A tale of two superpotentials

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Abstract. We compute the superpotential on the worldvolume theory of D-branes in the topological Landau-Ginzburg model associated with the cubic torus. An extended version of mirror symmetry relates this superpotential to the one on the mirror D-brane. We discuss the equivalence of these two superpotentials by explicitly constructing the open-string mirror map.

Keywords: Topological string theory, superpotentials, mirror symmetry
PACS: 11.25.Uv,11.30.Pb,11.25.Wx

The worldvolume theories of D-branes in the $\ell_s \to 0$ limit is typically an effective (quantum) field theory. The effective action for the field theory is obtained by computing open-string correlators involving vertex operators for the various modes that survive the $\ell_s \to 0$ limit. Closed string moduli appear as deformations of these open-string correlators[1]. A well-known example is that of the constant B-field – this leads to the non-commutative deformation of Yang-Mills theory[2].

Thus the effective action may be first computing open-string correlators without closed-string vertex operators inserted. The closed-string deformations are then realised by computing correlators with arbitrary closed-string insertions. In some examples, it may happen that all correlators with closed-string insertions may vanish. Closed-string moduli can nevertheless appear as non-perturbative deformations[3]. (c.f. M. Cvetic's talk at this conference)

In this talk, we report work done in collaboration with Hans Jockers[4].

The storyline

In type II compactifications of string theory, mirror symmetry relates compactifications involving two different manifolds. These two manifolds are called a mirror pair[5]. Adding D-branes to the picture, an extension of mirror symmetry leads to an even richer story[1, 6]. Focusing on the effective field theory associated with D-branes, mirror symmetry implies a map between two seemingly different effective theories. This talk is about perturbatively computing a superpotential in the topological B-model and comparing it with the non-perturbative superpotential on the corresponding brane in the mirror A-model.

Motivation

A nice intermediate step to obtaining the standard model from string theory is to look for $\mathscr{N}=1$ compactifications of string theory in four dimensions. The effective field theory at low energies is specified by the following functions

- The Kähler potential, $K(\phi,\bar{\phi})$,
- The superpotential $\mathscr{W}(\phi)$, and
- The (complexified) gauge coupling constants, $f_a(\phi)$.

The last two objects are holomorphic (in the chiral superfields) and can be computed in topological string theory. We will focus on computing $\mathscr{W}(\phi)$ in this talk.

Superpotentials can arise by turning on fluxes through compact cycles – usually these are computed using the Gukov-Vafa-Witten formula[7]. They can also arise from the worldvolume theory of branes that may be added to cancel tadpoles – say in orientifold theories. This is sometimes called the brane superpotential. The brane superpotential $\mathscr{W}$ has been computed for non-compact examples. Is there a systematic method to compute it in compact examples? Finally, the superpotential also appears as an obstruction to matrix factorizations[8, 9, 10].

For type II compactifications with $\mathscr{N}=2$ supersymmetry, mirror symmetry has proved useful in summing up non-perturbative contributions coming from worldsheet instantons[5]. One important ingredient in mirror symmetry is the closed-string mirror map. This is a highly non-trivial change of variables. An important ingredient in this computation is the observation of Candelas et. al. that the change of variables is given by a solution of a Picard-Fuchs differential equation. Is there an analogue for open-strings? Yes, for some non-compact examples[11, 12, 13]. Is there a differential equation for compact examples as well? (see [14] for a special example.)

LANDAU-GINZBURG MODELS

The computation discussed in this talk is carried out in the topological Landau-Ginzburg (LG) model. Landau-Ginzburg models are useful as they flow to non-trivial CFT's in the infrared limit. In this description one has a better handle on complex structure moduli appearing in the compactification. The main motivation in using this method is that the computation for a Calabi-Yau threefold is not different from that of a minimal model.

