Viable tensor-to-scalar ratio in a symmetric matter bounce

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Aspects of Early Universe Cosmology

Saha Institute of Nuclear Physics, Kolkata

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Plan of the talk

This talk is based on. . .

- D. Chowdhury, V. Sreenath and L. Sriramkumar, *The tensor bispectrum in a matter bounce*, JCAP **1511**, 002 (2015) [arXiv:1506.06475 [astro-ph.CO]].
- R. N. Raveendran, D. Chowdhury and L. Sriramkumar, *Viable tensor-to-scalar ratio in a symmetric matter bounce*, In preparation.

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$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\boldsymbol{x}^2 = a^2(\eta)\,\left(-\mathrm{d}\eta^2 + \mathrm{d}\boldsymbol{x}^2\right),$$

where t is the cosmic time, $a(t)$ is the scale factor and $\eta = \int \mathrm{d}t/a(t)$ denotes the conformal time coordinate.

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- We shall denote differentiation with respect to the cosmic and the conformal times t and η by an overdot and an overprime, respectively.
- Further, as usual, $H = \dot{a}/a$ shall denote the Hubble parameter associated with the FLRW universe.

Plan of the talk

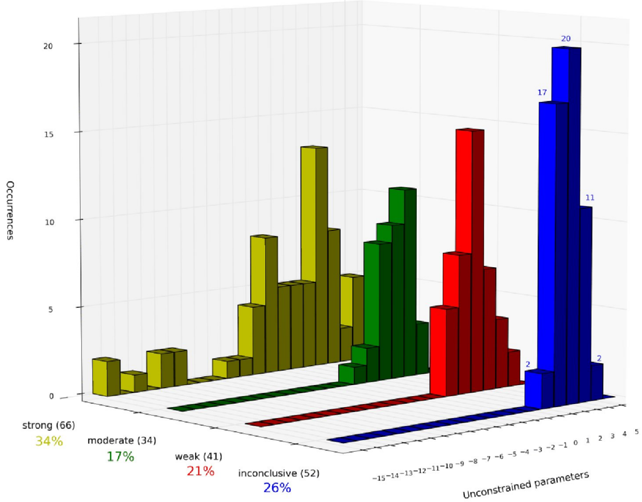
Proliferation of inflationary models

5-dimensional assisted inflation
anisotropic brane inflation
anomaly-induced inflation
assisted inflation
assisted chaotic inflation
boundary inflation
brane inflation
brane-assisted inflation
brane gas inflation
brane-antibrane inflation
braneworld inflation
Brans-Dicke chaotic inflation
Brans-Dicke inflation
bulky brane inflation
chaotic hybrid inflation
chaotic inflation
chaotic new inflation
D-brane inflation
D-term inflation
dilaton-driven inflation
dilaton-driven brane inflation
double inflation
double D-term inflation
dual inflation
dynamical inflation
dynamical SUSY inflation
eternal inflation
extended inflation
extended open inflation
extended warm inflation
extra dimensional inflation
F-term inflation
F-term hybrid inflation
false vacuum inflation
false vacuum chaotic inflation
fast-roll inflation
first order inflation
gauged inflation
generalised inflation
generalized assisted inflation
generalized slow-roll inflation
gravity driven inflation
Hagedorn inflation
higher-curvature inflation
hybrid inflation
hyperextended inflation
induced gravity inflation
induced gravity open inflation
intermediate inflation
inverted hybrid inflation
isocurvature inflation
K inflation
kinetic inflation
lambda inflation
large field inflation
late D-term inflation
late-time mild inflation
low-scale inflation
low-scale supergravity inflation
M-theory inflation
mass inflation
massive chaotic inflation
moduli inflation
multi-scalar inflation
multiple inflation
multiple-field slow-roll inflation
multiple-stage inflation
natural inflation
natural Chaotic inflation
natural double inflation
natural supergravity inflation
new inflation
next-to-minimal supersymmetric hybrid inflation
non-commutative inflation
non-slow-roll inflation
nonminimal chaotic inflation
old inflation
open hybrid inflation
open inflation
oscillating inflation
polynomial chaotic inflation
polynomial hybrid inflation
power-law inflation
pre-Big-Bang inflation
primary inflation
primordial inflation
quasi-open inflation
quintessential inflation
R-invariant topological inflation
rapid asymmetric inflation
running inflation
scalar-tensor gravity inflation
scalar-tensor stochastic inflation
Seiberg-Witten inflation
single-bubble open inflation
spinodal inflation
stable starobinsky-type inflation
steady-state eternal inflation
steep inflation
stochastic inflation
string-forming open inflation
successful D-term inflation
supergravity inflation
supernatural inflation
superstring inflation
supersymmetric hybrid inflation
supersymmetric inflation
supersymmetric topological inflation
supersymmetric new inflation
synergistic warm inflation
TeV-scale hybrid inflation

A (partial?) list of ever-increasing number of inflationary models[1]. Actually, it may not even be possible to rule out some of these models!

[1] From E. P. S. Shellard, *The future of cosmology: Observational and computational prospects*, in *The Future of Theoretical Physics and Cosmology*, Eds. G. W. Gibbons, E. P. S. Shellard and S. J. Rankin (Cambridge University Press, Cambridge, England, 2003).

Performance of inflationary models against the data



The efficiency of the inflationary paradigm leads to a situation wherein, despite the strong constraints, a variety of models continue to remain consistent with the data[2].

[2] J. Martin, C. Ringeval, R. Trotta and V. Vennin, JCAP **1403**, 039 (2014).

Can inflation be falsified?

The difficulty with the inflationary paradigm

A theory that predicts everything predicts nothing[a].

[a] P. J. Steinhardt, Sci. Am. **304**, 36 (2011).

Plan of the talk

Bouncing scenarios as an alternative paradigm[3]

- Bouncing models correspond to situations wherein the universe initially goes through a period of contraction until the scale factor reaches a certain minimum value before transiting to the expanding phase.

[3] See, for instance, M. Novello and S. P. Bergliaffa, Phys. Rep. **463**, 127 (2008); D. Battefeld and P. Peter, Phys. Rep. **571**, 1 (2015).

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- Bouncing models correspond to situations wherein the universe initially goes through a period of contraction until the scale factor reaches a certain minimum value before transiting to the expanding phase.
- They offer an alternative to inflation to overcome the horizon problem, as they permit well motivated, Minkowski-like initial conditions to be imposed on the perturbations at early times during the contracting phase.

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- They offer an alternative to inflation to overcome the horizon problem, as they permit well motivated, Minkowski-like initial conditions to be imposed on the perturbations at early times during the contracting phase.
- However, matter fields *will* have to violate the null energy condition near the bounce in order to give rise to such a scale factor. Also, there exist (genuine) concerns whether such an assumption about the scale factor is valid in a domain where general relativity can be supposed to fail and quantum gravitational effects are expected to take over.

