13 June 2012

Lecture 1 - Lecture 3

Introduction to Quantum Mechanics:

Basic Principles, Probabilities, Probability Amplitudes

· Quantum Behavior

- describes happenings on small scales (ie. atomic scales down). At small scales, the behavior of objects changes completely. For example, if I have a box of billiard balls that are free to collide with each other its motion is completely governed by laws of Classical Physics. If the balls were replaced with atoms, e^- or any small, it behaves in ways that cannot be explained by classical Physics. e^- for example behaves like a particle in some cases and a wave in other cases. Same is true of light (photons). All of this can be explained by Quantum Mechanics.

- Born from the works of Schrödinger, Heisenberg and Born.
- We will look at the main highlights through some examples in the first couple of lectures.
- Human intuition applies to large scale objects and small scale objects can be understood only thru' abstraction and imagination and not by connection with direct experience.

Example: Behavior of e^-:

Start by comparing the behavior

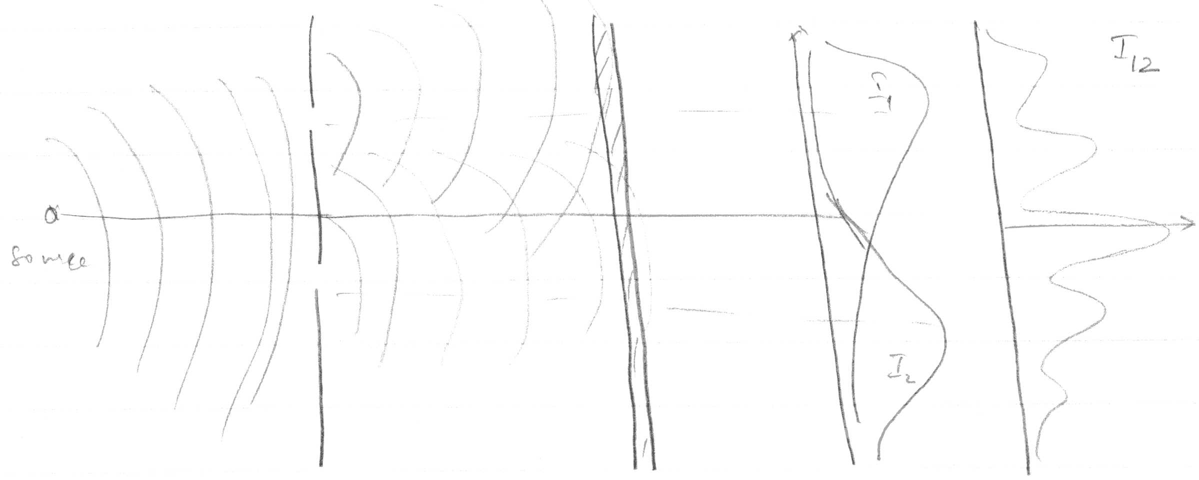
a) BULLETS:

Assume: • Bullets do not break
• Completely absorbed when they arrive at the detector
• Arrive one by one.
• Bullets are identical.

Measure: Prob. of arriving at a distance x from the center
(P_{12}) → could have arrived thru' 1 or 2.

• Cover 2, then you get P_1
• Cover 1: you get P_2.
• $P_{12} = P_1 + P_2$

2) WATER WAVES: Repeat with water



Absorber (sand beach)

Assume: • No reflection of waves
• Measure the intensity of the waves. ($|h|^2$ of wave)
(Measures the rate at which energy is carried to the detector).

$I_{12} \neq I_1 + I_2$

$I_1 = |h_1|^2 = |A_1|^2 \quad A_1 = h_1 e^{i\omega_1 t}$

$I_2 = |h_2|^2 = |A_2|^2, \quad A_2 = h_2 e^{i\omega_2 t}$

$I_{12} = |A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2A_1 \cdot A_2^*$

$$= h_1^2 + h_2^2 + \cancel{2}|h_1| e^{i\alpha_1 t} h_2 e^{-i\alpha_2 t} + h_1 h_2 e^{-i(\alpha_1 - \alpha_2)t}$$

$$= h_1^2 + h_2^2 + \cancel{2} h_1 h_2 \left[e^{i(\alpha_1 - \alpha_2)t} + e^{-i(\alpha_1 - \alpha_2)t} \right]$$

$$e^{i(\alpha_1 - \alpha_2)t} = \cos(\alpha_1 - \alpha_2)t + i \sin(\alpha_1 - \alpha_2)t$$
$$e^{-i(\alpha_1 - \alpha_2)t} = \cos(\alpha_1 - \alpha_2)t - i \sin(\alpha_1 - \alpha_2)t$$

$\therefore$

$$I_{12} = h_1^2 + h_2^2 + \underbrace{2 h_1 h_2 \cos \underbrace{(\alpha_1 - \alpha_2)}_{\delta} t}_{\text{Interference term.}}$$

3) <u>Electrons</u>: This expt. can only be imagined!