Two-dimensional LG models with $(2,2)$ supersymmetry are constructed from chiral and anti-chiral superfields. Chiral superfields have the following expansion ($\alpha=\pm$):

$$\Phi^i=\phi^i+\sqrt{2}\theta^\alpha\psi^i_\alpha+\theta^\alpha\theta_\alpha F^i\ . \tag{1}$$

The most general renormalizable action for such a theory has an action (see for instance [15])

$$\begin{aligned} S &= S_K+S_W \\ &= \int d^2x\left(\int d^4\theta K(\Phi,\bar{\Phi})-\lambda\int d^2\theta\; W(\Phi)-\bar{\lambda}\int d^2\bar{\theta}\;\bar{W}(\bar{\Phi})\right) \end{aligned} \tag{2}$$

where K is the worldsheet Kähler potential and W is the worldsheet superpotential (not to be confused with similar objects in spacetime!). We will find it useful to define the following combinations of fermions: $\tau = (\psi_+ - \psi_-)/\sqrt{2}$ and $\xi = (\psi_+ + \psi_-)/\sqrt{2}$.

We will assume that the worldsheet superpotential W is quasi-homogeneous, i.e.,

$$W(\lambda^{\alpha_i/2}\Phi^i) = \lambda\, W(\Phi^i)\ . \tag{3}$$

There is a lot of evidence that such LG models flow in the IR to CFT's with central charge $\hat{c} = \sum_i (1-\alpha_i)$. In models with several fields, we will be interested in LG orbifolds with projections onto states with (half-)integral R-charge.

Our working example: It involves three chiral superfields and a cubic superpotential $W = c_{ijk}\phi^i\phi^j\phi^k$ and a $\mathbb{Z}_3$ orbifolding. The CFT has $\hat{c} = 1$ and is the 1^3 Gepner model. Geometrically, this LG model is associated with the cubic torus.

Topological LG models

These are topologically twisted versions of the $(2,2)$ models[16, 15]. We will consider the topological B-twist. This has two BRST charges which we will denote by $\mathscr{Q}_\pm$. In LG models with boundary, we will assume that one linear combination, $\mathscr{Q}$, is preserved by the boundary conditions. Observables in the topological model are then given by the cohomology of $\mathscr{Q}$.

In the action, only the holomorphic part of S_W is non-trivial. S_K for instance, is $\mathscr{Q}$-exact. Thus, the topological partition function is independent of the Kähler potential and depends holomorphically on the parameters(moduli) in W.

The topological partition function is formally defined by the following path-integral:

$$Z_{top} \equiv \int_{disk} [d\Phi] e^{-S_K - S_W}\ P\left(e^{-S_\partial}\right) \tag{4}$$

with S_∂ representing the boundary perturbations and P indicating path-ordering of the boundary perturbation. We will treat S_W and S_∂ *perturbatively*. Let $\langle\langle \cdots \rangle\rangle$ denote correlation functions in the *free-theory* i.e., with $W = 0$. Then,

$$Z_{top} = \sum_{m,n=0}^{\infty} \left\langle\left\langle \frac{1}{n!}\,(S_W)^n\, \frac{1}{m!} P\,(S_\partial)^m \right\rangle\right\rangle\ , \tag{5}$$

is formally equivalent to the the path-integral.

$\mathscr{W} = Z_{top}$ in the topological LG model

It is known that the open-superstring partition function on a disk gives the open-string field theory action[17, 18, 19, 20]. For instance, this has been used by Kutasov, Marino and Moore to compute the *exact* action for the tachyons in order to verify Sen's conjectures on tachyon condensation. (see also [21]) For $\mathscr{N} = 1$ supersymmetric compactifications, Z_{top} can be identified with the the brane superpotential, $\mathscr{W}$. For non-geometric examples such as those that appear in matrix factorizations, Z_{top} can be identified with the obstruction superpotential – these encode higher order obstructions to marginality or obstructions to the existence of matrix factorizations[10].

Our goal is to compute $\mathscr{W} \equiv Z_{top}$ in the topological LG model as a function of both closed string moduli and open-string deformations. Such a computation was first done for the quintic(!), where the first correction from closed-string moduli was computed[22]. As we will see, the surprise is that the computation of $\mathscr{W}$ can be carried out to all orders in our example[4].

