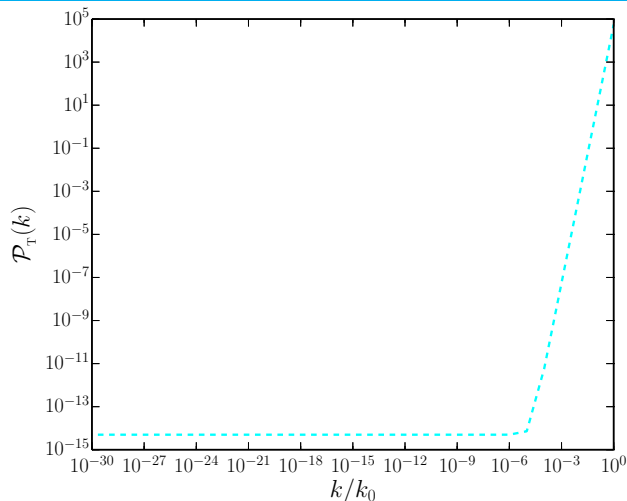
The behavior of the tensor power spectrum has been plotted as a function of k/k_0 for a wide range of wavenumbers. In plotting this figure, we have set $k_0 = M_{\mathrm{Pl}}$, $a_0 = 3 \times 10^{...}$ $\alpha = 10^5$ and $\beta = 10^2$. Note that the power spectrum is scale invariant for $k \ll k_0/\alpha$.

Plan of the talk

A new model for the completely symmetric matter bounce

As we had discussed, the matter bounce scenario described by the scale factor

$$a(\eta) = a_0 \left(1 + \eta^2/\eta_0^2\right) = a_0 \left(1 + k_0^2 \, \eta^2\right)$$

can be driven with the aid of two fluids, one which is matter and another fluid which behaves like radiation, but has negative energy density.

[15] R. N. Raveendran, D. Chowdhury and L. Sriramkumar, arXiv:1703.10061v1 [gr-qc].

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can be driven with the aid of two fluids, one which is matter and another fluid which behaves like radiation, but has negative energy density.

We find that the behavior can also be achieved with the help of two scalar fields, say, ϕ and χ, that are governed by the following action[15]:

$$S[\phi, \chi] = -\int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2}\,\partial_\mu \phi\,\partial^\mu \phi + V(\phi) + U_0 \left(-\frac{1}{2}\,\partial_\mu \chi\,\partial^\mu \chi\right)^2\right],$$

where U_0 is a constant and the potential $V(\phi)$ is given by

$$V(\phi) = \frac{6\,M_{\mathrm{Pl}}^2\,(k_0^2/a_0^2)}{\cosh^6[\phi/(\sqrt{12}\,M_{\mathrm{Pl}})]}.$$

[15]R. N. Raveendran, D. Chowdhury and L. Sriramkumar, arXiv:1703.10061v1 [gr-qc].

The scalar perturbations

When the scalar perturbations are taken into account, the FLRW line element can be written as

$$\mathrm{d}s^2 = -(1 + 2\,A)\,\mathrm{d}t^2 + 2\,a(t)\,(\partial_i B)\,\mathrm{d}t\,\mathrm{d}x^i + a^2(t)\,[(1 - 2\,\psi)\,\delta_{ij} + 2\,(\partial_i\,\partial_j E)]\,\mathrm{d}x^i\,\mathrm{d}x^j,$$

where, evidently, the quantities A, ψ, B and E represent the metric perturbations.

---

[16]R. N. Raveendran, D. Chowdhury and L. Sriramkumar, In preparation.

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The gauge invariant curvature and isocurvature perturbations $\mathcal{R}$ and $\mathcal{S}$ can be defined in terms of the above metric perturbations and the perturbations $\delta\phi$ and $\delta\chi$ in the scalar fields as follows[16]:

$$\mathcal{R} = \frac{H}{\dot{\phi}^2 - U_0\,\dot{\chi}^4}\,\left(\dot{\phi}\,\overline{\delta\phi} - U_0\,\dot{\chi}^3\,\overline{\delta\chi}\right), \quad \mathcal{S} = \frac{H\,\sqrt{\alpha\,\dot{\chi}^2}}{\dot{\phi}^2 - U_0\,\dot{\chi}^4}\,\left(\dot{\chi}\,\overline{\delta\phi} - \dot{\phi}\,\overline{\delta\chi}\right).$$

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The quantities $\overline{\delta\phi}$ and $\overline{\delta\chi}$ denote the gauge invariant versions of the perturbations in the scalar fields, and are given by

$$\overline{\delta\phi} = \delta\phi + \frac{\dot{\phi}\,\psi}{H}, \qquad \overline{\delta\chi} = \delta\chi + \frac{\dot{\chi}\,\psi}{H}.$$

---

[16] R. N. Raveendran, D. Chowdhury and L. Sriramkumar, In preparation.