- The apparatus should have very small scale for quantum phenomena

Source: $\bar{e}$ gun : tungsten wire surrounded by a metal box.

if the wire is at a -ve voltage w.r.t. the box, $\bar{e}$ will accelerate thru the walls and will be emitted thru' the hole.

Detector: Counter or $\bar{e}$ multiplier connected to an amplifier results in "clicks" when $\bar{e}$ arrives.

- obtain the average rate at which the $\bar{e}$ are arriving at the detector.

$\bar{e}$ arrive in lumps - like the bullets.

- Same size identical lumps.

- Result: Identical to water waves.
- What causes the interference?

Consider 2 quantities $\phi_1(x)$ $\phi_2(x)$.

$P_1 = |\phi_1(x)|^2$

$P_2 = |\phi_2(x)|^2$

$P_{12} = |\phi_1(x) + \phi_2(x)|^2$

$\bar{e}$: <u>Arrive as particles, but when detected behave like waves</u>

Watch the e^-:

Put a light source behind the screen with slit. e^- will scatter light, we will see a flash.

– Result: flash from either 1 or 2 never both.

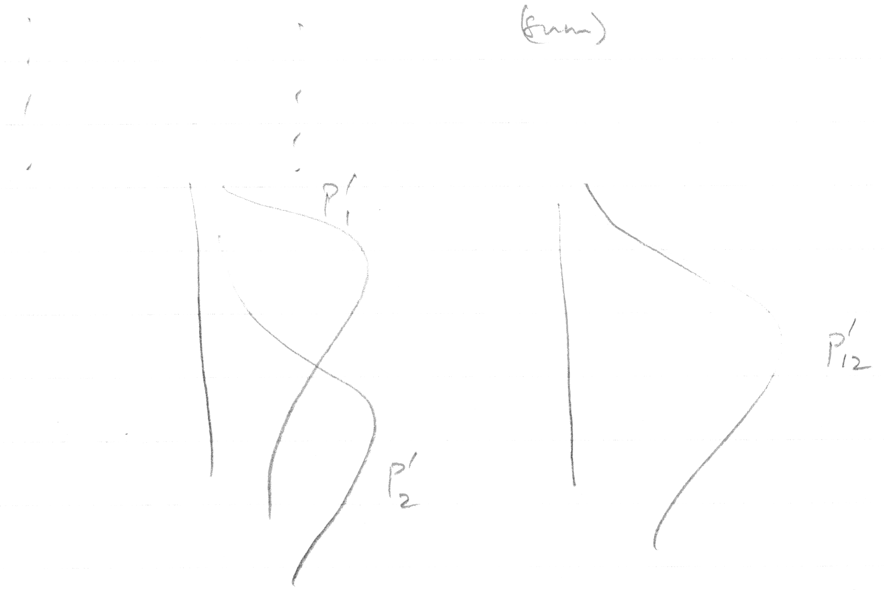
Then why the interference?

$P_{12} \neq P_1 + P_2$ flash near 1 ✓ C_1

P_{12} flash near 2 ✓ C_2

Col 1 (hole 1) Col 1 (hole 2)

(sum)



irrespective of hole 2 being opened or closed $P_1 = P_1'$

$P_2 = P_2'$

Switch off light, $P_{12}' \to P_{12}$.

- What if we decrease the intensity of light? May be the interference due to scattering of e^- from a light source would be reduced – But doing this, we see same intensity of flash, but at times only a click and no flash. What happens? light emits photons (particle nature) and these interact with e^-. Decreasing intensity $\Rightarrow$ decrease in the no. of photons emitted per sec (rate). If a click is heard and a photon was available, we see a flash, else only a click and no flash. Record the results in the following way:

flash @ 1   flash @ 2   only click (P_{12}) P'_{12} = flash @ 1 + flash @ 2

$P_{12} \rightarrow e^-$ are not observed (Interference pattern)
$P'_{12} \rightarrow e^-$ are observed (No pattern).

OBSERVING THEM DISTURBS THEM!!