The cubic torus

The LG description involves three chiral superfields Φ^i with a superpotential

$$W = c_{ijk}\phi^i\phi^j\phi^k = g_0\left[(\phi^1)^3+(\phi^2)^3+(\phi^3)^3\right]+g_1\,\phi^1\phi^2\phi^3 \tag{6}$$

There is an orbifold action:

$$\phi^i \to \omega\phi^i \quad \text{where} \quad \omega = e^{2\pi i/3}\ . \tag{7}$$

So one is dealing with an orbifold of a LG model. The IR fixed point of the LG model is the 1^3 Gepner model. Without the superpotential, one has a $\mathbb{C}^3/\mathbb{Z}_3$ orbifold.

Geometrically, the torus $\mathscr{T}$ is given by the hypersurface $W=0$ in $\mathbb{P}^2$ with complex structure modulus (and flat coordinate) τ implicitly given by $\frac{g_1}{g_0} = -3a(\tau)$. The relationship between a and τ is given through the j-function

$$j(\tau) = \left(\tfrac{3a(a^3+8)}{a^3-1}\right)^3\ .$$

While the precise value of g_0 does not have any special meaning, in obtaining the differential equation for the periods, one chooses g_0 to be[23]

$$(g_0)^{-1} = \sqrt{\tfrac{1-a^3(\tau)}{3a'(\tau)}} = \tfrac{1}{3\sqrt{2\pi i}}\,\tfrac{\eta(\tau)}{\eta^3(3\tau)}\ . \tag{8}$$

We will find that this choice is natural in the context of the open-string mirror map!

The mirror torus $\hat{\mathscr{T}}$ is one with complex structure $\hat{\tau} = e^{2\pi i/3}$ and (complexified) Kähler modulus $\hat{\rho} = \tau$. We shall focus on the situation where one imposes Dirichlet boundary conditions on all fields: $\phi^i = \tau^i = 0$. In the Gepner model, this boundary condition gets mapped to the $L_i = 0$ Recknagel-Schomerus states[24, 25, 26]. Figure 1 describes the equivalences between different descriptions.

Boundary deformations

On the boundary now there are several $\mathscr{Q}$-closed operators. The obvious one is $\bar{\xi}_i$ (R-charge $1/3$). There are two more – $\bar{\xi}_i\bar{\xi}_j$ and $\bar{\xi}_1\bar{\xi}_2\bar{\xi}_3$ with R-charges $2/3$ and 1,

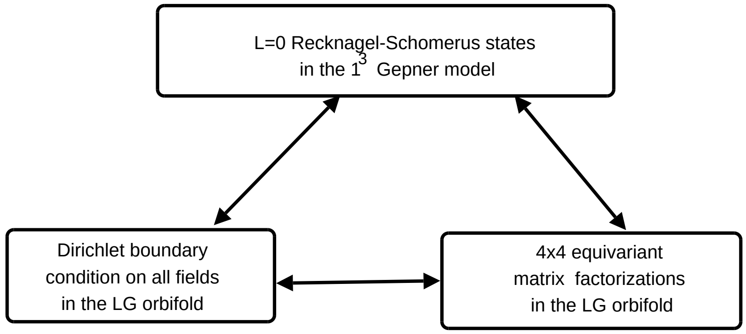

FIGURE 1. The relationship between the different constructions of the 'long' branes on the torus, $\mathscr{T}$, at the Gepner point in the Kähler moduli space.

respectively. We will focus on two boundary perturbations(corresponding to bosonic fields):

$$\begin{aligned} \Psi^{(0)} = X^i\ \bar{\xi}_i \quad &, \quad \Psi^{(1)} = X^i\ \partial_y \bar{\phi}_i\ , \\ \Omega^{(0)} = U\varepsilon^{ijk}\bar{\xi}_i\bar{\xi}_j\bar{\xi}_k \quad &, \quad \Omega^{(1)} = 3U\varepsilon^{ijk}\bar{\xi}_i\bar{\xi}_j\partial_y\bar{\phi}_k\ , \end{aligned} \tag{9}$$

where the superscript indicates the form number of the operator. Using the R-charge assignments, we see that the X-perturbation is a *relevant* one while the Ω-perturbation is a *marginal* one.