Equations governing the curvature and isocurvature perturbations

We obtain the equations of motion describing the gauge invariant perturbations $\mathcal{R}_k$ and $\mathcal{S}_k$ in our model to be

$$\mathcal{R}_k'' + \frac{2\left(7 + 9\,k_0^2\,\eta^2 - 6\,k_0^4\,\eta^4\right)}{\eta\,\left(1 - 3\,k_0^2\,\eta^2\right)^2\,\left(1 + k_0^2\,\eta^2\right)}\,\mathcal{R}_k' - \frac{k^2\left(5 + 9\,k_0^2\,\eta^2\right)}{3\left(1 - 3\,k_0^2\,\eta^2\right)}\,\mathcal{R}_k$$

$$= \frac{4\left(5 + 12\,k_0^2\,\eta^2\right)}{\sqrt{3}\,\eta\left(1 - 3\,k_0^2\,\eta^2\right)\sqrt{1 + k_0^2\,\eta^2}}\,\mathcal{S}_k' - \frac{4\left[5 - 22\,k_0^2\,\eta^2 - 24\,k_0^4\,\eta^4 + k^2\,\eta^2\left(1 + k_0^2\,\eta^2\right)^2\right]}{\sqrt{3}\,\eta^2\left(1 + k_0^2\,\eta^2\right)^{3/2}\left(1 - 3\,k_0^2\,\eta^2\right)}\,\mathcal{S}_k,$$

$$\mathcal{S}_k'' - \frac{2\left(9 + 7\,k_0^2\,\eta^2 + 6\,k_0^4\,\eta^4\right)}{\eta\,\left(1 - 3\,k_0^2\,\eta^2\right)\left(1 + k_0^2\,\eta^2\right)}\,\mathcal{S}_k'$$

$$- \frac{18 - 85\,k_0^2\,\eta^2 - 25\,k_0^4\,\eta^4 - 6\,k_0^6\,\eta^6 + k^2\,\eta^2\left(3 - k_0^2\,\eta^2\right)\left(1 + k_0^2\,\eta^2\right)^2}{\eta^2\left(1 - 3\,k_0^2\,\eta^2\right)\left(1 + k_0^2\,\eta^2\right)^2}\,\mathcal{S}_k$$

$$= \frac{4\sqrt{3}\left(3 - 2\,k_0^2\,\eta^2\right)}{\eta\,\sqrt{1 + k_0^2\,\eta^2}\left(1 - 3\,k_0^2\,\eta^2\right)}\,\mathcal{R}_k' + \frac{4\,k^2\,\sqrt{1 + k_0^2\,\eta^2}}{\sqrt{3}\left(1 - 3\,k_0^2\,\eta^2\right)}\,\mathcal{R}_k. \qquad \text{▸ Behavior of } \dot{H}$$

However, some of the coefficients diverge when $\dot{H}$ and/or H vanish.

The uniform-χ gauge

The issue of diverging coefficients can be avoided by working in a gauge wherein $\delta\chi = 0$[17].

[17]L. E. Allen and D. Wands, Phys. Rev. **70**, 063515 (2004).

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In this gauge, the equations of motion for the metric perturbations A_k and ψ_k can be obtained to be

$$A_k'' + 4\,\mathcal{H}\,A_k' + \left(\frac{k^2}{3} - \frac{20\,a_0^2\,k_0^2}{a^2}\right)\,A_k = -3\,\mathcal{H}\,\psi_k' + \frac{4\,k^2}{3}\,\psi_k,$$

$$\psi_k'' - 2\,\mathcal{H}\,\psi_k' + k^2\,\psi_k = 2\,\mathcal{H}\,A_k' - \frac{20\,a_0^2\,k_0^2}{a^2}\,A_k,$$

where $\mathcal{H} = a'/a$. These equations prove to be helpful in evolving the scalar perturbations across the bounce.

