Additional PS 1

1. Consider a system of angular momentum $\vec{J}$. Let us consider a system in 3d, spanned by the three eigenkets $|1\rangle$, $|0\rangle$, $|-1\rangle$, which are common eigenstates of J^2 and J_z.

The Hamiltonian of the system is

$$H_0 = a J_z + \frac{b}{\hbar} J_z^2$$

a, b are positive constants (dimensions of angular frequency).

a) What are the energy levels of this system? For what ratio of b/a is there degeneracy?

b. A static $\vec{B}_0$ field is applied in the direction of $\vec{u}$ with polar angles θ, φ. The interaction of the magnetic field with the magnetic moment of the system ($\vec{M} = \gamma \vec{J}$) is given by the Hamiltonian:

$$W = \omega_0 J_u$$

where $\omega_0 = -\gamma |\vec{B}_0|$ (Larmor angular frequency).

J_u: Component of $\vec{J}$ in the direction $\vec{u}$:

$$J_u = J_z \cos\theta + J_x \sin\theta \cos\varphi + J_y \sin\theta \sin\varphi.$$

Write the matrix that represents W in the basis of H_0.

b) Let $b = a$ and let $\hat{u}$ be parallel to the x axis.
Assume: $w_0 \ll a$.

Calculate the energies and eigenstates of the system. ~~By first order in w_2~~ [~~Energies to first order~~ to first order in w_0.

2. A SHO in 1d is subjected to a perturbation

$$\lambda H_1 = bx$$

b: real constant.

a) Calculate the energy shift of the ground state to lowest non-vanishing order.

b) Solve the problem exactly and compare with your result obtained in (a).