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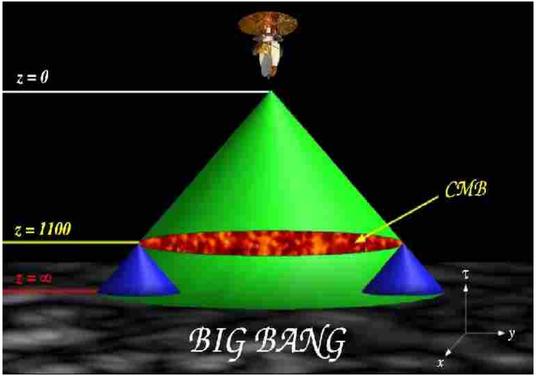
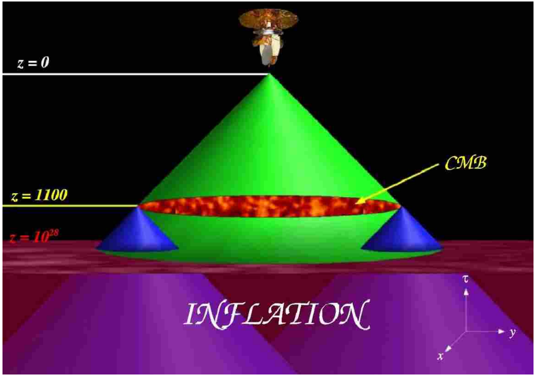
The resolution of the horizon problem in inflation





Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

[4] Images from W. Kinney, astro-ph/0301448.

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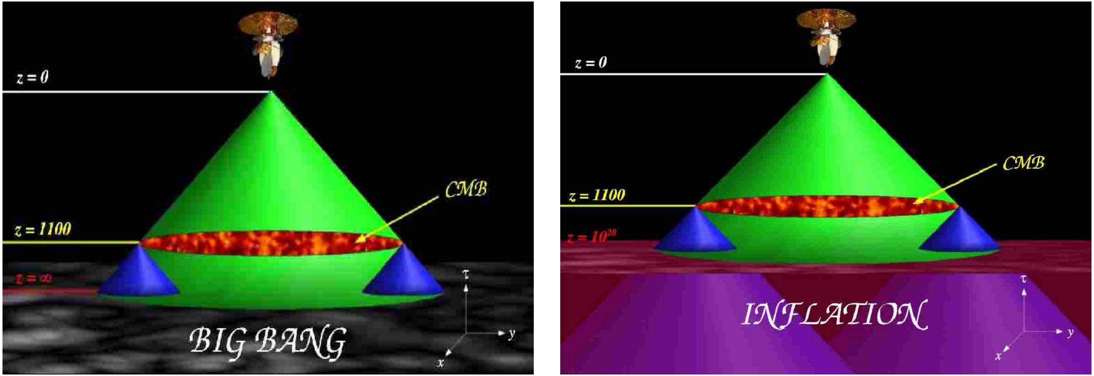

Left: The radiation from the CMB arriving at us from regions separated by more than the Hubble radius at the last scattering surface (which subtends an angle of about 1° today) could not have interacted before decoupling.

Right: An illustration of how an early and sufficiently long epoch of inflation helps in resolving the horizon problem[4].

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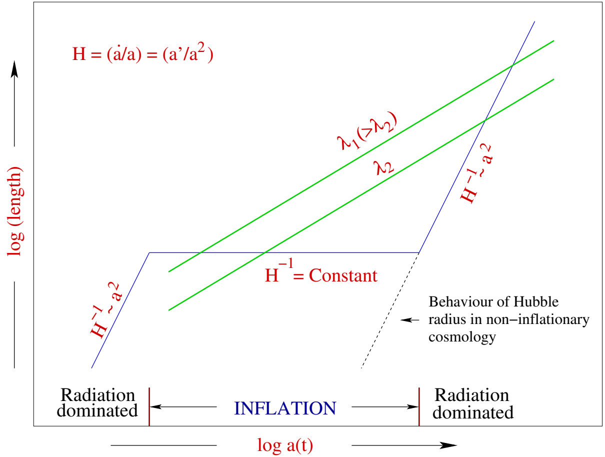
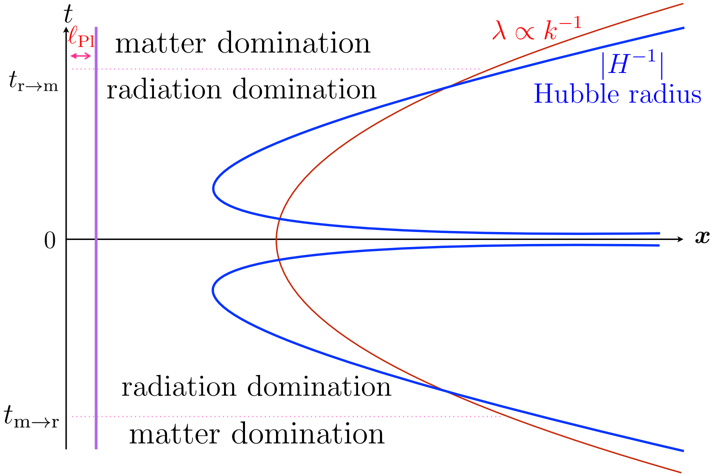
Bringing the modes inside the Hubble radius



The behavior of the physical wavelength $\lambda_{\rm P} \propto a$ (the green lines) and the Hubble radius H^{-1} (the blue line) during inflation and the radiation dominated epochs[5].

[5] See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, New York, 1990), Fig. 8.4.

Overcoming the horizon problem in bouncing models



Evolution of the physical wavelength and the Hubble radius in a bouncing scenario[6].

[6] Figure from, D. Battefeld and P. Peter, Phys. Rept. **571**, 1 (2015).

Violation of the null energy condition near the bounce

Recall that, according to the Friedmann equations

$$\dot{H} = -4\,\pi\,G\,(\rho + p)\,.$$

In any bouncing scenario, the Hubble parameter is negative before the bounce, crosses zero at the bounce and is positive thereafter.

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In such cases, one finds that $\dot{H}$ *will* be positive near the bounce, which implies that $(\rho + p)$ has to be negative in this domain. In other words, the null energy condition needs to be violated in order to achieve such bounces.

Classical bounces and sources

Consider for instance, bouncing models of the form

$$a(\eta) = a_0 \left(1 + \frac{\eta^2}{\eta_0^2}\right)^q = a_0 \left(1 + k_0^2\, \eta^2\right)^q,$$

where a_0 is the value of the scale factor at the bounce (*i.e.* when $\eta = 0$), $\eta_0 = 1/k_0$ denotes the time scale of the duration of the bounce and $q > 0$. We shall assume that the scale k_0 associated with the bounce is of the order of the Planck scale $M_{_{\rm Pl}}$.

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The above scale factor can be achieved with the help of two fluids with constant equation of state parameters $w_1 = (1-q)/(3\,q)$ and $w_2 = (2-q)/(3\,q)$. The energy densities of these fluids behave as $\rho_1 = M_1/a^{(2\,q+1)/q}$ and $\rho_2 = M_2/a^{2\,(1+q)/q}$, where $M_1 = 12\,k_0^2\,M_{_{\rm Pl}}^2\,a_0^{1/q}$ and $M_2 = -M_1\,a_0^{1/q}$.

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Note that, when $q = 1$, during very early times wherein $\eta \ll -\eta_0$, the scale factor behaves as in a matter dominated universe (*i.e.* $a \propto \eta^2$). Therefore, the $q = 1$ case is often referred to as the matter bounce scenario. In such a case, $\rho_1 = 12\, k_0^2\, M_{_{\rm Pl}}^2\, a_0/a^3$ and $\rho_2 = -12\, k_0^2\, M_{_{\rm Pl}}^2\, a_0^2/a^4$.