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where $\mathcal{H} = a'/a$. These equations prove to be helpful in evolving the scalar perturbations across the bounce.

Also, in the uniform-χ gauge, the curvature and isocurvature perturbations simplify to be

$$\mathcal{R}_k = \psi_k + \frac{2\,H\,M_{\text{Pl}}^2}{\dot{\phi}^2 - U_0\,\dot{\chi}^4}\left(\dot{\psi}_k + H\,A_k\right), \quad \mathcal{S}_k = \frac{2\,H\,M_{\text{Pl}}^2\,\sqrt{U_0\,\dot{\chi}^4}}{\left(\dot{\phi}^2 - U_0\,\dot{\chi}^4\right)\dot{\phi}}\left(\dot{\psi}_k + H\,A_k\right).$$

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Solutions for $\mathcal{R}_k$ and $\mathcal{S}_k$ in the first domain

As in the case of tensors, we shall be interested in evaluating the power spectrum after the bounce at $\eta = \beta\,\eta_0$. Also, to arrive at the analytical approximations, as earlier, we shall divide period of interest into two domains, *viz.* $-\infty < \eta < -\alpha\,\eta_0$ and $-\alpha\,\eta_0 < \eta < \beta\,\eta_0$.

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In the first domain, we find that the solution to the curvature perturbation can be arrived at as in the case of tensors and is given by

$$\mathcal{R}_k(\eta) \simeq \frac{1}{\sqrt{6 \, k} \, M_{_{\mathrm{Pl}}} \, a_0 \, k_0^2 \, \eta^2} \left(1 - \frac{i}{k \, \eta}\right) \, \mathrm{e}^{-i \, k \, \eta}.$$

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Using this solution, it is straightforward to obtain the following solution for the isocurvature perturbation at early times:

$$\begin{aligned}
\mathcal{S}_k(\eta) &\simeq \frac{1}{9\,\sqrt{2\,k^3}\,a_0\,k_0^3\,M_{_{\mathrm{Pl}}}\eta^4}\left(-12\,i\,(1 + i\,k\,\eta)\,\mathrm{e}^{-i\,k\,\eta} + \frac{9}{3^{1/4}}\,k\,k_0\,\eta^2\,\mathrm{e}^{-i\,k\,\eta/\sqrt{3}}\right. \\
&\quad \left. + 4\,k^2\,\eta^2\,\mathrm{e}^{-i\,k\,\eta/\sqrt{3}}\left\{\pi + i\,\mathrm{Ei}\left[\mathrm{e}^{-i\,(3-\sqrt{3})\,k\,\eta/3}\right]\right\}\right).
\end{aligned}$$

Solutions for ψ_k and A_k in the second domain

In the second domain, upon ignoring the k^2 dependent terms, one finds that the combination $A_k + \psi_k$ satisfies the same equation of motion as the tensor modes.

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This feature helps us obtain the solutions for A_k and ψ_k, and they are given by

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\begin{aligned}
A_k(\eta) + \psi_k(\eta) &\simeq \frac{\mathcal{C}_k}{2\,a_0^2}\,f(k_0\,\eta) + \mathcal{D}_k, \\
A_k(\eta) &\simeq \frac{\mathcal{C}_k\,k_0\,\eta}{4\,a_0^2\,(1 + k_0^2\,\eta^2)} + \mathcal{E}_k\,\mathrm{e}^{-2\sqrt{5}\,\tan^{-1}(k_0\,\eta)} + \mathcal{F}_k\,\mathrm{e}^{2\sqrt{5}\,\tan^{-1}(k_0\,\eta)},
\end{aligned}
$$

where $f(k_0\,\eta)$ is the same function that we had encountered earlier in the case of tensors, and $\mathcal{C}_k$, $\mathcal{D}_k$, $\mathcal{E}_k$ and $\mathcal{F}_k$ are four constants of integration. ▶ Function $f(k_0\,\eta)$