How about we do this: photons have a wavelength $\lambda = \frac{h}{p}$
$p = \frac{h}{\lambda}$. Change λ (ie. frequency of light) and

decrease p, then p_{e^-} might not change so much.
Make λ larger. keep repeating until $P_{12} = P'_{12}$.
$\rightarrow$ Unless $\lambda >$ dist between slits $P_{12} \neq P'_{12}$
when $\lambda >$ dist bet slits, then the 2 flashes cannot be resolved (limit of resolution). Therefore cannot distinguish whether the e^- came from hole 1 or hole 2!!

$\Rightarrow$ THERE IS A LIMIT ON ~~HOW~~ HOW PRECISELY ONE CAN DETERMINE STUFF $\longrightarrow$ Heisenberg Uncertainty Relation

- If you can precisely define the path an e^- takes, then that process results in disturbing the e^- and the interference pattern is lost.
- If quantum effects are to be observed, then we lose precision.

If all matter can be described by waves, then what about bullets? λ is very small, that interference pattern is extremely fine and we cannot resolve it!
(Path from Q. Mech $\rightarrow$ classical Physics).

Summarize Basic Quantum Rules:

(i) Any event happens with a prob. amplitude ϕ (complex no.)
$P = |\phi|^2$

(ii) When an event can occur in several alternate ways, then the prob. amplitudes add.
$\phi_{tot} = \phi_1 + \phi_2 + \cdots$

$P_{tot} = |\phi_1 + \phi_2 + \cdots|^2 \rightarrow$ interference term.

(iii) If we can narrow down the exact way for an event to occur, then interference is lost.
$P = P_1 + P_2$

Heisenberg Uncertainty Relation:

$\Delta x \, \Delta p_x \geq \hbar$ $\quad$ h: planck's Const: 6.63×10^{-34} Js

→ General statement: One cannot set up an experiment in a way to determine two alternatives accurately, without losing quantum effects.

In our interference expt. replace the plates by rollers, so that we can calculate the momentum after an e^- has passed thru' hole 1 or hole 2 (recoil momentum), and determine
$\Delta p = p_i - p_f$. $\quad$ $\Delta p > 0$ upwards $\rightarrow$ hole 1
$\Delta p < 0$ downwards $\rightarrow$ hole 2.

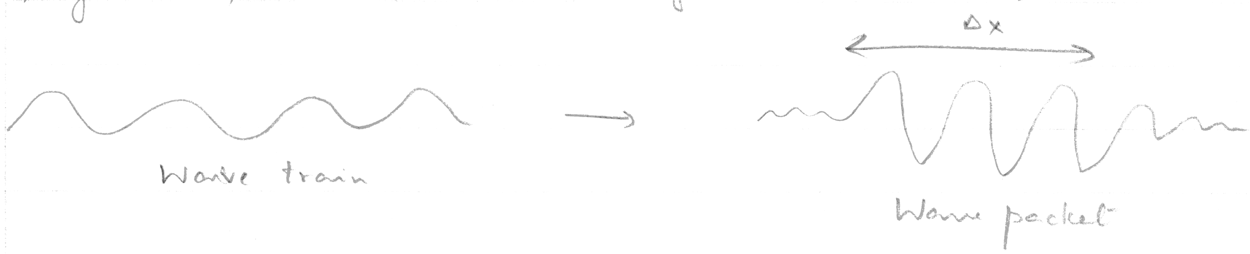
According to Heisenberg uncertainty if we know p_i accurately we will not know x of screen and hence cannot predict if $\bar{e}$ has passed thru' 1 or 2. Heinsenberg uncertainty leads to interesting quantum effects.

Probability Amplitudes

Any event in quantum mechanics is given a probability amplitude, which is necessarily a complex number. If there are more than one way of obtaining the same event, then the amplitudes add and the probability is the square of the total amplitude.
The amplitude in general is a function of position and time. $\Psi(\vec{r}, t)$
In some special cases: $\Psi(\vec{r}, t) = e^{i(\omega t - \vec{k}\cdot\vec{r})}$ ω: frequency, k: wave number, $\Rightarrow$ sinusoidal propagation. This then can be interpreted as a particle with definite energy $E = \omega\hbar$ and momentum $\vec{p} = \hbar\vec{k}$ $\quad (\hbar = \frac{h}{2\pi})$