E-$\mathcal{N}$-folds

The conventional e-fold N is defined $N = \log(a/a_{\rm i})$ so that $a(N) = a_{\rm i} \exp N$. However, the function ${\rm e}^N$ is a monotonically increasing function of N.

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In completely symmetric bouncing scenarios, an obvious choice for the scale factor seems to be[7]

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with $\mathcal{N}$ being the new time variable that we shall consider for integrating the differential equation governing the background as well as the perturbations.

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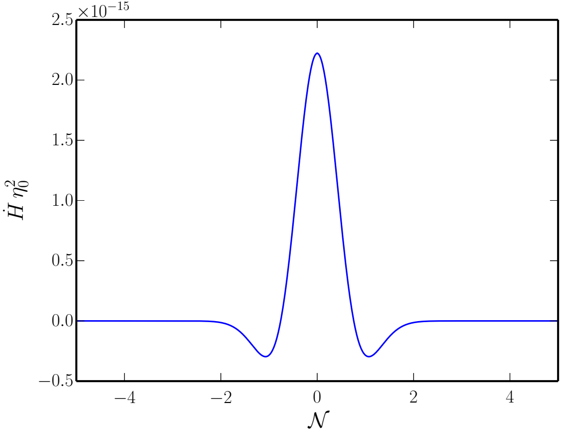
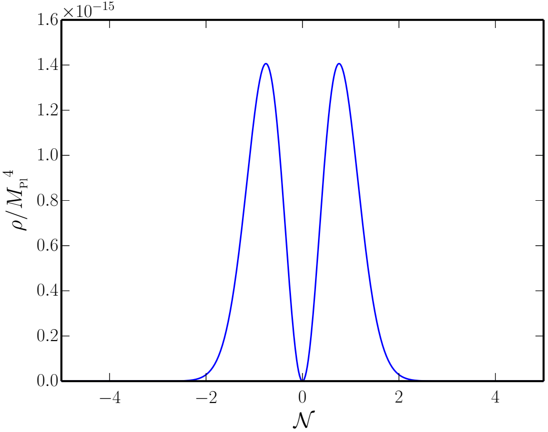
We shall refer to the variable $\mathcal{N}$ as e-$\mathcal{N}$-fold since the scale factor grows roughly by the amount $\mathrm{e}^{\mathcal{N}}$ between $\mathcal{N}$ and $(\mathcal{N}+1)$.

[7] L. Sriramkumar, K. Atmjeet and R. K. Jain, JCAP **1509**, 010 (2015).

Behavior of $\dot{H}$ and ρ in a matter bounce





The behavior of $\dot{H}$ (on the left) and the total energy density ρ (on the right) in a symmetric matter bounce scenario has been plotted as a function of $\mathcal{N}$. Note that the maximum value of ρ is much smaller than $M_{_{\rm Pl}}^4$, which suggests that the bounce can be treated completely classically.

Duality between de Sitter inflation and matter bounce

It is known that the solutions to the equations of motion governing the scalar and tensor perturbations are invariant under a certain transformation referred to as the duality transformation[8].

[8] D. Wands, Phys. Rev. D **60**, 023507 (1999).

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For instance, recall that the Mukhanov-Sasaki variable corresponding to the tensor perturbations satisfies the differential equation

$$u_k'' + \left(k^2 - \frac{a''}{a}\right)\, u_k = 0.$$

Given a scale factor a, the corresponding dual, say, $\tilde{a}$, which leads to the same equation for the variable u_k is given by

$$a(\eta) \to \tilde{a}(\eta) = C\, a(\eta) \int_{\eta_*}^{\eta} \frac{\mathrm{d}\bar{\eta}}{a^2(\bar{\eta})},$$

where C and η_* are constants.

It is straightforward to show that the dual solution to de Sitter inflation corresponds to the matter bounce. Both these cases lead to scale invariant spectra.

[8] D. Wands, Phys. Rev. D **60**, 023507 (1999).

Plan of the talk

Equation governing the tensor perturbations

Upon quantization, the tensor perturbations can be written in terms of the corresponding modes, say, h_k, as follows:

$$\begin{aligned}\hat{\gamma}_{ij}(\eta, \boldsymbol{x}) &= \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}}\, \hat{\gamma}_{ij}^{\boldsymbol{k}}(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} \\ &= \sum_s \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\,\pi)^{3/2}}\, \left(\hat{b}_{\boldsymbol{k}}^s\, \varepsilon_{ij}^s(\boldsymbol{k})\, h_k(\eta)\, \mathrm{e}^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{b}_{\mathbf{k}}^{s\dagger}\, \varepsilon_{ij}^{s*}(\boldsymbol{k})\, h_k^{*}(\eta)\, \mathrm{e}^{-i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right),\end{aligned}$$

where $\hat{b}_{\boldsymbol{k}}^s$ and $\hat{b}_{\boldsymbol{k}}^{s\,\dagger}$ are the usual creation and annihilation operators that satisfy the standard commutation relations, while $\varepsilon_{ij}^s(\boldsymbol{k})$ represents the transverse and traceless polarization tensor describing gravitational waves.

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The modes h_k are governed by the differential equation

$$h_k'' + 2\,\mathcal{H}\,h_k' + k^2\,h_k = 0$$

where $\mathcal{H} = a'/a$ and, in terms of the variable $u_k = M_{_{\rm Pl}}\,a\,h_k/\sqrt{2}$, the above equation reduces to

$$u_k'' + \left(k^2 - \frac{a''}{a}\right)\,u_k = 0.$$

The tensor power spectrum: Definition

The tensor power spectrum $\mathcal{P}_{_{\rm T}}(k)$ is defined through the relation

$$\langle\hat{\gamma}_{m_1n_1}^{\boldsymbol{k}}\,\hat{\gamma}_{m_2n_2}^{\boldsymbol{p}}\rangle=\frac{(2\,\pi)^2}{8\,k^3}\,\Pi_{m_1n_1,m_2n_2}^{\boldsymbol{k}}\,\mathcal{P}_{_{\rm T}}(k)\,\delta^3\,(\boldsymbol{k}+\boldsymbol{p})\,,$$

where

$$\Pi_{m_1n_1,m_2n_2}^{\boldsymbol{k}}=\sum_s\,\varepsilon_{m_1n_1}^{s}(\boldsymbol{k})\,\varepsilon_{m_2n_2}^{s*}(\boldsymbol{k}).$$

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where

$$\Pi^{\boldsymbol{k}}_{m_1 n_1, m_2 n_2} = \sum_{s} \varepsilon^{s}_{m_1 n_1}(\boldsymbol{k})\; \varepsilon^{s*}_{m_2 n_2}(\boldsymbol{k}).$$

In terms of the quantities h_k and u_k, the tensor power spectrum $\mathcal{P}_{_{\rm T}}(k)$ in the Bunch-Davies vacuum is given by

$$\mathcal{P}_{_{\rm T}}(k) = 4\,\frac{k^3}{2\,\pi^2}\,\vert h_k\vert^2 = \frac{8}{M_{_{\rm Pl}}^2}\,\frac{k^3}{2\,\pi^2}\,\left(\frac{\vert u_k\vert}{a}\right)^2,$$

with the right hand side being evaluated at suitably late times[9].