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$$A_k(\eta) \simeq \frac{\mathcal{C}_k\,k_0\,\eta}{4\,a_0^2\,(1 + k_0^2\,\eta^2)} + \mathcal{E}_k\,\mathrm{e}^{-2\sqrt{5}\,\tan^{-1}(k_0\,\eta)} + \mathcal{F}_k\,\mathrm{e}^{2\sqrt{5}\,\tan^{-1}(k_0\,\eta)},$$

where $f(k_0\,\eta)$ is the same function that we had encountered earlier in the case of tensors, and $\mathcal{C}_k$, $\mathcal{D}_k$, $\mathcal{E}_k$ and $\mathcal{F}_k$ are four constants of integration. ▶ Function $f(k_0\,\eta)$

The four constants, *viz.* $\mathcal{C}_k$, $\mathcal{D}_k$, $\mathcal{E}_k$ and $\mathcal{F}_k$, are determined by matching the above solutions with the solutions for $\mathcal{R}_k$ and $\mathcal{S}_k$ in the first domain at $\eta = -\alpha\,\eta_0$.

Evolution of $\mathcal{R}_k$, $\mathcal{S}_k$ and h_k
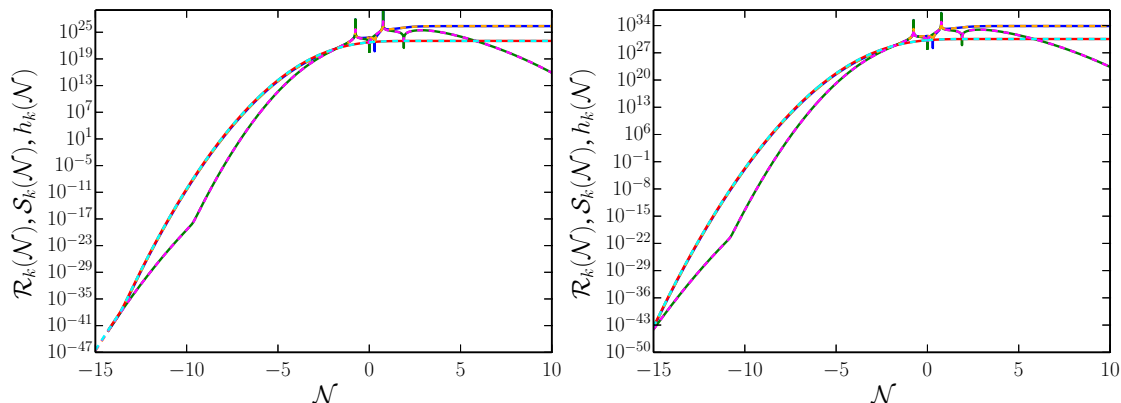
The evolution of the curvature, isocurvature and tensor perturbations, *viz.* $\mathcal{R}_k$ (in blue and orange), $\mathcal{S}_k$ (in green and magenta) and h_k (in red and cyan) across the bounce for the modes $k/k_0 = 10^{-20}$ (on the left) and $k/k_0 = 10^{-25}$ (on the right). We have set $k_0/(a_0\,M_{\mathrm{Pl}}) \simeq 3 \times 10^{-8}$, $\alpha = 10^5$ and $\beta = 10^2$. The solid lines denote the results obtained numerically, while the dashed lines represent the analytical approximations.

The scalar and tensor power spectra in the matter bounce
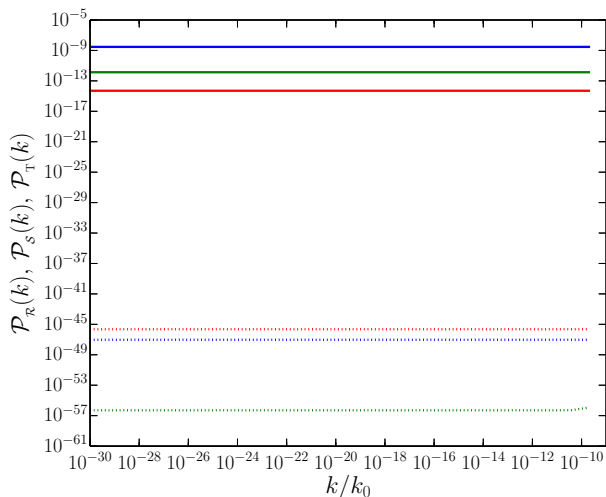
The scalar (curvature as blue and isocurvature as green) and tensor (as red) power spectra have been plotted before (as dotted lines) as well as after (as solid lines) the bounce.[18]

[18] R. N. Raveendran, D. Chowdhury and L. Sriramkumar, arXiv:1703.10061v1 [gr-qc].