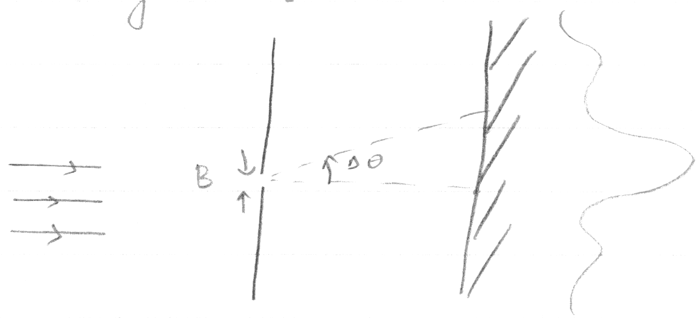
Now the prob. of finding a particle at $\vec{r}$ = const. $\Rightarrow$ We do not know exactly where the particle is!

On the other hand if we know where the particle is within a length Δx, then outside this region $P(x) = 0$. $\Rightarrow$



Difficulty: Cannot define a unique wavelength to a wave packet (short wave train) $\Rightarrow$ there is an indefiniteness in momentum.

Let us think of another experiment to demonstrate uncertainty principle. Consider a beam of e^- passing through a slit B.



We know their momenta ($\vec{p}$) and this is along the horizontal direction. When it passes thru' the slit $\Delta x \sim B$, it acquires a vertical component (due to diffraction). Before passing thru' the slit, the e^- had precise momenta, but one could not pin down their position, after passing thru' the slit, one knows their position but not their momenta

$$\Rightarrow \Delta p_y \Delta y \approx \hbar \quad \text{or} \quad \Delta p_y \Delta y \geq \hbar$$

Let us formalize all that we have learned with some mathematics.

FIRST PRINCIPLE OF Q.MECH:

→ Any event is described by a probability amplitude and the prob for the event ocurring is the square of the amplitude. The prob amplitude is a complex number.

→ A short hand for such an amplitude (Dirac Notation).

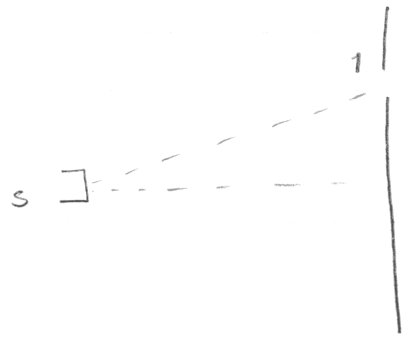
$$\langle \text{final state} \mid \text{initial state} \rangle = \text{Complex number.}$$

Bracket (Dirac's):

Right of the vertical line - initial state.

Left of the vertical line - final state.

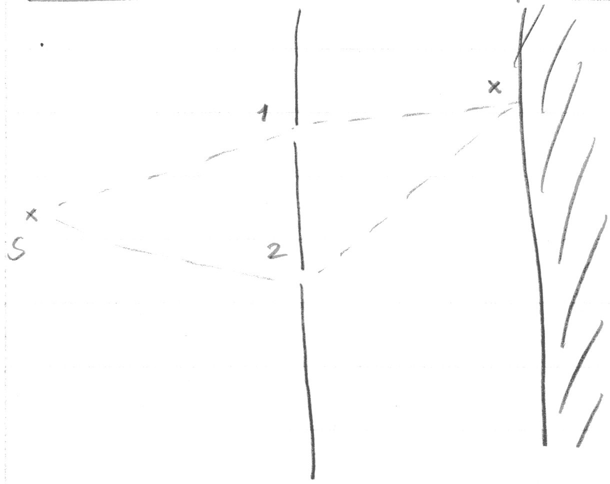
Example:



Prob ampl. that e^- arrives at the slit 1: $\langle 1 | s \rangle$

final ↙ ↘ initial

Look at double slit experiment:



prob. of e^- at x:

$$\langle x | s \rangle = \underbrace{\langle x | s \rangle_1 + \langle x | s \rangle_2}$$