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The matter bounce

We shall assume that the scale factor describing the bouncing scenario is given in terms of the conformal time coordinate η by the relation

$$a(\eta) = a_0\,\left(1 + \eta^2/\eta_0^2\right) = a_0\,\left(1 + k_0^2\,\eta^2\right).$$

As we had discussed earlier, at very early times, *viz.* when $\eta \ll -\eta_0$, the scale factor behaves as in a matter dominated epoch[10].

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The quantity a''/a corresponding to the above scale factor is given by

$$\frac{a''}{a} = \frac{2\,k_0^2}{1 + k_0^2\,\eta^2},$$

which is essentially a Lorentzian profile.

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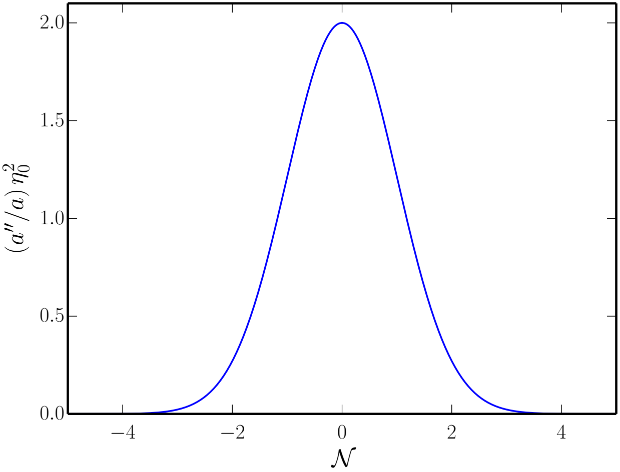
The behavior of a''/a



The behavior of the quantity a''/a has been plotted as a function of $\mathcal{N}$ for the matter bounce scenario of interest. Note that the maximum value of a''/a is of the order of k_0^2.

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In the first domain, we can assume that the scale factor behaves as $a(\eta) \simeq a_0\, k_0^2\, \eta^2$, so that $a''/a \simeq 2/\eta^2$. Since the condition $k^2 = a''/a$ corresponds to, say, $\eta_k = -\sqrt{2}/k$, the initial conditions can be imposed when $\eta \ll \eta_k$.

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In the first domain, we can assume that the scale factor behaves as $a(\eta) \simeq a_0\, k_0^2\, \eta^2$, so that $a''/a \simeq 2/\eta^2$. Since the condition $k^2 = a''/a$ corresponds to, say, $\eta_k = -\sqrt{2}/k$, the initial conditions can be imposed when $\eta \ll \eta_k$.

The modes h_k can be easily obtained in such a case and the positive frequency modes that correspond to the vacuum state at early times are given by

$$h_k(\eta) = \frac{\sqrt{2}}{M_{_{\rm Pl}}}\,\frac{1}{\sqrt{2\,k}}\,\frac{1}{a_0\,k_0^2\,\eta^2}\,\left(1-\frac{i}{k\,\eta}\right)\,{\rm e}^{-i\,k\,\eta}.$$

The modes in the second domain

Let us now consider the behavior of the modes in the domain $-\alpha\,\eta_0 < \eta < \beta\,\eta_0$, where, say, $\beta \simeq 10^2$. Since we are interested in scales much smaller than k_0, we shall assume that $\eta_k \ll -\alpha\,\eta_0$, which corresponds to $k \ll k_0/\alpha$.

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In such a case, upon ignoring the k^2 term, the equation governing h_k can be immediately integrated to yield

$$h_k(\eta) \simeq h_k(\eta_*) + h_k'(\eta_*)\,a^2(\eta_*)\,\int_{\eta_*}^{\eta} \frac{\mathrm{d}\tilde{\eta}}{a^2(\tilde{\eta})},$$

where η_* is a suitably chosen time and the scale factor $a(\eta)$ is given by the complete expression.

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where $\eta_\ast$ is a suitably chosen time and the scale factor $a(\eta)$ is given by the complete expression.

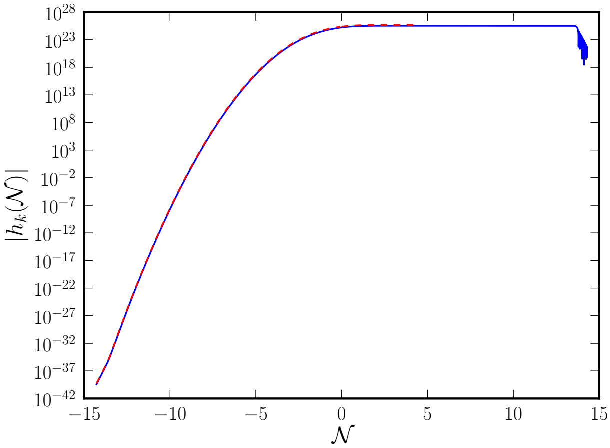
If we choose $\eta_\ast = -\alpha\,\eta_0$, we can make use of the solution in the first domain to determine the constants and express the solution in the second domain as follows:

$$h_k = \mathcal{A}_k + \mathcal{B}_k\, f(k_0\,\eta),$$

where

$$f(k_0\,\eta) = \frac{k_0\,\eta}{1 + k_0^2\,\eta^2} + \tan^{-1}(k_0\,\eta)\,.$$

Evolution of the tensor modes across the bounce



A comparison of the numerical results (in blue) with the analytical results (in red) for the amplitude of the tensor mode $|h_k|$ corresponding to the wavenumber $k/k_0 = 10^{-20}$. We have set $a_0 = 10^5$, and we have chosen $\alpha = 10^5$ for plotting the analytical results[11].

[11] D. Chowdhury, V. Sreenath and L. Sriramkumar, JCAP **1511**, 002 (2015)

The tensor power spectrum after the bounce

The quantities $\mathcal{A}_k$ and $\mathcal{B}_k$ are given by

$$\mathcal{A}_k = \frac{\sqrt{2}}{M_{_{\rm Pl}}} \frac{1}{\sqrt{2\,k}} \frac{1}{a_0\,\alpha^2} \left(1 + \frac{i\,k_0}{\alpha\,k}\right) {\rm e}^{i\,\alpha\,k/k_0} + \mathcal{B}_k\, f(\alpha),$$

$$\mathcal{B}_k = \frac{\sqrt{2}}{M_{_{\rm Pl}}} \frac{1}{\sqrt{2\,k}} \frac{1}{2\,a_0\,\alpha^2} \left(1+\alpha^2\right)^2 \left(\frac{3\,i\,k_0}{\alpha^2\,k} + \frac{3}{\alpha} - \frac{i\,k}{k_0}\right) {\rm e}^{i\,\alpha\,k/k_0}.$$