The evolution of the tensor-to-scalar ratio
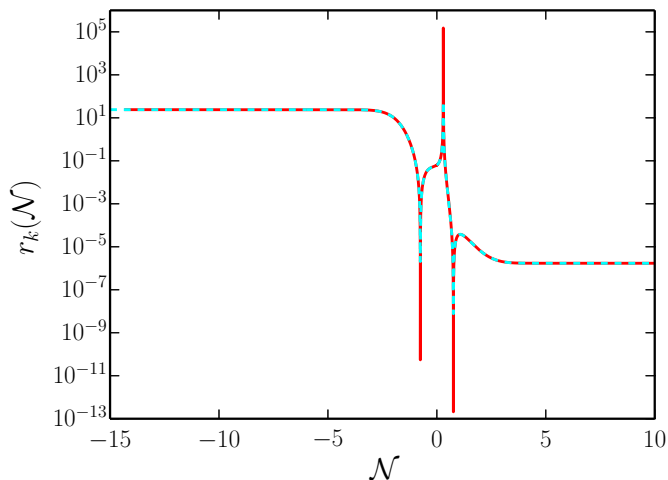
The evolution of the tensor-to-scalar ratio r across the symmetric matter bounce for a typical mode of cosmological interest. The solid (in red) and dashed (in cyan) lines represent the numerical and analytical results, respectively.

Plan of the talk

Near-matter bounces

Near-matter bounces can be described by the scale factor

$$a(\eta) = a_0 \left(1 + k_0^2 \, \eta^2\right)^{(1+\lambda)}$$

and, one finds that the index λ leads to a tilt in the power spectra.

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Such a scale factor can also be achieved with the aid of two scalar fields, say, ϕ and χ, governed by the action[19]

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where U_0 is a constant, $\alpha = (2 + \lambda)/(1 + \lambda)$ and the potential $V(\phi)$ is given by

$$V(\phi) = \frac{2 \, (3 + 4 \, \lambda)}{1 + \lambda} \left(\frac{M_{\mathrm{Pl}} \, k_0 \, (1 + \lambda)}{a_0}\right)^2 \cosh^{-2 \, (3 + 2 \, \lambda)} \left(\frac{\phi}{2 \, \sqrt{(1 + \lambda) \, (3 + 2 \, \lambda)} \, M_{\mathrm{Pl}}}\right).$$

---

[19] R. N. Raveendran and L. Sriramkumar, in preparation.

Red-tilted scalar and tensor power spectra in a near-matter bounce
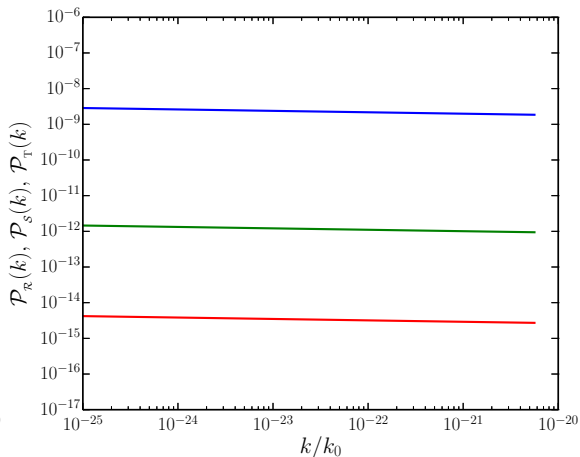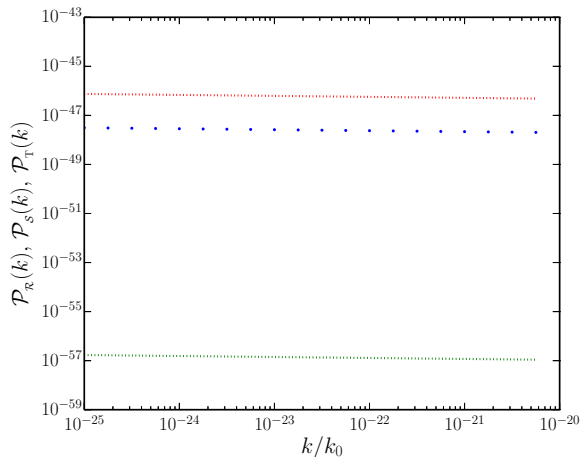
The scalar (curvature as blue and isocurvature as green) and tensor (as red) power spectra for $\lambda \simeq 10^{-2}$ (corresponding to a scalar spectral tilt of $n_s = 0.96$) have been plotted before (as dotted lines, on the left) and after (as solid lines, on the right) the bounce[20].