However, since the *free* theory (i.e., when the worldsheet superpotential is set to zero) is an orbifold, there are three boundary states corresponding to fractional zero-branes. This is incorporated by using Chan-Paton factors which take into account the spectrum of open-strings connecting the various fractional zero-branes.

$$X^i = \begin{pmatrix} 0 & x^i_{12} & 0 \\ 0 & 0 & x^i_{23} \\ x^i_{31} & 0 & 0 \end{pmatrix} \quad , \quad U = \begin{pmatrix} u_1 & 0 & 0 \\ 0 & u_2 & 0 \\ 0 & 0 & u_3 \end{pmatrix} . \tag{10}$$

Thus the X^i are boundary condition changing operators while U is a boundary condition preserving operator.

The topological partition function

There are a few issues to fix before actually carrying out the computation of the partition function. The worldsheet is taken to be the upper-half plane with the coordinates (x,y) and $x > 0$.

- We fix $SL(2,\mathbb{R})$ invariance by choosing one bulk operator as a zero-form located at the point (x_0,y_0) and one boundary operator as a zero-form located at $x = +\infty$. All other operators are chosen to be integrated ones. In the absence of a bulk insertion, we choose three boundary operators as zero-forms.

- R**-charge selection rule** The only non-vanishing correlators $\langle\langle \cdots \rangle\rangle$ occur when the sum of the R-charges of all operators equals 1.
- **Fermion zero-modes:** There is one fermion zero-mode coming from every $\bar{\xi}^i$ which we indicate by $\bar{\xi}^{\mathbf{i}}$. The bulk topological theory has more zero-modes which are removed by the boundary conditions.
- All the fields must be contracted with some other field.

It is useful to separate the partition function by the number of bulk insertions:

$$\mathscr{W} = \sum_{n=0}^{\infty} \mathscr{W}_n \ .$$

R-charge considerations imply that $\mathscr{W}_n$ (for $n \neq 0$) equals

$$\left\langle\!\left\langle \frac{1}{n!} V_W^{(0)} \left(\int V_W^{(2)} \right)^{n-1} \frac{P}{n!3!} \left[\left(\int \Omega^{(1)} \right)^n \left(\int \Psi^{(1)} \right)^3 \right] \Omega^{(0)}(\infty) \right\rangle\!\right\rangle \ .$$

When there are no bulk insertions, one has

$$\mathscr{W}_0 = \left\langle\!\left\langle \Psi^{(0)}(0) \Psi^{(0)}(1) \Psi^{(0)}(\infty) \right\rangle\!\right\rangle = \mathrm{Tr}(X^i X^j X^k) \left\langle\!\left\langle \bar{\xi}_{\mathbf{i}} \bar{\xi}_{\mathbf{j}} \bar{\xi}_{\mathbf{k}} \right\rangle\!\right\rangle = \varepsilon_{ijk} \mathrm{Tr}(X^i X^j X^k) \ .$$

This is known to be the $\mathbb{C}^3/\mathbb{Z}_3$ superpotential. We now move on to the situations with one or more insertions of the bulk operators, V_W. The $\mathscr{W}_n$ can be written as

$$\mathscr{W}_n = \mathscr{I}_n \, \mathscr{C}_n \, g_0^n (u_1 + u_2 + u_3)^n \ ,$$

where

- $\mathscr{I}_n$ includes the contribution from the integrals,
- $\mathscr{C}_n$ contains the contractions of the n copies of the totally symmetric tensor of $SU(3)$, c_{ijk}, with the boundary X's and the antisymmetric tensor ε^{ijk} from the Ω's.
- One can show that only the combination $g_0 u \equiv g_0(u_1 + u_2 + u_3)$ appears.

The integrals that appear simplify in the limit when we take the bulk zero-form operator close to the boundary. For $\mathscr{W}_1$, we obtain

$$\begin{aligned} \mathscr{C}_1 &= 3 c_{ijk} \mathrm{Tr}\left(X^i X^j X^k \right) \ , \\ \mathscr{W}_1 &= 3 \mathscr{I}_0 \left(3\kappa_{111} - \frac{3}{2} a (\kappa_{123} + \kappa_{132}) \right) g_0 u \end{aligned} \qquad (11)$$

where we define the following useful combinations:

$$\kappa_{111} = \frac{1}{3} \sum_i \mathrm{Tr}\left(X^i X^i X^i \right) \ , \quad \kappa_{123} = \mathrm{Tr}\left(X^1 X^2 X^3 \right) \ , \quad \kappa_{132} = \mathrm{Tr}\left(X^1 X^3 X^2 \right) \ .$$

The next term, i.e., $\mathscr{W}_2$ vanishes since $\mathscr{C}_2$ vanishes.