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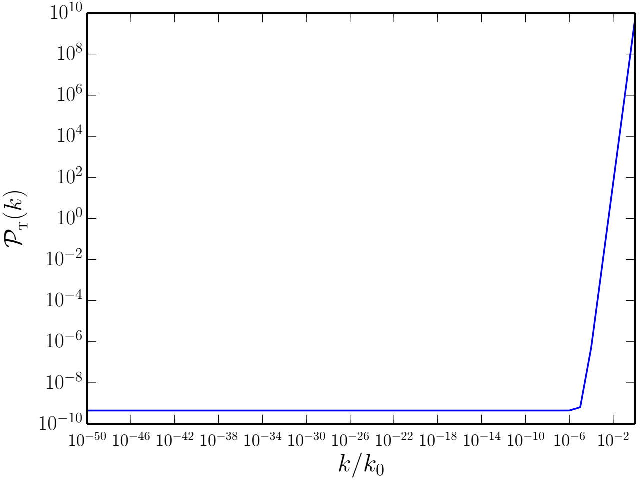
If we evaluate the tensor power spectrum after the bounce at $\eta = \beta\,\eta_0$, we find that it can be expressed as

$$\mathcal{P}_{_{\rm T}}(k) = 4\,\frac{k^3}{2\,\pi^2}\,\vert\mathcal{A}_k + \mathcal{B}_k\, f(\beta)\vert^2.$$

The tensor power spectrum



The behavior of the tensor power spectrum has been plotted as a function of k/k_0 for a wide range of wavenumbers. In plotting this figure, we have set $k_0/M_{\rm Pl} = 1$, $a_0 = 10^5$, $\alpha = 10^5$ and $\beta = 10^2$. Note that the power spectrum is scale invariant for $k \ll k_0/\alpha$.

Plan of the talk

Modeling the matter bounce with scalar fields

As we had discussed, the matter bounce scenario described by the scale factor

$$a(\eta) = a_0\,\left(1 + \eta^2/\eta_0^2\right) = a_0\,\left(1 + k_0^2\,\eta^2\right)$$

can be driven with the aid of two fluids, one which is matter and another fluid which behaves like radiation, but has negative energy density.

[12] R. N. Raveendran, D. Chowdhury and L. Sriramkumar, In preparation.

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We find that the behavior can also be achieved with the help of two scalar fields, say, ϕ and χ, that are governed by the following action[12]:

$$S[\phi,\chi] = -\int \mathrm{d}^4x\,\sqrt{-g}\,\left[-\frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + V(\phi) + \mathcal{V}\,\left(-\frac{1}{2}\,\partial_\mu\chi\,\partial^\mu\chi\right)^2\right],$$

where $\mathcal{V}$ is a constant and the potential $V(\phi)$ is given by

$$V(\phi) = \frac{6\,M_{_{\mathrm{Pl}}}^2\,k_0^2/a_0^2}{\cosh^6(\sqrt{12}\,\phi/M_{_{\mathrm{Pl}}})}.$$

[12] R. N. Raveendran, D. Chowdhury and L. Sriramkumar, In preparation.

Plan of the talk

The scalar perturbations

When the scalar perturbations are taken into account, the FLRW line element can be written as

$$\mathrm{d}s^2 = -(1+2\,A)\,\mathrm{d}t^2 + 2\,a(t)\,(\partial_i B)\;\mathrm{d}t\;\mathrm{d}x^i + a^2(t)\;\left[(1-2\,\psi)\;\delta_{ij} + 2\;(\partial_i\,\partial_j E)\right]\,\mathrm{d}x^i\,\mathrm{d}x^j,$$

where, evidently, the quantities A, ψ, B and E represent the metric perturbations.

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The gauge invariant curvature and isocurvature perturbations $\mathcal{R}$ and $\mathcal{S}$ can be defined in terms of the above metric perturbations and the perturbations $\delta\phi$ and $\delta\chi$ in the scalar fields as follows[13]:

$$\mathcal{R} = \frac{H}{\dot{\phi}^2 - \alpha\,\dot{\chi}^4}\,\left(\dot{\phi}\,\overline{\delta\phi} - \alpha\,\dot{\chi}^3\,\overline{\delta\chi}\right), \quad \mathcal{S} = \frac{H\,\sqrt{\alpha\,\dot{\chi}^2}}{\dot{\phi}^2 - \alpha\,\dot{\chi}^4}\,\left(\dot{\chi}\,\overline{\delta\phi} - \dot{\phi}\,\overline{\delta\chi}\right).$$

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The quantities $\overline{\delta\phi}$ and $\overline{\delta\chi}$ denote the gauge invariant versions of the perturbations in the scalar fields, and are given by

$$\overline{\delta\phi} = \delta\phi + \frac{\dot{\phi}\,\psi}{H}, \qquad \overline{\delta\chi} = \delta\chi + \frac{\dot{\chi}\,\psi}{H}.$$

[13] R. N. Raveendran, D. Chowdhury and L. Sriramkumar, In preparation.

Equations governing the curvature and isocurvature perturbations

We obtain the equations of motion describing the gauge invariant perturbations $\mathcal{R}$ and $\mathcal{S}$ in our model to be

$$\mathcal{R}'' + \frac{2\,\left(7+9\,k_0^2\,\eta^2-6\,k_0^4\,\eta^4\right)}{\eta\,\left(1-2\,k_0^2\,\eta^2-3\,k_0^4\,\eta^4\right)}\,\mathcal{R}' + \frac{k^2\,\left(5+9\,k_0^2\,\eta^2\right)}{\left(-3+9\,k_0^2\,\eta^2\right)}\,\mathcal{R}$$

$$= -\frac{4\,\left(5+12\,k_0^2\,\eta^2\right)}{\eta\,\left(-1+3\,k_0^2\,\eta^2\right)\sqrt{3+3\,k_0^2\,\eta^2}}\,\mathcal{S}' - \frac{4\,\left[5-22\,k_0^2\,\eta^2-24\,k_0^4\,\eta^4+k^2\,\eta^2\,\left(1+k_0^2\,\eta^2\right)^2\right]}{\sqrt{3}\,\eta^2\,\left(1+k_0^2\,\eta^2\right)^{3/2}\,\left(-1+3\,k_0^2\,\eta^2\right)}\,\mathcal{S},$$

$$\mathcal{S}'' + \frac{2\,\left(9+7\,k_0^2\,\eta^2+6\,k_0^4\,\eta^4\right)}{\eta\,\left(-1+2\,k_0^2\,\eta^2+3\,k_0^4\,\eta^4\right)}\,\mathcal{S}'$$

$$+\frac{-18+85\,k_0^2\,\eta^2+25\,k_0^4\,\eta^4+6\,k_0^6\,\eta^6+k^2\,\left(-3+k_0^2\,\eta^2\right)\,\left(\eta+k_0^2\,\eta^3\right)^2}{\left(-1+3\,k_0^2\,\eta^2\right)\,\left(\eta+k_0^2\,\eta^3\right)^2}\,\mathcal{S}$$

$$= -\frac{4\,\sqrt{3}\,\left(-3+2\,k_0^2\,\eta^2\right)}{\eta\,\sqrt{1+k_0^2\,\eta^2}\,\left(-1+3\,k_0^2\,\eta^2\right)}\,\mathcal{R}' - \frac{4\,k^2\,\sqrt{1+k_0^2\,\eta^2}}{\sqrt{3}\,\left(-1+3\,k_0^2\,\eta^2\right)}\,\mathcal{R}.$$

However, some of the coefficients diverge when $\dot{H}$ vanishes and/or at the bounce.