[20] R. N. Raveendran, and L. Sriramkumar, in preparation.

Plan of the talk

Tensor bispectrum and non-Gaussianity parameter

The tensor bispectrum, evaluated at the conformal time, say, η_{e}, is defined as

$$\langle \hat{\gamma}^{\boldsymbol{k}_1}_{m_1 n_1}(\eta_{\mathrm{e}})\, \hat{\gamma}^{\boldsymbol{k}_2}_{m_2 n_2}(\eta_{\mathrm{e}})\, \hat{\gamma}^{\boldsymbol{k}_3}_{m_3 n_3}(\eta_{\mathrm{e}}) \rangle = (2\,\pi)^3\, \mathcal{B}^{m_1 n_1 m_2 n_2 m_3 n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$$
$$\times\, \delta^{(3)}\left(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3\right)$$

and, for convenience, we shall set

$$\mathcal{B}^{m_1 n_1 m_2 n_2 m_3 n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = (2\,\pi)^{-9/2}\, G^{m_1 n_1 m_2 n_2 m_3 n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3).$$

---

[21] V. Sreenath, R. Tibrewala and L. Sriramkumar, JCAP **1312**, 037 (2013).

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As in the scalar case, one can define a dimensionless non-Gaussianity parameter to characterize the amplitude of the tensor bispectrum as follows[21]:

$$h_{\mathrm{NL}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -\left(\frac{4}{2\,\pi^2}\right)^2 \left[k_1^3 \, k_2^3 \, k_3^3 \, G^{m_1 n_1 m_2 n_2 m_3 n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\right]$$
$$\times \left[\Pi^{\boldsymbol{k}_1}_{m_1 n_1, m_2 n_2} \, \Pi^{\boldsymbol{k}_2}_{m_3 n_3, \bar{m}\bar{n}} \, k_3^3 \, \mathcal{P}_{\mathrm{T}}(k_1) \, \mathcal{P}_{\mathrm{T}}(k_2) + \text{five permutations}\right]^{-1}.$$

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[21] V. Sreenath, R. Tibrewala and L. Sriramkumar, JCAP **1312**, 037 (2013).

The third order action and the tensor bispectrum

The third order action that leads to the tensor bispectrum is given by[22]

$$S_{\gamma\gamma\gamma}^3[\gamma_{ij}] = \frac{M_{_{\mathrm{Pl}}}^2}{2} \int \mathrm{d}\eta \int \mathrm{d}^3\boldsymbol{x} \left[\frac{a^2}{2} \gamma_{lj} \gamma_{im} \partial_l\partial_m\gamma_{ij} - \frac{a^2}{4} \gamma_{ij} \gamma_{lm} \partial_l\partial_m\gamma_{ij} \right].$$

[22] J. Maldacena, JHEP **0305**, 013 (2003).

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The tensor bispectrum calculated in the perturbative vacuum using the Maldacena formalism, can be written in terms of the modes h_k as follows:

$$G_{\gamma\gamma\gamma}^{m_1 n_1 m_2 n_2 m_3 n_3}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$$

$$= M_{\mathrm{Pl}}^2 \left[\left(\Pi_{m_1 n_1, ij}^{\boldsymbol{k}_1} \Pi_{m_2 n_2, im}^{\boldsymbol{k}_2} \Pi_{m_3 n_3, lj}^{\boldsymbol{k}_3} - \frac{1}{2} \Pi_{m_1 n_1, ij}^{\boldsymbol{k}_1} \Pi_{m_2 n_2, ml}^{\boldsymbol{k}_2} \Pi_{m_3 n_3, ij}^{\boldsymbol{k}_3}\right) k_{1m} k_{1l}\right.$$

$$\left. + \text{ five permutations}\right]$$

$$\times \left[h_{k_1}(\eta_{\mathrm{e}}) h_{k_2}(\eta_{\mathrm{e}}) h_{k_3}(\eta_{\mathrm{e}}) \mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + \text{complex conjugate}\right],$$

where $\mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ is described by the integral

$$\mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -\frac{i}{4} \int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}} \mathrm{d}\eta \, a^2 \, h_{k_1}^* \, h_{k_2}^* \, h_{k_3}^*,$$

with η_{i} denoting the time when the initial conditions are imposed on the perturbations.