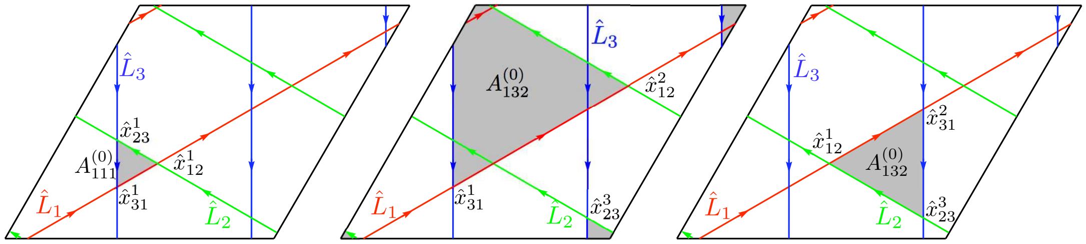

FIGURE 2. For the $D1$-branes, $\hat{L}_1$, $\hat{L}_2$ and $\hat{L}_3$, the non-trivial correlators $\langle \hat{x}^1_{31}\hat{x}^1_{12}\hat{x}^1_{23}\rangle$, $\langle \hat{x}^1_{31}\hat{x}^2_{12}\hat{x}^3_{23}\rangle$ and $\langle \hat{x}^1_{12}\hat{x}^3_{23}\hat{x}^2_{31}\rangle$ arise from disk instantons. The figures show the disk instantons ($k=0$) for these three correlation functions respectively.

Simple considerations show that the combination $(\kappa_{123}-\kappa_{132})$ appears only in $\mathscr{W}_{2n}$ while $(\kappa_{123}+\kappa_{132})$ and κ_{111} appear in $\mathscr{W}_{2n+1}$ alone. This implies that the the full superpotential defined below

$$\mathscr{W}_B = \Delta^B_{111}(\tau, g_0 u)\kappa_{111}(X) + \Delta^B_{123}(\tau, g_0 u)\kappa_{123}(X) + \Delta^B_{132}(\tau, g_0 u)\kappa_{132}(X) \tag{12}$$

is such that

$$\Delta^B_{111}(\tau, -g_0 u) = -\Delta^B_{111}(\tau, -g_0 u) \quad \text{and} \quad \Delta^B_{123}(\tau, -g_0 u) = -\Delta^B_{132}(\tau, -g_0 u)\ . \tag{13}$$

We will see that these properties are compatible with the computation done on the mirror torus. The Δ^B can be viewed as an open-string three-point function deformed by the bulk and boundary modulus, τ and u.

The topological A-model

Under mirror symmetry, the topological B-model gets mapped to the topological A-model on the mirror. Superpotentials are classical objects in the B-model while they are quantum objects in the A-model. All contributions arise from worldsheet instantons[3]. The simplicity of our example enables us to easily write out the disk instanton contributions and we use it as a check of our computation.

Under the mirror transform, B-branes get mapped to A-branes. The three branes that we considered thus get mapped to branes that are special Lagrangian one-cycles (labelled $\hat{L}_1$, $\hat{L}_2$ and $\hat{L}_3$) on the mirror torus $\hat{\mathscr{T}}$. The boundary changing operators (x^i_{ab}) are operators located at the intersection points (see Fig. 2) – there are nine of them. The boundary moduli, u_i are the positions of the three branes.