The uniform-χ gauge

The above issue can be avoided by working in a gauge wherein $\delta\chi = 0$[14]. In this gauge, the curvature and isocurvature perturbations simplify to be

$$\mathcal{R} = \psi + \frac{2\,H\,M_{_{\rm Pl}}^2}{\dot{\phi}^2 - \alpha\,\dot{\chi}^4}\,\left(\dot{\psi} + H\,A\right), \quad \mathcal{S} = \frac{2\,H\,M_{_{\rm Pl}}^2\,\sqrt{\alpha\,\dot{\chi}^2}}{\dot{\phi}^2 - \alpha\,\dot{\chi}^4}\,\left(\frac{\dot{\chi}}{\dot{\phi}}\right)\,\left(\dot{\psi} + H\,A\right).$$

[14] L. E. Allen and D. Wands, Phys. Rev. **70**, 063515 (2004).

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The equations of motion for $\mathcal{R}$ and $\mathcal{S}$ then lead to the following equations for the metric perturbations A and ψ:

$$\begin{aligned} A'' + 4\,\mathcal{H}\,A' + \left(\frac{k^2}{3} - \frac{5}{4}\,\frac{\alpha\,\chi'^4}{a^2\,M_{\rm Pl}^2}\right)\,A &= -3\,\mathcal{H}\,\psi' + \frac{4\,k^2}{3}\,\psi, \\ \psi'' - 2\,\mathcal{H}\,\psi' + k^2\,\psi &= -2\,\mathcal{H}\,A' - \frac{5\,\alpha\chi'^4}{4\,M_{\rm Pl}^2\,a^2}\,A, \end{aligned}$$

where $\mathcal{H} = a'/a$. These equations prove to be helpful in evolving the scalar perturbations across the bounce.

[14] L. E. Allen and D. Wands, Phys. Rev. **70**, 063515 (2004).

Solutions for $\mathcal{R}_k$ and $\mathcal{S}_k$ in the first domain

As in the case of tensors, we shall be interested in evaluating the power spectrum after the bounce at $\eta = \beta\,\eta_0$. Also, to arrive at the analytical approximations, as earlier, we shall divide period of interest into two domains, *viz.* $-\infty < \eta < -\alpha\,\eta_0$ and $-\alpha\,\eta_0 < \eta < \beta\,\eta_0$.

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In the first domain, we find that the solution to the curvature perturbation can be arrived at as in the case of tensors and is given by

$$\mathcal{R}_k(\eta) \simeq \frac{1}{\sqrt{6\,k}\,M_{_{\rm Pl}}\,a_0\,k_0^2\,\eta^2}\,\left(1-\frac{i}{k\,\eta}\right)\,{\rm e}^{-i\,k\,\eta}.$$

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Using this solution, it is straightforward to obtain the following solution for the isocurvature perturbation at early times:

$$\begin{aligned}\mathcal{S}_k(\eta) \quad \simeq \quad & \frac{{\rm e}^{-i\,(3+\sqrt{3})\,k\,\eta/3}}{9\,\sqrt{2\,k^3}\,a_0\,k_0^3\,M_{_{\rm Pl}}\,\eta^4}\biggl\{-12\,i\,{\rm e}^{i\,k\,\eta/\sqrt{3}}+12\,k\eta\,{\rm e}^{i\,k\,\eta/\sqrt{3}}+\left(9/3^{1/4}\right)\,k\,k_0\,\eta^2\,{\rm e}^{i\,k\,\eta}\\ & +4\,\pi\,k^2\,\eta^2\,{\rm e}^{i\,k\,\eta}+4\,i\,k^2\,\eta^2\,{\rm e}^{i\,k\,\eta}\,{\rm Ei}\left[{\rm e}^{i\,(-3+\sqrt{3})\,k\,\eta/3}\right]\biggr\}.\end{aligned}$$

Solutions for ψ_k and A_k in the second domain

In the second domain, upon ignoring the k^2 dependent terms, one finds that the combination $A_k + \psi_k$ satisfies the same equation of motion as the tensor modes.

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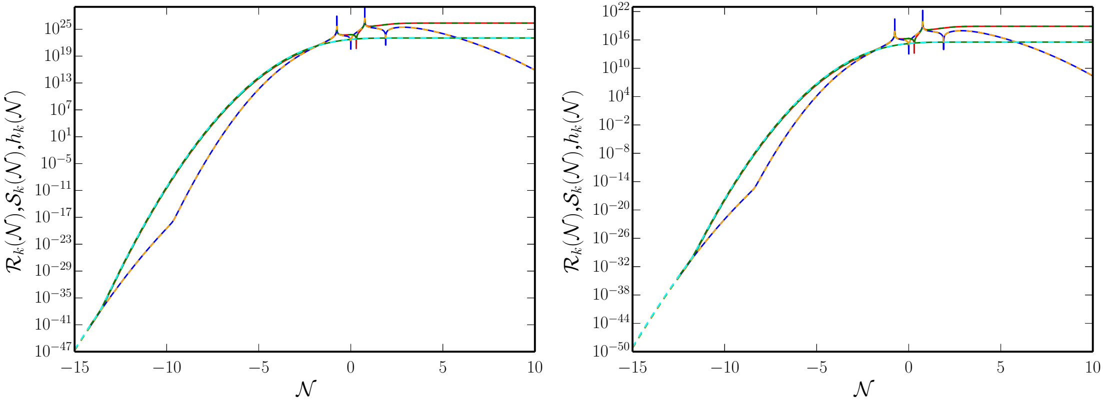
This feature helps us obtain the solutions for A_k and ψ_k, and they are given by

$$
\begin{aligned}
A_k(\eta) &\simeq \frac{\mathcal{C}_k^1\, \eta}{4\, a_0^2\, (1 + k_0^2\, \eta^2)} + \mathcal{C}_k^2\, \mathrm{e}^{-2\sqrt{5}\, \tan^{-1}(k_0\, \eta)} + \mathcal{C}_k^3\, \mathrm{e}^{2\sqrt{5}\, \tan^{-1}(k_0 \eta)}, \\
\psi_k(\eta) &\simeq \mathcal{C}_k^1 \left[\frac{\eta}{4\, a_0^2\, (1 + k_0^2 \eta^2)} + \frac{\tan^{-1}(k_0\, \eta)}{2\, k_0\, a_0^2}\right] \\
&\quad - \mathcal{C}_k^2\, \mathrm{e}^{-2\sqrt{5}\, \tan^{-1}(k_0\, \eta)} - \mathcal{C}_k^3\, \mathrm{e}^{2\sqrt{5}\, \tan^{-1}(k_0 \eta)} + \mathcal{C}_k^4.
\end{aligned}
$$

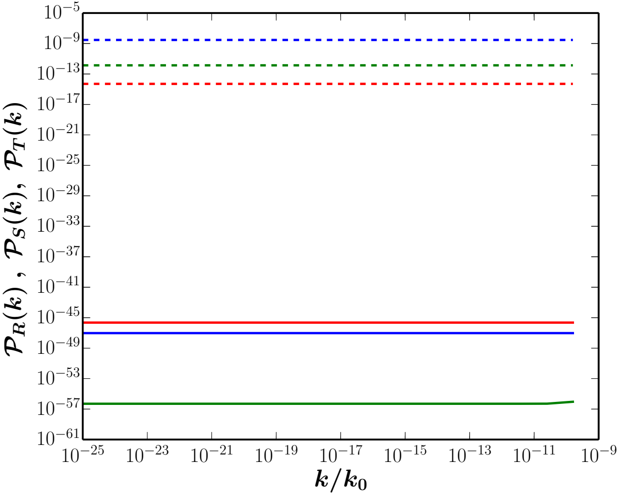
The four constants, *viz.* $\mathcal{C}_k^1$, $\mathcal{C}_k^2$, $\mathcal{C}_k^3$ and $\mathcal{C}_k^4$, are fixed by matching the above solutions with the solutions for $\mathcal{R}_k$ and $\mathcal{S}_k$ in the first domain at $\eta = -\alpha\, \eta_0$.