[22] J. Maldacena, JHEP **0305**, 013 (2003).

The contributions due to the three domains
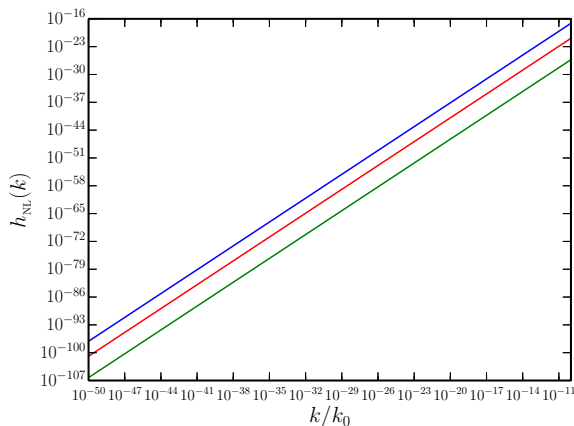
The contributions to the non-Gaussianity parameter h_{NL} in the equilateral limit from the first (in green), the second (in red) and the third (in blue) domains have been plotted as a function of k/k_0 for $k \ll k_0/\alpha$. Clearly, the third domain gives rise to the maximum contribution to h_{NL} [23].

[23] D. Chowdhury, V. Sreenath and L. Sriramkumar, JCAP **1511**, 002 (2015)

The effect of the long wavelength tensor modes

Since the amplitude of a long wavelength mode freezes on super-Hubble scales during inflation, such modes can be treated as a background as far as the smaller wavelength modes are concerned. Let us denote the constant amplitude of the long wavelength tensor mode as γ_{ij}^{B}.

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[24] S. Kundu, JCAP **1404**, 016 (2014).

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In the presence of such a long wavelength mode, the background FLRW metric can be written as

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,[\mathrm{e}^{\gamma^{\mathrm{B}}}]_{ij}\,\mathrm{d}\boldsymbol{x}^i\,\mathrm{d}\boldsymbol{x}^j,$$

i.e. the spatial coordinates are modified according to a spatial transformation of the form $\boldsymbol{x}' = \Lambda\,\boldsymbol{x}$, where $\Lambda_{ij} = [\mathrm{e}^{\gamma^{\mathrm{B}}/2}]_{ij}$.

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Under such a spatial transformation, the small wavelength tensor perturbation transforms as[24]

$$\gamma_{ij}^{\boldsymbol{k}} \to \det{(\Lambda^{-1})} \, \gamma_{ij}^{\Lambda^{-1}\,\boldsymbol{k}},$$

where $\det{(\Lambda^{-1})} = 1$.

---

[24] S. Kundu, JCAP **1404**, 016 (2014).

The behavior of the two and three-point functions

On using the above results, one finds that the tensor two-point function in the presence of a long wavelength mode denoted by, say, the wavenumber k, can be written as

$$\langle \hat{\gamma}^{\boldsymbol{k}_1}_{m_1 n_1} \hat{\gamma}^{\boldsymbol{k}_2}_{m_2 n_2} \rangle_k = \frac{(2\pi)^2}{2 k_1^3} \frac{\Pi^{\boldsymbol{k}_1}_{m_1 n_1, m_2 n_2}}{4} \mathcal{P}_{\mathrm{T}}(k_1) \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2)$$
$$\times \left[1 - \left(\frac{n_{\mathrm{T}} - 3}{2} \right) \gamma^{\mathrm{B}}_{ij} \hat{n}_{1i} \hat{n}_{1j} \right],$$

where $\hat{n}_{1i} = k_{1i}/k_1$.

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[25] V. Sreenath and L. Sriramkumar, JCAP **1410**, 021 (2014).