The three-point functions in the topological A-model vanish perturbatively and get contributions only from worldsheet instantons – these are disk instantons. Schematically, one finds

$$\Delta_{ijk} \sim \sum_l e^{2\pi A^{(l)}_{ijk}(\hat{\beta})} e^{2\pi i W^{(l)}_{ijk}(\hat{\alpha})}$$

where $\hat{u} = \sum_i \hat{u}_i = \hat{\alpha} + \hat{\rho}\beta$ is the position modulus, $A^{(l)}_{ijk}$ is the area of the disk instanton (see Fig. 2) and $W^{(l)}_{ijk}$ is the Wilson line contribution. This result has been computed in references [27, 28]. We quote the result as adapted by Brunner et. al. in [29].

$$
\begin{aligned}
\Delta_{111} &= \sum_{m\in\mathbb{Z}} q^{\frac{3}{2}\left(m-\frac{1}{2}\right)^2} e^{2\pi i\left(m-\frac{1}{2}\right)\left(u_A-\frac{1}{2}\right)} \ , \\
\Delta_{123} &= e^{\frac{2}{3}i\pi} \sum_{m\in\mathbb{Z}} q^{\frac{3}{2}\left(-\frac{1}{3}+m-\frac{1}{2}\right)^2} e^{2\pi i\left(-\frac{1}{3}+m-\frac{1}{2}\right)\left(u_A-\frac{1}{2}\right)} \ , \\
\Delta_{132} &= e^{-\frac{2}{3}i\pi} \sum_{m\in\mathbb{Z}} q^{\frac{3}{2}\left(-\frac{2}{3}+m-\frac{1}{2}\right)^2} e^{2\pi i\left(-\frac{2}{3}+m-\frac{1}{2}\right)\left(u_A-\frac{1}{2}\right)} \ .
\end{aligned}
$$

These are θ-functions of characteristic three.

Finding the open-string mirror map

We need to figure out the change of variables that is needed to match our computation to the A-model result. We make the following ansatz

$$
\begin{aligned}
u_A &= \mathscr{N}_u(\tau)\, u_B + u_0(\tau) \\
X_A &= \mathscr{N}_X(\tau, u_B)\, X_B
\end{aligned}
\tag{14}
$$

The normalizations depend on the closed-string modulus τ. The additivity of the u's implies that the change of variable from u_A to u_B must be linear. R-charge considerations imply that X_A must be proportional to X_B – however, its normalization can depend on u_B.

We match the two results by requiring

$$
\mathscr{N}_X^3\ \mathscr{W}_A(\tau, \mathscr{N}_u u_B + u_0, X_B) = \mathscr{W}_B(\tau, g_0 u_B)\ . \tag{15}
$$

If g_0 is taken to equal the value as given in Eqn. (8), then $\mathscr{N}_u$ becomes a τ independent constant. This implies that u_B transforms like a point on the torus.

$$
\mathscr{N}_X = \frac{3i\mathscr{I}_0}{\eta(\tau)} \exp\left(2G_2(\tau)\mathscr{N}_u^2 u^2/3\right)\ f(\tau, u^2)\ .
$$

where $G_{2k} = \sum'_{m,n\in\mathbb{Z}}(m\tau+n)^{-2k}$ is the Eisenstein series of weight $2k$ and $f(\tau,u^2) = 1+\mathscr{O}(u^4)$ is a modular invariant function that is determined order by order.

The open-string mirror map is highly overdetermined and hence its very existence is a non-trivial check of the perturbative treatment. From a practical viewpoint, we have checked terms to several orders. We have also made use of the modular properties to obtain additional checks. The Eisenstein series $G_2(\tau)$ is actually not a modular form. However, $\hat{G}_2 = G_2 - \pi/\mathrm{Im}(\tau)$, has nice modular transformation properties but is not holomorphic. Is this related to the *holomorphic anomaly*? Using matrix factorizations, Brunner et. al. have also computed the three-point function which matches our results[29].

CONCLUSIONS

These methods are applicable to more realistic examples such as the intersecting brane models for supersymmetric extensions of standard model. The tricky bit, as always, is to find the open-string mirror map. The models of Dijkstra et. al. provide us with a rich set of examples where one can hope to make progress[30]. A much harder problem is to push these results to the non-topological theory and compute the Kähler potential.

It turns out that the superpotential that we computed is identical to the general Leigh-Strassler deformations of $\mathscr{N} = 4$ SYM theory. Can we reinterpret our computation in this context? Does it help in obtaining the gravity dual for the general Leigh-Strassler deformation?

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