Evolution of $\mathcal{R}_k$, $\mathcal{S}_k$ and h_k



The evolution of the curvature, isocurvature and tensor perturbations, *viz.* $\mathcal{R}_k$ (in red), $\mathcal{S}_k$ (in blue) and tensor h_k (in green) across the bounce for the mode $k/k_0 = 10^{-20}$ (on the left) and the mode $k/k_0 = 10^{-15}$ (on the right). We have set $k_0/M_{\rm Pl} = 1$, $a_0 = 3 \times 10^7$ and $\mathcal{V}\, M_{\rm Pl}^4 = 1$. The solid lines denote the results obtained numerically, while the dashed lines represent the analytical approximations.

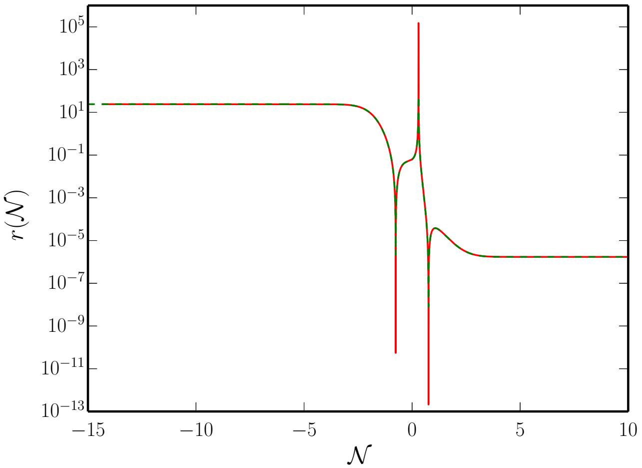
The scalar and tensor power spectra



The scalar and tensor power spectra have been plotted before (as solid lines) as well as after (as dashed lines) the bounce[15].

[15] R. N. Raveendran, D. Chowdhury and L. Sriramkumar, In preparation.

The evolution of the tensor-to-scalar ratio



The evolution of the tensor-to-scalar ratio across the symmetric matter bounce for a typical mode of cosmological interest. The solid and dashed lines represent the numerical and analytical results, respectively.

Plan of the talk

Tensor bispectrum and non-Gaussianity parameter

The tensor bispectrum, evaluated at the conformal time, say, $\eta_{\rm e}$, is defined as

$$\langle \hat{\gamma}^{\boldsymbol{k}_1}_{m_1 n_1}(\eta_{\rm e})\, \hat{\gamma}^{\boldsymbol{k}_2}_{m_2 n_2}(\eta_{\rm e})\, \hat{\gamma}^{\boldsymbol{k}_3}_{m_3 n_3}(\eta_{\rm e})\rangle = (2\,\pi)^3\, \mathcal{B}^{m_1 n_1 m_2 n_2 m_3 n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \times \delta^{(3)}\left(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3\right)$$

and, for convenience, we shall set

$$\mathcal{B}^{m_1 n_1 m_2 n_2 m_3 n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = (2\,\pi)^{-9/2}\, G^{m_1 n_1 m_2 n_2 m_3 n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3).$$

[16] V. Sreenath, R. Tibrewala and L. Sriramkumar, JCAP **1312**, 037 (2013).

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As in the scalar case, one can define a dimensionless non-Gaussianity parameter to characterize the amplitude of the tensor bispectrum as follows[16]:

$$h_{\rm NL}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -\left(\frac{4}{2\,\pi^2}\right)^2 \left[k_1^3\, k_2^3\, k_3^3\, G^{m_1 n_1 m_2 n_2 m_3 n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\right] \times \left[\Pi^{\boldsymbol{k}_1}_{m_1 n_1, m_2 n_2}\, \Pi^{\boldsymbol{k}_2}_{m_3 n_3, \bar{m}\bar{n}}\, k_3^3\, \mathcal{P}_{_{\rm T}}(k_1)\, \mathcal{P}_{_{\rm T}}(k_2) + \text{five permutations}\right]^{-1}.$$

[16] V. Sreenath, R. Tibrewala and L. Sriramkumar, JCAP **1312**, 037 (2013).

The third order action and the tensor bispectrum

The third order action that leads to the tensor bispectrum is given by[17]

$$S^3_{\gamma\gamma\gamma}[\gamma_{ij}] = \frac{M_{_{\rm Pl}}^2}{2} \int {\rm d}\eta \int {\rm d}^3\boldsymbol{x} \left[\frac{a^2}{2}\, \gamma_{lj}\, \gamma_{im}\, \partial_l \partial_m \gamma_{ij} - \frac{a^2}{4}\, \gamma_{ij}\, \gamma_{lm}\, \partial_l \partial_m \gamma_{ij}\right].$$

[17] J. Maldacena, JHEP **0305**, 013 (2003).

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The tensor bispectrum calculated in the perturbative vacuum using the Maldacena formalism, can be written in terms of the modes h_k as follows:

$$\begin{aligned} G^{m_1n_1m_2n_2m_3n_3}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) &= M_{\rm Pl}^2\,\Bigg[\left(\Pi^{\boldsymbol{k}_1}_{m_1n_1,ij}\,\Pi^{\boldsymbol{k}_2}_{m_2n_2,im}\,\Pi^{\boldsymbol{k}_3}_{m_3n_3,lj} - \frac{1}{2}\,\Pi^{\boldsymbol{k}_1}_{m_1n_1,ij}\,\Pi^{\boldsymbol{k}_2}_{m_2n_2,ml}\,\Pi^{\boldsymbol{k}_3}_{m_3n_3,ij}\right)k_{1m}\,k_{1l} \\ &\quad + \text{five permutations}\Bigg] \\ &\quad \times \left[h_{k_1}(\eta_{\rm e})\,h_{k_2}(\eta_{\rm e})\,h_{k_3}(\eta_{\rm e})\,\mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \text{complex conjugate}\right], \end{aligned}$$

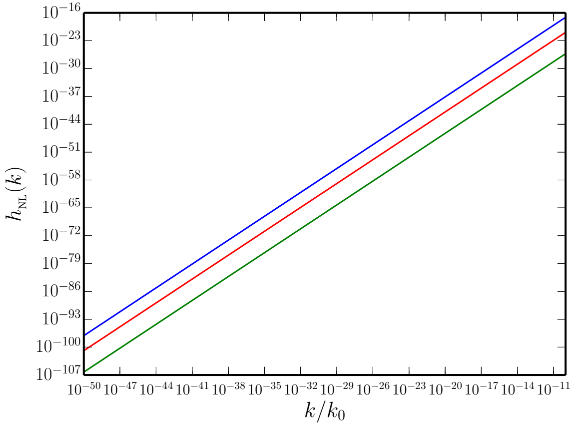
where $\mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ is described by the integral

$$\mathcal{G}_{\gamma\gamma\gamma}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = -\frac{i}{4}\int_{\eta_{\rm i}}^{\eta_{\rm e}} {\rm d}\eta\; a^2\, h^\ast_{k_1}\, h^\ast_{k_2}\, h^\ast_{k_3},$$

with $\eta_{\rm i}$ denoting the time when the initial conditions are imposed on the perturbations.