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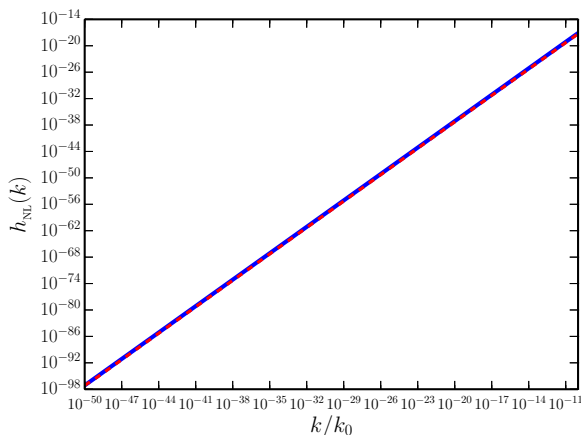
where $\hat{n}_{1i} = k_{1i}/k_1$.

One can also show that, in the presence of a long wavelength mode, the tensor bispectrum can be written as[25]

$$\langle \hat{\gamma}^{\boldsymbol{k}_1}_{m_1 n_1} \hat{\gamma}^{\boldsymbol{k}_2}_{m_2 n_2} \hat{\gamma}^{\boldsymbol{k}_3}_{m_3 n_3} \rangle_{k_3} = -\frac{(2\pi)^{5/2}}{4 k_1^3 k_3^3} \left(\frac{n_{\mathrm{T}} - 3}{32} \right) \mathcal{P}_{\mathrm{T}}(k_1) \, \mathcal{P}_{\mathrm{T}}(k_3)$$
$$\times \Pi^{\boldsymbol{k}_1}_{m_1 n_1, m_2 n_2} \Pi^{\boldsymbol{k}_3}_{m_3 n_3, ij} \, \hat{n}_{1i} \, \hat{n}_{1j} \, \delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2).$$

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[25]V. Sreenath and L. Sriramkumar, JCAP **1410**, 021 (2014).

The complete contribution to $h_{\rm NL}$

The behavior of $h_{\rm NL}$ in the equilateral (in blue) and the squeezed (in red) limits plotted as a function of k/k_0 for $k \ll k_0/\alpha$. The resulting $h_{\rm NL}$ is considerably small when compared to the values that arise in de Sitter inflation wherein $3/8 \lesssim h_{\rm NL} \lesssim 1/2$. Moreover, we find that $h_{\rm NL}$ behaves as k^2 in the equilateral and the squeezed limits, with similar amplitudes[26].

[26] D. Chowdhury, V. Sreenath and L. Sriramkumar, JCAP **1511**, 002 (2015).

Plan of the talk

Summary

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● In this work, we have been able to construct a *completely* symmetric matter bounce scenario that leads to nearly scale invariant spectra and a tensor-to-scalar ratio that is consistent with the observations.

● It is also important to examine if the non-Gaussianities generated in such models are in agreement with the recent constraints from Planck.

Issues confronting bouncing models

✦ In inflation, any classical perturbations present at the start will decay. In contrast, they grow strongly in bouncing models. So, these need to be assumed to be rather small if smooth bounces have to begin.

[27] L. E. Allen and D. Wands, Phys. Rev. **70**, 063515 (2004).

[28] Y-F. Cai, R. Brandenberger and X. Zhang, Phys. Letts. B **703**, 25 (2011).

[29] J. Quintin, Z. Sherkatghanad, Y-F. Cai and R. Brandenberger, Phys. Rev. D **92**, 062532 (2015).

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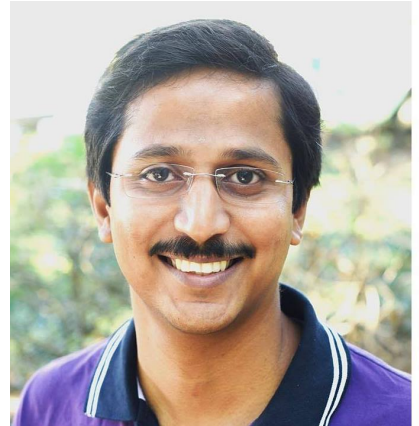
Collaborators: current and former students

Rathul Nath Raveendran   Debika Chowdhury   V. Sreenath

Thank you for your attention