I. EXERCISES - SESSION 7

A. Comments on Sessions:

The goals of this session are the following:

1. Using gsl libraries
2. Handling Matrices
3. Linear Algebra routines

Everyone attempts all the problems!

B. Preparations:

Preparations: You have to write all the codes from scratch, so make a directory called session07 using the following command:

```
mkdir session07
```

Move into the correct directory and follow the rest of the work sheet

C. Problems:

1. Let us solve the Poisson's Equation in 1D using linear algebra techniques. The 1D equation we will solve is:

$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\epsilon_0}$$

As a first step we will cast this equation in the following form:

$$-u''(x) = f(x)$$

where we assume that all variables are dimensionless. Let $f(x) = (3x + x^2)e^x$. This can be solved analytically to give:

$$u(x) = x(1-x)e^x$$

We will solve the Poisson Equation in terms of simultaneous linear equation.

(a) Using the numerical second derivative and the following definition for $h = 1/(N+1)$ and choosing $x \in [0, 1]$ with the boundary conditions $u(0) = u(1) = 0$, cast the Poisson's Equation (dimensionless) in the form of a matrix equation $AX = B$ and show that

$$A = \begin{pmatrix} 2 & -1 & 0 & 0 & 0\cdots \\ -1 & 2 & -1 & 0 & 0\cdots \\ 0 & -1 & 2 & -1 & 0\cdots \\ \vdots & & & & \end{pmatrix} \qquad B = \begin{pmatrix} h^2 f(x_1) \\ h^2 f(x_2) \\ \vdots \end{pmatrix}$$

(b) Obtain $u(x)$ numerically using the gsl library functions gsl_linalg_LU_decomp and gsl_linalg_LU_solve. Compare the relative error between the analytic and the numerical solutions as a function of x for different N values, which determines the size of the matrix A and h. The larger the value of N, the smaller the step size h. Tabulate your results for $N = 10, \cdots 10000$. You could increment N as mutiples of 2, i.e. $N* = 2$. Plot your results for $N = \mathrm{O}(10), \mathrm{O}(100), \mathrm{O}(1000)$.

(c) Using the C clock function clock() determine the time taken for solving the linear equation as a function of N. You will need to include the header file time.h. The clock function can be used as follows:

```
clock_t start, stop; /*Declaring variables start and stop of type clock_t,
which is defined in time.h*/

start = clock();

        - Do something complicated -

stop = clock();

printf("time in seconds: %e \n", (double)(stop - start)/(CLOCKS_PER_SEC));
/*CLOCKS_PER_SEC is the number of ticks in a second and the clock() function
measures time in units of ticks.*/
```

Plot the time taken in seconds for the Linear Equation solver that uses LU decomposition followed by solving the factorized system as a function of N and find the slope. Do you think you can use LU Decomposition for matrices which are of size $10^5 \times 10^5$? Interpret your plot.

2. (Bonus): Without using gsl library functions for matrices, write a code that allocates matrices dynamically (for example using the C function malloc) and use it to allocate a 4×4 unit matrix. Print out the elements. Please list the steps below. You should remember to free the matrix at the end of the calculation.