[17] J. Maldacena, JHEP **0305**, 013 (2003).

The contributions due to the three domains



The contributions to the non-Gaussianity parameter $h_{\rm NL}$ in the equilateral limit from the first (in green), the second (in red) and the third (in blue) domains have been plotted as a function of k/k_0 for $k \ll k_0/\alpha$. Clearly, the third domain gives rise to the maximum contribution to $h_{\rm NL}$[18].

[18] D. Chowdhury, V. Sreenath and L. Sriramkumar, JCAP **1511**, 002 (2015)

The effect of the long wavelength tensor modes

Since the amplitude of a long wavelength mode freezes on super-Hubble scales during inflation, such modes can be treated as a background as far as the smaller wavelength modes are concerned. Let us denote the constant amplitude of the long wavelength tensor mode as γ_{ij}^{B}.

[19] S. Kundu, JCAP **1404**, 016 (2014).

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In the presence of such a long wavelength mode, the background FLRW metric can be written as

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,[\mathrm{e}^{\gamma^{\mathrm{B}}}]_{ij}\,\mathrm{d}\boldsymbol{x}^i\,\mathrm{d}\boldsymbol{x}^j,$$

i.e. the spatial coordinates are modified according to a spatial transformation of the form $\boldsymbol{x}' = \Lambda\,\boldsymbol{x}$, where $\Lambda_{ij} = [\mathrm{e}^{\gamma^{\mathrm{B}}/2}]_{ij}$.

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Under such a spatial transformation, the small wavelength tensor perturbation transforms as[19]

$$\gamma_{ij}^{\boldsymbol{k}} \to \det\left(\Lambda^{-1}\right)\gamma_{ij}^{\Lambda^{-1}\boldsymbol{k}},$$

where $\det\left(\Lambda^{-1}\right) = 1$.

[19] S. Kundu, JCAP **1404**, 016 (2014).

The behavior of the two and three-point functions

On using the above results, one finds that the tensor two-point function in the presence of a long wavelength mode denoted by, say, the wavenumber k, can be written as

$$\langle\hat{\gamma}^{\boldsymbol{k}_1}_{m_1n_1}\,\hat{\gamma}^{\boldsymbol{k}_2}_{m_2n_2}\rangle_k = \frac{(2\,\pi)^2}{2\,k_1^3}\,\frac{\Pi^{\boldsymbol{k}_1}_{m_1n_1,m_2n_2}}{4}\,\mathcal{P}_{_{\rm T}}(k_1)\,\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2) \times\left[1-\left(\frac{n_{_{\rm T}}-3}{2}\right)\gamma^{\rm B}_{ij}\,\hat{n}_{1i}\,\hat{n}_{1j}\right],$$

where $\hat{n}_{1i} = k_{1i}/k_1$.

[20] V. Sreenath and L. Sriramkumar, JCAP **1410**, 021 (2014).

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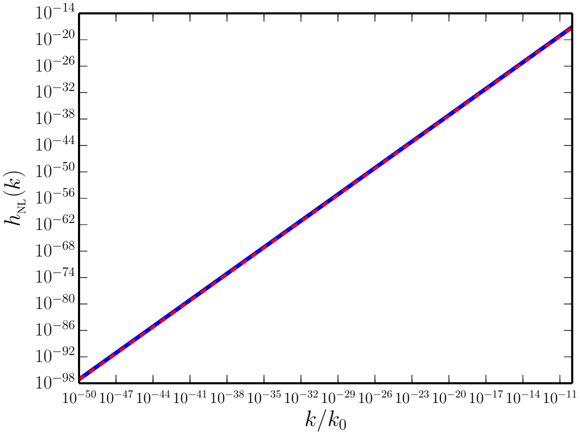
One can also show that, in the presence of a long wavelength mode, the tensor bispectrum can be written as[20]

$$\langle\,\hat{\gamma}^{\boldsymbol{k}_1}_{m_1n_1}\,\hat{\gamma}^{\boldsymbol{k}_2}_{m_2n_2}\,\hat{\gamma}^{\boldsymbol{k}_3}_{m_3n_3}\,\rangle_{k_3} = -\frac{(2\,\pi)^{5/2}}{4\,k_1^3\,k_3^3}\left(\frac{n_{_{\rm T}}-3}{32}\right)\mathcal{P}_{_{\rm T}}(k_1)\,\mathcal{P}_{_{\rm T}}(k_3) \times\Pi^{\boldsymbol{k}_1}_{m_1n_1,m_2n_2}\,\Pi^{\boldsymbol{k}_3}_{m_3n_3,ij}\,\hat{n}_{1i}\,\hat{n}_{1j}\,\delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2).$$

[20] V. Sreenath and L. Sriramkumar, JCAP **1410**, 021 (2014).

The complete contribution to $h_{\rm NL}$



The behavior of $h_{\rm NL}$ in the equilateral (in blue) and the squeezed (in red) limits plotted as a function of k/k_0 for $k \ll k_0/\alpha$. The resulting $h_{\rm NL}$ is considerably small when compared to the values that arise in de Sitter inflation wherein $3/8 \lesssim h_{\rm NL} \lesssim 1/2$. Moreover, we find that $h_{\rm NL}$ behaves as k^2 in the equilateral and the squeezed limits, with similar amplitudes[21].

[21] D. Chowdhury, V. Sreenath and L. Sriramkumar, JCAP **1511**, 002 (2015).

Plan of the talk

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- We are currently working on constructing symmetric bouncing models that lead to scalar power spectra with a tilt as suggested by the cosmological data.
- It is also important to examine if the non-Gaussianities generated in such models are in agreement with the recent constraints from Planck.

Issues confronting bouncing models

- In inflation, any classical perturbations present at the start will decay. In contrast, they grow strongly in bouncing models. So, these need to be assumed to be rather small if smooth bounces have to begin.

[22] L. E. Allen and D. Wands, Phys. Rev. **70**, 063515 (2004).

[23] Y-F. Cai, R. Brandenberger and X. Zhang, Phys. Letts. B **703**, 25 (2011).

[24] J. Quintin, Z. Sherkatghanad, Y-F. Cai and R. Brandenberger, Phys. Rev. D **92**, 062532 (2015).

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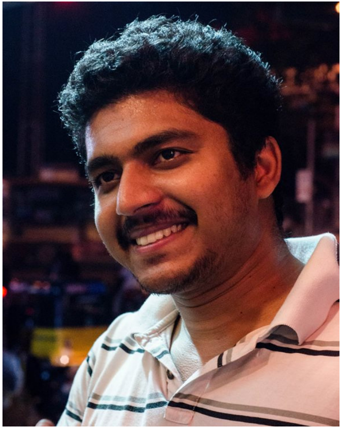
Collaborators: current and former students

Debika Chowdhury



Rathul Nath Raveendran

V. Sreenath

Thank you for your attention