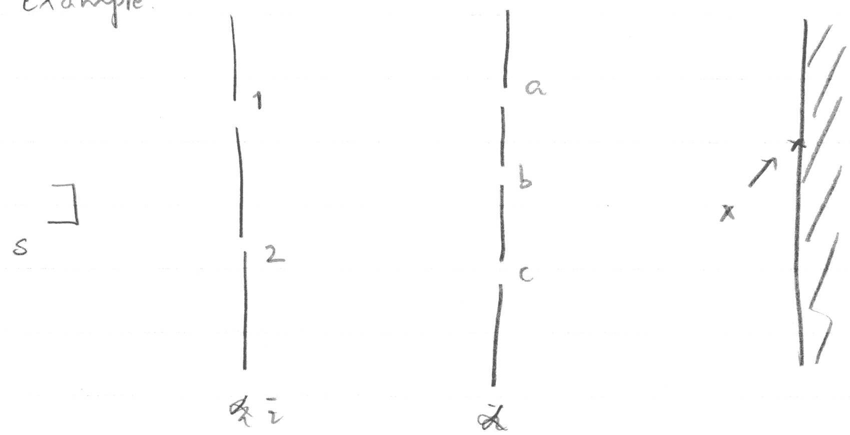
Sum prob. ampl. for all possible ways an event can occur: SECOND PRINCIPLE.

Now:
$$\left.\begin{aligned} \langle x | s \rangle_1 &= \langle x | 1 \rangle \langle 1 | s \rangle \\ \langle x | s \rangle_2 &= \langle x | 2 \rangle \langle 2 | s \rangle \end{aligned}\right\}$$

THIRD PRINCIPLE:
Prob. ampl. of a route = pdt of small pieces of the route

→ Analogy: Picking up black ball at first try from 10 bins

Example:



$$\langle x | s \rangle = \langle x | a \rangle \langle a | 1 \rangle \langle 1 | s \rangle + \langle x | b \rangle \langle b | 1 \rangle \langle 1 | s \rangle + \cdots$$
$$= \sum_{i,\alpha}{}' \langle x | \alpha \rangle \langle \alpha | i \rangle \langle i | s \rangle \qquad i = 1,2 \quad \alpha = a, b, c$$

Free particle: prob. for a particle at $\vec{r}_1$ to end up at $\vec{r}_2$:

$$\langle \vec{r}_2 | \vec{r}_1 \rangle = \frac{e^{i\vec{p}\cdot\vec{r}_{12}/\hbar}}{r_{12}}$$

$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$ $\quad\quad \vec{p}$: momentum

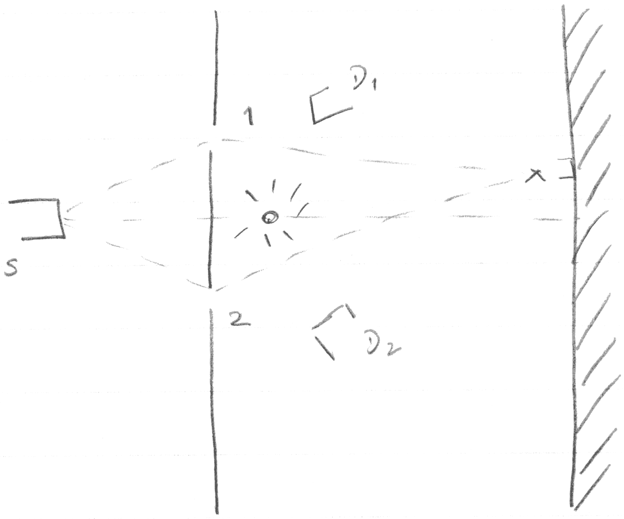
→ Wave like prop.

→ Satisfies S. eqn.: Wave eqn.

→ Can be interpreted as "particle waves" but there is a catch. Two waves at $\vec{r}_1$ & $\vec{r}_2$: Classical: Linear superposition.

Two ~~wa~~ particle waves: prob amp. multiply!!

Two-slit interference with observation:



$$\langle x|s\rangle_1 = \langle x|1\rangle a \langle 1|s\rangle + \langle x|2\rangle\langle 2|s\rangle b$$

$$\langle x|s\rangle_2 = \langle x|2\rangle a \langle 2|s\rangle + \langle x|1\rangle\langle 1|s\rangle b$$

$$\phi_1 = \langle x|1\rangle\langle 1|s\rangle$$
$$\phi_2 = \langle x|2\rangle\langle 2|s\rangle$$

$\therefore \langle x|s\rangle_1 = a\phi_1 + b\phi_2$

$\langle x|s\rangle_2 = a\phi_2 + b\phi_1$

Choose source such that $b \simeq 0$

$\langle x|s\rangle_1$, $\langle x|s\rangle_2$ independent events

$\wr$

$s \to x$ via 1 or 2 and detected by D_1

$\langle x|s\rangle_2$: $s \to x$ via 1 or 2 and detected by D_2

$\therefore P' = a^2\phi_1^2 + a^2\phi_2^2$

$= a^2 [P_1' + P_2']$